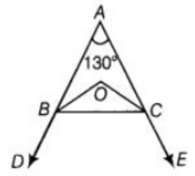
Solution

QUESTION BANK

Class 09 - Mathematics

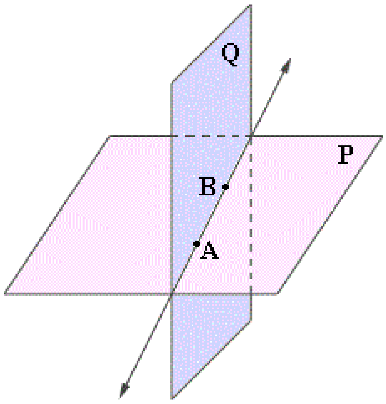
1. **(b)** 155°
   Explanation:
   Let angles of a triangle be ∠A, ∠B and ∠C.
   

   In △ABC,
   $\angle A+\angle B+\angle C=180^{\circ}$ [sum of all interior angles of a triangle is 180^{o}]
   $\Rightarrow \frac{1}{2}\angle A+\frac{1}{2}\angle B+\frac{1}{2}\angle C=\frac{180^{\circ}}{2}=90^{\circ}$ [dividing both sides by 2]
   $\Rightarrow \frac{1}{2}\angle B+\frac{1}{2}\angle C=90^{\circ}-\frac{1}{2}\angle A$ [∵ In △OBC, ∠OBC +∠BCO + ∠COB = 180^{o}]
   ⇒ Since, $\frac{\angle B}{2}+\frac{\angle C}{2}+\angle BOC=180^{\circ}$ as BO and OC are the angle bisectors of ∠ABC and ∠BCA, respectively
   $\Rightarrow 180^{\circ}-\angle BOC=90^{\circ}-\frac{1}{2}\angle A$
   ∴ ∠BOC = 180^{o} - 90^{o} + $\frac{1}{2}\angle A$
   = $90^{\circ}+\frac{1}{2}\times 130^{\circ}$ = 90^{o} + 65^{o} [∴ ∠A = 130^{o} (given)]
   = 155^{o}
   Hence, the required angle is 155^{o}.
2. **(a)** 75°
   Explanation: Let the measure of the required angle be x°
   Then, the measure of its complement will be (90 − x)°
   ∴ x = 5 (90 - x)
   ⇒ x = 450 - 5x
   ⇒ 6x = 450
   ⇒ x = 75°
3. **(a)** 54°
   Explanation: Let the measure of the required angle be x°
   Then, the measure of its complement will be(90 − x)°
   ∴ 2x = 3 (90 - x)
   ⇒ 2x = 270 - 3x
   ⇒ 5x = 270
   ⇒ x = 54°
4. **(c)** (ii) and (iii) are correct
   Explanation: When two straight lines intersect them, Adjacent angles are supplementary and opposite angles are equal.
5. **(a)** 90°
   Explanation: Given that,
   AB and CD intersect at O
   $\angle AOC$ + $\angle COB$ +$\angle BOD$ = 270° (i)
   ∠COB + ∠BOD = 180^{o} (Linear pair) (ii)
   Using (ii) in (i), we get
   ∠AOC + 180^{o} = 270^{o}
   ∠AOC = 90^{o}
6. **(d)** An acute angled triangle
   Explanation: Let the angles of the triange be 5x, 3x and 7x
   We know that the sum of the angles of a triangle is 180º

$5x + 3x + 7x = 180^o$
$15x = 180^o$
$x = 12^o$
Therefore the angles are
$5x = 5 \times 12^0 = 60^\circ$
$3x = 3 \times 12^0 = 36^0$
$7x = 7 \times 12^0 = 84^0$
Since all the angles are less than 90^o there fore it is a acute angled triangle.

7. **(d)** 12 : 3 : 2
Explanation: Let A be x
B = $\frac{1}{4}$x
C = $\frac{1}{6}$ x
A : B : C
x : $\frac{1}{4}$x : $\frac{1}{6}$ x
LCM of 4 and 6 is 12
12 : 3 : 2

8. **(a)** 117°
Explanation: $\angle$BOD + $\angle$BOC = 180^o (Linear pair)
63^o + $\angle$BOC = 180^o
$\angle$BOC = 117^o

9. **(d)** Straight line
Explanation: 

As can be seen from the above diagram, the two planes "P" and "Q" are intersecting in a line, which is AB.

10. **(d)** 30^0
Explanation: Let one angle be x^o
Its supplementary angle will be 180^o - x^o
According to question
x = $\frac{1}{5}$(180^o - x)
5x + x = 180^o
6x = 180^o
x = $\frac{180}{6}$
x = 30^0

11. Supplement of an angle = 180 - x
Complement of an angle = 90 - x
According to question,
180 - x = 3(90 - x)
180 - x = 270 - 3x
-x + 3x = 270 - 180
2x = 90

$x = \frac{90}{2}$
x = 45

12. Let the measure of an angle be x^0, then the measure of its complement is also x^0.
    We know that the sum of the measures of complementary angles is 90°.
    Therefore, $x^0 + x^0 = 90^\circ$
    $\Rightarrow 2x^0 = 90^\circ$
    $\Rightarrow x^0 = 45^\circ$
13. Sum of any two angles equals to 90 degrees then they are called complementary angles
    Let one angle = x
    It's complementary angle = x + 14
    x + x + 14 = 90
    2x = 90 - 14
    2x = 76
    $x = \frac{76}{2}$
    x = 38
    One angle = x = 38
    Second angle = x + 14 = 38 + 14 = 52.
14. Given angle is 138^o
    Since the sum of an angle and its supplement is 180^o
    Therefore, its supplement will be:
    $180^o - 138^o = 42^o$
15. Let the angle be x.
    Its supplement = 180° - x
    according to question,
    $180^\circ - x = \frac{2}{3}x$
    $\Rightarrow 3(180^\circ - x) = 2x$
    $\Rightarrow 540^\circ - 3x = 2x$
    $\Rightarrow 540^\circ = 5x$
    $\Rightarrow x = \frac{540}{5}$
    $\Rightarrow x = 108^\circ$.
    Now, the angle's supplement = $180^\circ - x = 180^\circ - 108^\circ = 72^\circ$
    Hence, the angle is 108 and its supplement is 72°.
16. Given angle is 20^o
    Since the sum of an angle and its compliment is 90^o
    Therefore, its compliment will be:
    $90^o - 20^o = 70^o$
17. Two angles whose sum is 180° are called supplementary angles.
    Supplement of 42° = 180° - 42° = 138°
    Supplement of 42° = 138°
18. Let the measure of an angle be x, then measure of its supplement is also x.
    Since the sum of supplementary angles is 180°.
    $\therefore x + x = 180^\circ \Rightarrow \quad 2x = 180^\circ$
    $\Rightarrow \quad x = 90^\circ$
19. The measure of the supplementary angle x = $(180^o - r^o)$
    Where r^o = given measurement
    $\therefore x = (180^o - 68^o) = 112^o$

