Solution

REAL NUMBER QUESTION BANK

Class 09 - Mathematics

1. **(b)** $\frac{1}{8}$

   Explanation: $\sqrt[4]{(64)^{-2}}$

   $\Rightarrow (64)^{\frac{-2}{4}}$

   $\Rightarrow (64)^{\frac{-1}{2}}$ or $\frac{1}{\sqrt{64}}$

   $\Rightarrow \frac{1}{8}$

2. **(c)** $13^{-2/15}$

   Explanation: $\frac{13^{1/5}}{13^{1/3}}$

   = $13^{1/5+1/3}$

   =$13^{-2/15}$

3. **(d)** 25

   Explanation: $(0.00032)^{\frac{-2}{5}}$

   $=\left(\frac{32}{100000}\right)^{\frac{-2}{5}}$

   $=\left(\frac{2}{10}\right)^{5\times\frac{-2}{5}}$

   $=\left(\frac{1}{5}\right)^{-2}=25$

4. **(d)** $\frac{\sqrt{7}+2}{3}$

   Explanation: After rationalising:

   $\frac{1}{\sqrt{7}-2}=\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}$

   $=\frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$

   = $\frac{\sqrt{7}+2}{7-4}$

   = $\frac{\sqrt{7}+2}{3}$

5. **(a)** 0

   Explanation: 0, [Since 0 is in between -3 and 3 (ie -3<0<3)]

6. **(b)** 0.5

   Explanation: 9^{x+2} = 240 + 9^x

   $\Rightarrow$ 9^{x+2} - 9^x = 240

   $\Rightarrow$ $9^x \cdot 9^2$ - 9^x = 240

   $\Rightarrow$ $9^x(9^2$ - 1) = 240

   $\Rightarrow$ 9^x(81 - 1) = 240

   $\Rightarrow$ $9^x \times$ 80 = 240

   $\Rightarrow$ 9^x = $\frac{240}{80}$

   $\Rightarrow$ $(3^2)^x$ = 3^1

   $\Rightarrow$ 3^{2x} = 3^1

   $\therefore$ 2x = 1 $\Rightarrow$ x = $\frac{1}{2}$ = 0.5

7. **(a)** $\frac{2}{9}$

   Explanation: Let x= 0.222...---(i)

   multiply eq. (i) by 10, we get

   10 x = 2.222....---(ii)

   10 x - x = 2.222...-0.222...

9x=2

x= $\frac{2}{9}$

8. **(b)** $(56)^{1/2}$

Explanation: $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

= $(7 \cdot 8)^{\frac{1}{2}}$

$(56)^{1/2}$

9. **(c)** 2

Explanation: 1.9999 can be written as 2,

2is taken as approx vlaue .

10. **(b)** $\sqrt{12}+\sqrt{5}$

Explanation: $\frac{1}{2\sqrt{3}-\sqrt{5}}$

= $(2\sqrt{3}-\sqrt{5})(2\sqrt{3}+\sqrt{5})$

=12-5

= 7

Rational number

$(2\sqrt{3}+\sqrt{5})$ = $(\sqrt{4\times 3}+\sqrt{5})$= $\sqrt{12}+\sqrt{5}$

11. **(b)** $2\sqrt{2}+\sqrt{3}$

Explanation: $2\sqrt{2}+\sqrt{3}$

12. **(a)** $\frac{26}{45}$

Explanation: $0.5\overline{7}$ = $\frac{57-5}{90}$

= $\frac{52}{90}$ = $\frac{26}{45}$

13. **(a)** $\sqrt{\frac{1}{7}\times\frac{2}{7}}$

Explanation: An irrational number between a and b is given by $\sqrt{ab}$.

So, an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$ is $\sqrt{\frac{1}{7}\times\frac{2}{7}}$.

14. **(d)** $6^{1/4}$

Explanation: $\sqrt{2}$ and $\sqrt{3}$

= $2^{\frac{1}{2}}$ and $3^{\frac{1}{2}}$

= $2^{\frac{2}{4}}$ and $3^{\frac{2}{4}}$

= $4^{\frac{1}{4}}$ and $9^{\frac{1}{4}}$

irrational between $\sqrt{2}$ and $\sqrt{3}$ is $6^{1/4}$

15. **(c)** 0.4142

Explanation: Given $\sqrt{2}$ =1.4142

$\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$

= $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}}$

=$\sqrt{\frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-(1)^2}}$

=$\sqrt{\frac{(\sqrt{2}-1)^2}{2-1}}$

= $(\sqrt{2}-1)$

=1.4142-1

= 0.4142