20. Given angle is 54^o

    Since the sum of an angle and its supplement is 180^o
    Therefore, its compliment will be:

    $180^o - 54^o = 126^o$

21. The measure of the supplementary angle x = $(180^o - r^o)$

    Where r^o = given measurement

    $\therefore x = (180^o - 125) = 55^o$

22. The measure of the complementary angle x = $(90^o - r^o)$

    Where r^o = given measurement

    $\therefore x = (90^o - 72^o) = 18^o$

    hence, measure of the complementary angle of 72^o.= 18^o

23. Let the measure of the required angle be x°.
    Then, the measure of its complement is (90 - x)°.
    It is given that
    (90 - x)° - x° = 20°
    ⇒ 90° - 2x° = 20°
    ⇒ -2x° = 20° - 90°
    ⇒ -2x° = -70°
    ⇒ x° = 35°
    Hence, the measure of the angle is 35°

24. Given angle is 30^o

    Since the sum of an angle and its compliment is 90^o
    Therefore, its compliment will be,

    $90^o - 30^o = 60^o$

25. Let x be the angle
    then according to the question,
    90 - x = 180 - 3x
    $\Rightarrow x = \frac{90}{2}$
    $\therefore x = 45$

26. Since OA and OB are opposite rays.
    Therefore, AB is a line. Since ray OC
    stands on line AB.
    ∴∠AOC+∠COB=180° [Linear Pairs]
    ⇒∠AOC+∠COD+∠BOD=180° [∵ ∠COB=∠COD+∠BOD]
    ⇒ (∠AOC+∠BOD)+∠COD=180°
    ⇒ 90° +∠ COD=180° [∵ ∠AOC+∠BOD=90° Given)]
    ⇒ ∠COD=180° - 90° = 90°.

27. x = y (Given)
    Therefore, l || m (Corresponding angles) (i)
    Also, a = b (Given)
    Therefore, n || m (Corresponding angles) (ii)
    From (i) and (ii), l || n (Lines parallel to the same line).

28. ∠BOC = ∠AOB + ∠AOC

    $= 90^o + 90^o = 180^o$ . . . [Given : ∠AOB and ∠AOC = 90^o]
    ∴ BOD is a line . . . [Linear pair axiom]

29. When a ray falls on a mirror, it is reflected and angle of incidence = angle of reflection = x° (say).
    QM is drawn normal to AB and therefore,we have,
    angle of incidence = ∠PQM,
    angle of reflection = ∠MQR
    and $\angle AQM = 90^\circ$

Now,we have, $\angle PQM + \angle MQR = \angle PQR = 112^\circ$ (given)
$\therefore 2\angle PQM = 112^\circ$
$\therefore \angle PQM = 56^\circ$
Therefore, $\angle PQA = \angle AQM - \angle PQM = 90^\circ - 56^\circ = 34^\circ$

30. AOB will be a straight line, if
$\angle AOC + \angle BOC = 180^\circ$
$\therefore$ (3x + 5)° + (2x - 25)° = 180°
$\Rightarrow$ 5x° = 200° $\Rightarrow$ x = 40°
Therefore, x = 40° will make AOB a straight line

31. Given: AB and CD are two lines intersect each other at O.
To prove:
    i. $\angle 1 = \angle 2$
    ii. $\angle 3 = \angle 4$

Proof:
$\angle 1 + \angle 4 = 180^0$ ...(i) [By linear pair]
$\angle 4 + \angle 2 = 180^0$ ...(ii) [By linear pair]
$\angle 1 + \angle 4 = \angle 4 + \angle 2$ [By eq (i) and (ii)]
$\angle 1 = \angle 2$
Similarly,
$\angle 3 = \angle 4$

32. $\angle$POC = $\angle$DOQ = 2y . . . . [Vertically opposite angles]
$\angle$AOB = 180^o . . . . . [A straight angle = 180^o]
$\angle$AOB + $\angle$POC + $\angle$BOC = 180^o
$\angle$ 5y + 2y + 5y = 180^o
$\angle$ 12y = 180^o
$\angle\, y = \frac{180^0}{12} = 15^0$

33. Let the two complementary angles be 2x and 3x.
We know that, sum of complementary angles is 90°.
$\therefore 2x + 3x = 90^\circ$
$\Rightarrow 5x = 90^\circ$
$\Rightarrow \quad x = 18^\circ$
$\therefore$ The angles are $2 \times 18^\circ = 36^\circ$ and $3 \times 18^\circ = 54^\circ$.

34. AOB is a straight line. Therefore,by linear pair axiom,
$\angle AOC + \angle COD + \angle BOD = 180^\circ$
$\Rightarrow$ (3x + 7)° + (2x - 19)° + x° = 180°
$\Rightarrow$ 6x = 192°
$\Rightarrow$ x = 32°
Therefore,
$\angle AOC = 3 \times 32^\circ + 7 = 103^\circ$
$\angle COD = 2 \times 32^\circ - 19 = 45^\circ$ and
$\angle BOD = 32^\circ$

35. a + b = 180^o . . . . [Linear pair axiom] . . . . (1)
a – b = 80^o . . . . [Given] (2)
2a = 180^o + 80^o . . . . [Adding (1) and (2)]
$\therefore$ 2a = 260^o
$\therefore a = \frac{260^0}{2} = 130^0$
Subtracting (2) and (1), we get
$\therefore$ 2b = 180^o – 80^o
$\therefore$ 2b = 100^o
$\therefore b = \frac{100^0}{2} = 50^0$

36. Let the two angles be 4x and 5x, respectively

Then,

$4x° + 5x° = 90°$

$\Rightarrow 9x° = 90°$

$\Rightarrow x° = 10°$

Then the two anlges are $4x = 4 \times 10° = 40°$ and $5x = 5 \times 10° = 50°$

Therefore, one angle is 40° and its complemetry angle is 50°

37. Here, $\angle AOC$ and $\angle BOC$ form a linear pair.

$\therefore \angle AOC + \angle BOC = 180°$

$\Rightarrow x° + 125° = 180°$

$\Rightarrow x° = 180° - 125° = 55°$

Now,

$\angle \mathrm{AOD} = \angle \mathrm{BOC} = 125^{\circ}$ (Vertically opposite angles)

$\therefore y° = 125°$

$\angle BOD = \angle AOC = 55^{\circ}$ (Vertically opposite angles)

$\therefore z° = 55°$

38. Let the measure of the required angle be x°

Then, the measure of its complement = (90° - x°)

And then measure of its supplement = (180 - x)°

given that,

$(90° - x°) = \frac{1}{3}(180° - x°)$

$\Rightarrow 3(90° - x°) = (180° - x°)$

$\Rightarrow 270° - 3x° = 180° - x°$

$\Rightarrow 2x° = 90°$

$\Rightarrow x° = 45°$

Hence, the measure of the required angle is 45°

39. Given that,

$(2x – 10)^{o}$ and $(x – 5)^{o}$ are compliment angles.

Since, angles are complimentary

Therefore,

$(2x – 10)^{o} + (x – 5)^{o} = 90^{0}$

$3x – 15^{o} = 90^{o}$

$x = 35^{o}$

40. Let the measure of the required angle be x°,

Then, its complement = (90 - x)°

and its supplement = (180 - x)°.

$\therefore 7(90° - x°) = 3(180° - x°) - 10°$

$\Rightarrow 630° - 7x° = 540° - 3x° -10°$

$\Rightarrow 4x° = 100°$

$\Rightarrow x° = 25°$

Hence, the measure of the required angle is 25°.

41. AB is a line

$\therefore \angle AOB = 180^{o}$ (Linear Pair)

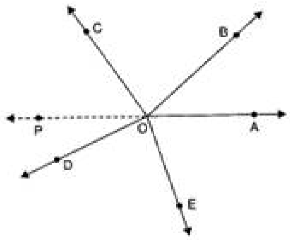
$\therefore \angle AOC + \angle COD + \angle BOD = 180^{o}$ (Linear Pair)

$\therefore (\angle AOC + \angle BOD) + \angle COD = 180^{o}$

$\therefore 70^{o} + \angle COD = 180^{o}$ . . . . [Given : $\angle AOC + \angle BOD = 70^{o}$]

$\therefore \angle COD = 180^{o} – 70^{o} = 110^{o}$

42. Draw a ray OP opposite to ray OA



$\angle$AOB + $\angle$BOC + $\angle$COP = 180^o . . . [A straight angle = 180^o] . . .(1)

$\angle$POD + $\angle$DOE + $\angle$EOA = 180^o . . . [A straight angle = 180^o] . . .(2)

$\angle$AOB + $\angle$BOC + ($\angle$COP + $\angle$POD) + $\angle$DOE + $\angle$EOA = 180^o + 180^o = 360^o

$\angle$AOB + $\angle$BOC + $\angle$COD + $\angle$DEO + $\angle$EOA = 360^o

43. We know that total angle on a line is equal to 180°.

$\therefore$ 2x + 3x + 4x = 180° (angles on same line)

$\Rightarrow 9x = 180^\circ$

$\therefore x = 20^\circ$.

44. AOB will be a straight line if

3x + 20 + 4x - 36 = 180°

$\Rightarrow$ 7x = 196°

$\Rightarrow$ x = 25°

Therefore, x = 28 will make AOB a straight line

45. Since AOB is a straight line, the sum of all the angles on the same side of AOB at a point O on it, is 180°.

Therefore,we have,

x° + 65° + (2x - 20)° = 180°

$\Rightarrow$ 3x° = 135°

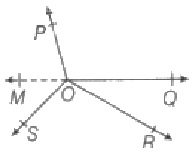
$\Rightarrow$ x° = 45°

$\therefore$ $\angle$AOC = x° = 45° and $\angle$BOD = (2 $\times$ 45 - 20)° = 70°

46. Let us produce a ray OQ backwards to a point M, then MOQ is a straight line.

Now, OP is a ray on the line MOQ. Then, by linear pair axiom, we have

$\angle$MOP + $\angle$POQ = 180^o ......(i)



Similarly, OS is a ray on the line MOQ. Then, by linear pair axiom, we have

$\angle$MOS + $\angle$SOQ = 180^o ....(ii)

Also, $\angle$SOR and $\angle$ROQ are adjacent angles.

$\therefore$ $\angle$SOQ = $\angle$SOR + $\angle$ROQ ...(iii)

On putting the value of $\angle$SOQ from Eq.(iii) in Eq.(ii), we get

$\angle$MOS + $\angle$SOR + $\angle$ROQ = 180^o ....(iv)

Now, on adding Eqs.(i) and (iv), we get

$\angle$MOP + $\angle$POQ + $\angle$MOS + $\angle$SOR + $\angle$ROQ = 180^o + 180^o

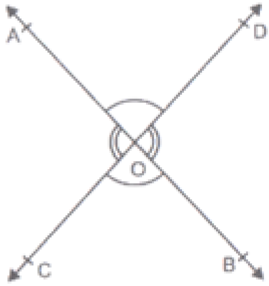
$\Rightarrow$ $\angle$MOP + $\angle$MOS + $\angle$POQ + $\angle$SOR + $\angle$ROQ = 360^o ....(iv)

But $\angle$MOP + $\angle$MOS = $\angle$POS

Then, from Eq.(v), we get

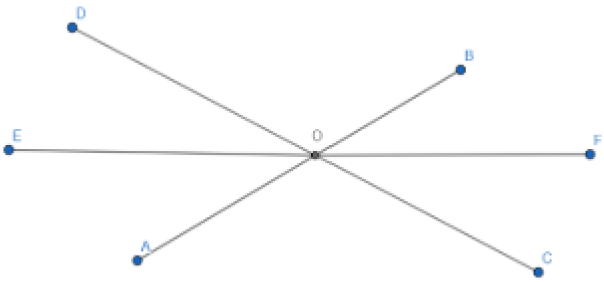
$\angle$POS + $\angle$POQ + $\angle$SOR + $\angle$ROQ = 360^o

Hence proved.

47. 

Let two lines AB and CD intersect at point O.
To prove: $\angle AOC = \angle BOD$ (vertically opposite angles)
$\angle AOD = \angle BOC$ (vertically opposite angles)
Proof: (i) Since, ray OA stands on the line CD.
$\Rightarrow \angle AOC + \angle AOD$= 180 °…(1)[Linear pair axiom]
Also, ray OD stands on the line AB.
$\angle AOD + \angle BOD = 180^\circ$ …(2) [Linear pair axiom]
From equations (1) and (2), we get
$\angle AOC + \angle AOD = \angle AOD + \angle BOD$
$\Rightarrow \angle AOC = \angle BOD$
Hence, proved.
(ii) Since, ray OD stands on the line AB.
$\therefore \angle AOD + \angle BOD = 180^\circ$ …(3) [Linear pair axiom]
Also, ray OB stands on the line CD.
$\therefore \angle DOB + \angle BOC = 180^\circ$ …(4) [linear pair axiom]
From equations (3) and (4), we get
$\angle AOD + \angle BOD = \angle BOD + \angle BOC$
$\Rightarrow \angle AOD = \angle BOC$
Hence, proved.

48. AB and CD are straight lines intersecting at O. OE the bisector of angles ∠AOD and OF is the bisector of ∠BOC.



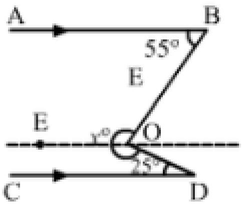
$\angle AOC = \angle BOD$ *(vertically opposite angles)*
Also,
OE is the bisector of $\angle AOD$ and OF is the bisector of $\angle BOC$
To prove: EOF is a straight line.
$\angle AOD = \angle BOC = 2x$ (Vertically opposite angle) ...(i)
As OE and OF are bisectors.So $\angle AOE = \angle BOF = x$ .....*(ii)*
$\angle AOD + \angle BOD = 180^\circ$ *(linear pair)*
$\angle AOE + \angle EOD + \angle DOB = 180^\circ$
From (ii)
$\angle BOF + \angle EOD + \angle DOB = 180^\circ$
$\angle EOF = 180^o$
EF is a straight line.

49. We are given that $AB \parallel CD, CD \parallel EF$ and $y : z = 3 : 7$
We need to find the value of x in the figure given below.
We know that lines parallel to the same line are also parallel to each other.
We can conclude that $AB \parallel EF$

Let $y=3a$ and $z=7a$
We know that angles on the same side of a transversal are supplementary.
$\therefore x+y=180^{\circ}$
$x=z$ Alternate interior angles
$z+y=180^{\circ}$
or $7a+3a=180^{\circ}$
$\Rightarrow 10a=180^{\circ}$
$a=18^{\circ}$.
$z=7a=126^{\circ}$
$y=3a=54^{\circ}$.
Now, as $x=z$
=> $x=126^{\circ}$.
Therefore, we can conclude that $x=126^{\circ}$

50. ∠AOF + ∠FOG = 180^{o} . . . . [Linear pair axiom]
$\Rightarrow$ ∠AOG = 180^{o}
$\Rightarrow$ ∠AOB + ∠EOB + ∠FOE + ∠FOG = 180^{o}
$\Rightarrow$ 30^{o} + 90^{o} + ∠FOE + 30^{o} = 180^{o}
$\Rightarrow$ ∠FOE + 150^{o} = 180^{o}
$\Rightarrow$ ∠FOE = 180^{o} – 150^{o} = 30^{o}
∠AOF + ∠FOG = 180^{o} . . . . [Linear pair axiom]
$\Rightarrow$ ∠AOG = 180^{o}
$\Rightarrow$ ∠AOB + ∠COB + ∠FOC + ∠FOG = 180^{o}
$\Rightarrow$ 30^{o} + ∠COB + 90^{o} + 30^{o} = 180^{o}
$\Rightarrow$ ∠COB + 150^{o} = 180^{o}
$\Rightarrow$ ∠COB = 180^{o} – 150^{o} = 30^{o}
∠FOC = 90^{o}
$\Rightarrow$ ∠FOE + ∠DOE + ∠DOC = 90^{o}
$\Rightarrow$ 30^{o} + ∠DOE + 30^{o} = 90^{o}
$\Rightarrow$ ∠DOE + 60^{o} = 90^{o}
$\Rightarrow$ ∠DEO = 30^{o}

51. We know that if two lines intersect, then the vertically-opposite angles are equal.
Let $\angle BOC=\angle AOD=x^{\circ}$
$\angle BOC+\angle AOD$ = 280°
x + x = 280°
$\Rightarrow$ 2x = 280°
$\Rightarrow$ x = 140°
$\therefore \angle BOC=\angle AOD=140^{\circ}$
Also, let $\angle AOC=\angle BOD=y^{\circ}$
We know that the sum of all angles around a point is 360°
$\therefore \angle AOC+\angle BOC+\angle BOD+\angle AOD=360^{\circ}$
$\Rightarrow$ y + 140 + y + 140 = 360°
$\Rightarrow$ 2y = 80°
$\Rightarrow$ y = 40°
Hence, $\angle AOC=\angle BOD=40^{\circ}$
$\therefore \angle BOC=\angle AOD=140^{\circ}$ and $\angle AOC=\angle BOD=40^{\circ}$

52. 

Draw EO ∥ AB ∥ CD

Then, $\angle EOB + \angle EOD = x^\circ$

Now, EO ∥ AB and *BO* is the transversal.

$\therefore \angle EOB + \angle ABO = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle EOB + 55^\circ = 180^\circ$

$\Rightarrow \angle EOB = 125^\circ$

Again, EO ∥ CD and DO is the transversal.

$\therefore \angle EOD + \angle CDO = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle EOD + 25^\circ = 180^\circ$

$\Rightarrow \angle EOD = 155^\circ$

Therefore,

$x^\circ = \angle EOB + \angle EOD$

x° = (125 + 155)°

x° = 280°

53. i. In △BOD,

$\angle OBD + \angle BOD + \angle ODB = 180^o$

(The sum of the three angles of a triangle is 180^o)

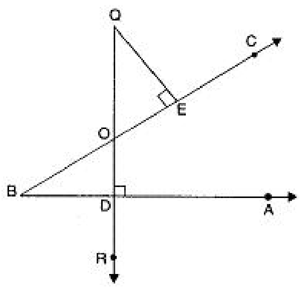
$\Rightarrow \angle OBD + \angle BOD + 90^o = 180^o$

$\Rightarrow \angle OBD + \angle BOD = 90^o$ .... (1)

In△OEQ,

$\angle EQO + \angle QOE + \angle OEQ = 180^o$ .... (2)

(The sum of the three angles of a triangle is 180^o)



$\Rightarrow \angle EQO + \angle QOE + 90^o = 180^o$

$\Rightarrow \angle EQO + \angle QOE = 90^o$ .... (2)

From (1) and (2), we get

$\angle OBD + \angle BOD = \angle EQO + \angle QOE$

But $\angle BOD = \angle QOE$ (Vertically Opposite Angles)

$\therefore \angle OBD = \angle EQO$

ii. Join BQ

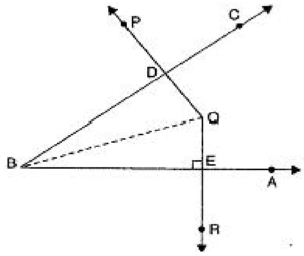
In△BDQ,

$\angle DBQ + \angle BQD + \angle QDB = 180^o$

(The sum of the three angles of a triangle is 180^o)

$\Rightarrow \angle DBQ + \angle BQD + 90^o = 180^o$

$\Rightarrow \angle DBQ + \angle BQD = 90^o$ .... (1)



In△BQE,

$\angle EBQ + \angle BQE + \angle BEQ = 180^{o}$

(The sum of the three angles of a triangle is 180^{o})

$\Rightarrow \angle EBQ + \angle BQE + 90^{o} = 180^{o}$

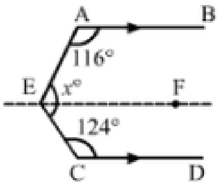
$\Rightarrow \angle EBQ + \angle BQE = 90^{o}$

Adding (1) and (2), we get

$(\angle DBQ + \angle EBQ) + (\angle BQD + \angle BQE) = 90^{o} + 90^{o} = 180^{o}$

$\Rightarrow \angle DBE + \angle EQD = 180^{o}$

$\Rightarrow \angle DBE$ and $\angle EQD$ are supplementary.

54. 

Draw EF || AB || CD

Then, $\angle AEF + \angle CEF = x^\circ$

Now, EF || AB and AE is the transversal

$\therefore \angle AEF + \angle BAE = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle AEF + 116 = 180$

$\Rightarrow \angle AEF = 64^\circ$

Again, EF || CD and CE is the transversal.

$\angle CEF + \angle ECD = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle CEF + 124 = 180$

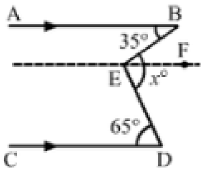
$\Rightarrow \angle CEF = 56^\circ$

Therefore,

$x^\circ = \angle AEF + \angle CEF$

$x^\circ = (64 + 56)^\circ$

$x^\circ = 120^\circ$

55. 

Draw EF || AB || CD

Now, AB || EF and BE is the transversal.

Then,

$\angle ABE = \angle BEF$ [Alternate Interior Angles]

$\Rightarrow \angle BEF = 35^\circ$

Again, EF || CD and DE is the transversal

Then,

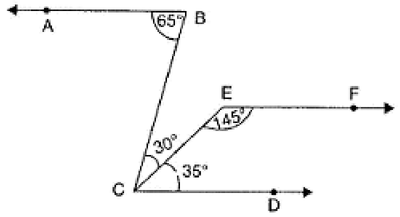
$\angle DEF = \angle FED$

$\Rightarrow \angle FED = 65^\circ$

$\therefore x^\circ = \angle BEF + \angle FED$

$x^\circ = 35^\circ + 65^\circ$

$x^\circ = 100^\circ$

56. 

$\angle$ABC = 65^o

$\angle$BCD = $\angle$BCE + $\angle$ECD = 30^o + 35^o = 65^o

$\therefore$ $\angle$ABC = $\angle$BCD

These angles form a pair of equal alternate angles

$\therefore$ AB || CD . . . (1)

$\angle$FEC + $\angle$ECD = 145^o + 35^o = 180^o

These angles are consecutive interior angles formed on the same side of the transversal.

$\therefore$ CD || EF . . . . (2)

AB || EF . . . [From (1) and (2)]

57. To Prove: $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Given: OR is perpendicular to PQ, or $\angle$QOR = 90°

From the given figure, we can conclude that $\angle$POR and $\angle$QOR form a linear pair.

We know that sum of the angles of a linear pair is 180°.

$\therefore$ $\angle$POR + $\angle$QOR = 180°

or $\angle$POR = 90°

From the figure, we can conclude that

$\angle$POR = $\angle$POS + $\angle$ROS

$\Rightarrow$ $\angle$POS + $\angle$ROS = 90°

$\Rightarrow$ $\angle$ROS = 90° - $\angle$POS...(i)

Again,

$\angle$QOS + $\angle$POS = 180°

$\Rightarrow \frac{1}{2}(\angle QOS + \angle POS) = 90^\circ$.(ii)

Substitute (ii) in (i), to get

$\angle ROS = \frac{1}{2}(\angle QOS + \angle POS) - \angle POS$

$= \frac{1}{2}(\angle QOS - \angle POS)$.

Therefore, the desired result is proved.

58. Through O, draw EO $\parallel$ AB $\parallel$ CD

Then, $\angle EOB + \angle EOD = x^\circ$,

Now, AB $\parallel$ EO and BO is the transversal

$\therefore \angle ABO + \angle BOE = 180^\circ$ [consecutive interior angles]

$\Rightarrow 40^\circ + \angle BOE = 180^\circ$

$\Rightarrow \angle BOE = (180^\circ - 40^\circ) = 140^\circ$

$\Rightarrow$ $\angle$BOE =140°

Again CD $\parallel$ EO and OD is the transversal.

$\therefore \angle EOD + \angle ODC = 180^\circ$

$\Rightarrow \angle EOD + 35^\circ = 180^\circ$

$\Rightarrow \angle EOD = (180^\circ - 35^\circ) = 145^\circ$

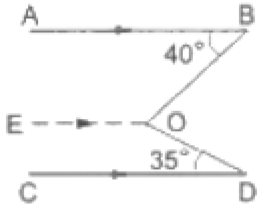
$\Rightarrow$ $\angle$EOD = 145°

$\therefore$ reflex $\angle BOD = x^\circ = (\angle BOE + \angle EOD)$

= (140° + 145°) = 285°

Hence, x° = 285°

$\Rightarrow \angle$BOD = x° = 285°



59. Through O draw OE $\|$ AB $\|$ CD

Then, $\angle AOE + \angle COE = x^\circ$

Now, AB $\|$ OE and AO is the transversal

$\therefore \angle OAB + \angle AOE = 180^\circ$

$\Rightarrow 104^\circ + \angle AOE = 180^\circ$

$\Rightarrow \angle AOE = (180 - 104)^\circ = 76^\circ$ .....(1)

Again, CD $\|$ OE and OC is the transversal

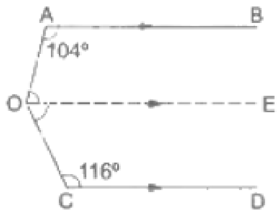
$\therefore \angle COE + \angle OCD = 180^\circ$

$\Rightarrow \angle COE + 116^\circ = 180^\circ$

$\Rightarrow \angle COE = (180^\circ - 116^\circ) = 64^\circ$ ....(2)

$\therefore \angle AOC = \angle AOE + \angle COE$ = (76° + 64°) = 140° [from (1) and (2)]

Hence, x° = 140°



60. Since corresponding angles are equal.

$\therefore$ x = y ... (i)

We know that the interior angles on the same side of the transversal are supplementary.

$\therefore$ y + 55° = 180°

$\Rightarrow$ y = 180° - 55° = 125°

So, x = y = 125°

Since AB || CD and CD || EF.

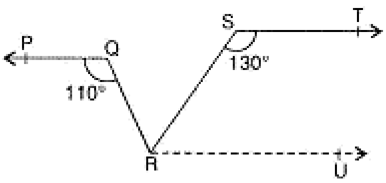
$\therefore$ AB || EF

$\Rightarrow \angle$EAB + $\angle$FEA = 180° [$\because$ Interior angles on the same side of the transversal EA are supplementary]

$\Rightarrow$ 90° + z + 55° = 180°

$\Rightarrow$ z = 35°

61. Draw a line RU parallel to ST through point R.



$\angle$RST + $\angle$SRU = 180°

$\therefore$ 130° + $\angle$SRU = 180°

$\therefore \angle$SRU = 180° – 130° = 50° . . . (1)

$\angle$QRU = $\angle$PQR = 110° . . . . [Alternate interior angles]

$\therefore \angle$QRS + $\angle$SRU = 110°

$\therefore \angle$QRS + 50° = 110° . . . [Using (1)]

$\therefore \angle$QRS = 110° – 50° = 60°

62. Lines AB and CD intersect at O

$\therefore \angle AOC = \angle BOD$ . . . . [Vertically opposite angles]

But $\angle BOD = 40^o$ . . . . [Given] . . . (1)

$\therefore \angle AOC = 40^o$ . . . . (2)

Now, $\angle AOC + \angle BOE = 70^o$

$\Rightarrow 40^o + \angle BOE = 70^o$

$\therefore \angle BOE = 70^o - 40^o$

$\therefore \angle BOE = 30^o$

Again,

Reflex $\angle COE = \angle COD + \angle BOD + \angle BOE$

$= \angle COD + 40^o + 30^o$ . . . . [Using (1) and (2)]

$= 180^o + 40^o + 30^o$ . . . . [As ray OA stands on the line CD]

$= 250^o$

$\therefore \angle AOC + \angle AOD = 180^o$. . . . . [Linear Pair Axiom]

$\therefore \angle COD = 180^o$

$\Rightarrow$ a = 2k, b = 3k

Putting the values of a and b in (1), we get

$2k + 3k = 90^o$

$5k = 90^o = k = \frac{90^0}{5}$

$\Rightarrow k = 18^o$

$a = 2k = 2\ (18^o) = 36^o$ and $b = 3k = 3(18^o) = 54^o$ . . . . . (2)

As ray OX is perpendicular to line MN

$\therefore \angle XOM + \angle XON = 180^o$ . . . . [Linear Pair Axiom]

$b + c = 180^o$

$\therefore 54^o + c = 180^o$ . . . [Using (2)]

$\therefore c = 180^o - 54^o \therefore c = 126^o$

63. d = a . . . [Vertically opposite angles]

$= 50^o$

$b + c + d = 180^o$ . . . [A straight line angle = 180^o]

$\therefore 90^o + c + 50^o = 180^o$

$\therefore c + 140^o = 180^o$

$\therefore c = 180^o - 140^o$

$\therefore c = 40^o$

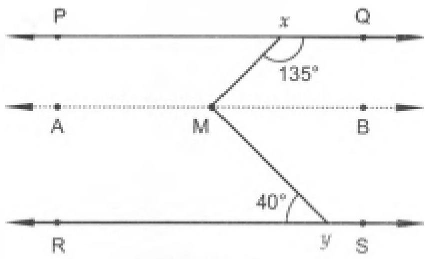
e = b . . . [Vertically opposite angles]

$= 90^o$

f = c . . . [Vertically opposite angles]

$= 40^o$

64. Through point M draw a line AB parallel to the line PQ as shown in Fig. Thus, we have



AB || PQ and PQ || RS

$\Rightarrow$ AB || RS

Now, AB || PQ and $\angle QXM$ and $\angle XMB$ are interior angles on the same side of the transversal XM.

$\therefore \angle QXM + \angle XMB = 180^o$

$\Rightarrow 135^o + \angle XMB = 180^o$

$\Rightarrow \angle XMB = 180^o - 135^o = 45^o$

Now, AB || RS and $\angle BMY$ and $\angle MYR$ are alternate angles.

$\therefore \angle BMY = \angle MYR$

$\Rightarrow \angle BMY = 40^o$

Hence, $\angle XMY = \angle XMB + \angle BMY = 45^o + 40^o = 85^o$

65. Given AD is transversal intersect two lines PQ and RS

To prove PQ $\|$ RS

Proof: BE bisects ABQ

$\angle 1 = \angle ABE = \angle EBQ = \frac{1}{2}\angle ABQ$ ...(i)

Similarity CG bisects $\angle$ BCS

$\therefore \angle 2 = \frac{1}{2}\angle BCS$ ...(ii)

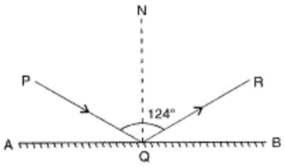
But BE $\|$ CG and AD is the transversal

$\therefore \angle 1 = \angle 2$

$\therefore \frac{1}{2}\angle ABQ = \frac{1}{2}\angle BCS$ [by (i) and (ii)]

$\Rightarrow \angle \mathrm{ABQ} = \angle \mathrm{BCS}$ [$\because$ corresponding angles are equal]

$\therefore$ PQ $\|$ RS

66. 

Draw QN $\perp$ AB

Angle of incident = Angle of reflection . . . [by law of reflection]

$\therefore \angle PQN = \angle NQR$

$\angle PQR = 124^o$ . . . . [Given]

$\therefore \angle PQN + \angle NQR = 124^o$

$\therefore \angle NQR + \angle NQR = 124^o$ . . . . [As $\angle PQN = \angle NQR$]

$\therefore 2\angle NQR = 124^o$

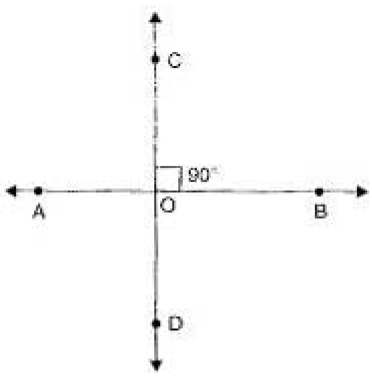
$\therefore \angle NQR = \frac{124^0}{2} = 62^o$

$\therefore \angle NQB - \angle RQB = 62^o$

$\therefore 90^o - \angle RQB = 62^o$

$\therefore \angle RQB = 90^o - 62^o = 28^o$

67. Let AB and CD be two intersecting lines intersects at O.



Let $\angle BOC = 90^o$

We have to prove that each other angles i.e. $\angle AOC$ $\angle AOD$ and $\angle BOD$ is a right angle.

$\angle AOD = \angle BOC$ . . . [Vertically opposite angles]

$= 90^o$

$\angle$AOC + $\angle$BOC = 90^o . . . [A straight line angle = 180^o]

$\angle$AOC + 90^o = 180^o

$\angle$AOC = 180^o – 90^o = 90^o

$\angle$BOD = $\angle$AOC . . . .[Vertically opposite angles]

= 90^o

68. a + b = 180^o . . . [Linear Pair Axiom] . . . (1)

a = b$+\frac{1}{3}$(a right angle) . . . [Given]

$a = b + \frac{1}{3}(90^0)$ . . . . [right angle = 90^o]

$\therefore$ a + b = 30^o

$\therefore$ a – b = 30^o . . . . . (2)

2a = 180^o + 30^o . . . . . [Adding (1) and (2)]

$\therefore$ 2a = 210^o

$\therefore a = \frac{210^0}{2} = 105^0$

2b = 180^o – 30^o . . . . . [Subtracting (2) from (1)]

$\therefore$ 2b = 150^o

$\therefore b = \frac{150^0}{2} = 75^0$

69. We need to prove that $\angle$PQS = $\angle$PRT

We are given that $\angle$PQR = $\angle$PRQ

From the given figure, we can conclude that $\angle$PQS and $\angle$PQR, and $\angle$PRQ and $\angle$PRT form a linear pair.

We know that sum of the angles of a linear pair is 180°

$\therefore \angle PQS + \angle PQR = 180^\circ$, and ...(i)

$\angle PRQ + \angle PRT = 180^\circ$. ...(ii)

From equation (i) and (ii), we can conclude that

$\angle PQS + \angle PQR = \angle PRQ + \angle PRT$.

But, $\angle$PQR = $\angle$PRQ

$\therefore$ $\angle$PQS = $\angle$PRT

Hence, proved.

70. $\angle$AOC + $\angle$BOC = 180^o . . . . [Linear pair]

$\angle$AOC + $\angle$BOE + $\angle$COE = 180^o . . . [As $\angle$BOC = $\angle$BOE + $\angle$COE]

$\therefore$ $2x^o + x^o + 90^o = 180^o$

$\therefore$ $3x^o + 90^o = 180^o$

$\therefore$ $3x^o = 180^o - 90^o = 90^o$

$\therefore$ $x^o = \frac{90^0}{3} = 30^o$ $\therefore$ x = 30

$\angle$BOD = $\angle$AOC . . . [Vertically opposite angles]

$\therefore$ $y^o = 2x^o = 2(30^o) = 60^o$

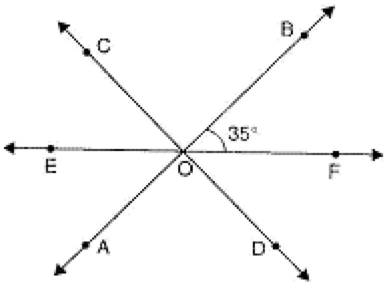
$\therefore$ y = 60

$\angle$AOD = $\angle$COB . . . [Vertically opposite angles]

$\therefore$ $\angle$AOD = $\angle$COE + $\angle$EOB

$\therefore$ $z^o = 90^o + x^o = 90^o + 30^o = 120^o$

$\therefore$ z = 120

71. 

OF bisects $\angle$BOD . . . [Given]

$\angle$BOF = $\angle$DOF = 35^o

$\angle$COE = $\angle$DOF = 35^o

$\angle$EOF = 180^o . . . . [A straight angle = 180^o]

$\therefore$ $\angle$EOC + $\angle$BOC + $\angle$BOF = 180^o

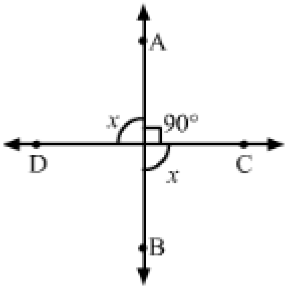
$\therefore$ 35^o + $\angle$BOC + 35^o = 180^o

$\therefore$ $\angle$BOC = 180^o – 70^o = 110^o

$\angle$AOD = $\angle$BOC . . . [Vertically opposite angles]

= 110^o

72. We know that if two lines intersect, then the vertically-opposite angles are equal.



$\angle AOC = 90^\circ$, Then $\angle AOC = \angle BOD = 90^\circ$

And let $\angle BOC = \angle AOD$ = x°

Also, we know that the sum of all angles around a point is 360°

$\Rightarrow$ 90° + 90° + x° + x° = 360°

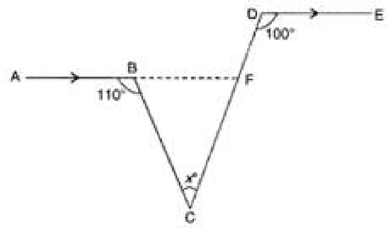
$\Rightarrow$ $2x^\circ$ = 180°

$\Rightarrow$ x° = 90°

Hence, $\angle BOC = \angle AOD$ = x° = 90°

$\therefore \angle AOC = \angle BOD = \angle BOC = \angle AOD = 90^\circ$

Hence, the measure of each of the remaining angles is 90°.

73. 

Draw AB to meet CD in F.

As AF || DE and transversal DF intersects them

$\therefore$ $\angle$DFB = $\angle$EDF = 100^o . . . [Alternate Angles]

$\angle$DFB + $\angle$BFC = 180^o . . . [Linear pair axiom]

$\therefore$ 100^o + $\angle$BFC = 180^o

$\therefore$ $\angle$BFC = 180^o – 100^o = 80^o

$\angle$ABC + $\angle$FBC = 180^o . . . [Linear pair axiom]

$\therefore$ 110^o + $\angle$FBC = 180^o

$\therefore$ $\angle$FBC = 180^o – 110^o = 70^o

In $\triangle$BFC,

$\angle$BCF + $\angle$BFC + $\angle$FBC = 180^o . . . [Sum of all the angles of a triangle]

$\therefore$ x^o + 80^o + 70^o = 180^o

$\therefore$ x^o + 150^o = 180^o

$\therefore$ x^o = 180^o – 150^o = 30^o

$\therefore x = 30^0$

74. Given: In figure, OD $\perp$ OE (i.e. $\angle DOE = 90^0$), OD and OE are the bisectors of $\angle$AOC and $\angle$BOC.

To prove: points A, O and B are collinear i.e., AOB is a straight line.

Proof: From the fig. we have ∠AOB comprising ∠AOC and ∠BOC such that OD and OE are the bisectors of these two angles

∠AOB = ∠AOC + ∠BOC

Since, OD and OE bisect angles ∠AOC and ∠BOC respectively.

$\therefore \angle AOC = 2\angle DOC$ …...(1)

And $\angle COB = 2\angle COE$ …..(2)

On adding equations (1) and (2), we get

$\angle AOC + \angle COB = 2\angle DOC + 2\angle COE$

$\Rightarrow \angle AOC + \angle COB = 2(\angle DOC + \angle COE)$

$\Rightarrow \angle AOC + \angle COB = 2\angle DOE$

$\Rightarrow \angle AOC + \angle COB = 2 \times 90^\circ$ $[\because OD \perp OE]$

$\Rightarrow \angle AOC + \angle COB = 180^\circ$

$\therefore \angle AOB = 180^\circ$

So, $\angle AOC\ and\ \angle COB$ are forming linear pair or AOB is a straight line. Hence, points A, O and B are collinear.

75. Since , $OP \| RS$ and transversal RN intersects them at N and R respectively

$\therefore \quad \angle RNP = \angle SRN$ (Alternate interior angles)

$\Rightarrow \quad \angle RNP = 130^\circ$

$\therefore \quad \angle PNQ = 180^\circ - 130^\circ = 50^\circ$ (Linear pair)

$\angle OPQ = \angle PNQ + \angle PQN$ (Exterior angle property)

$\Rightarrow \quad 110^\circ = 50^\circ + \angle PQN$

$\Rightarrow \quad \angle PQN = 110^\circ - 50^\circ = 60^\circ$

Also, $\angle PQN = \angle PQR$ (see figure)

$\therefore \angle PQR = 60^\circ$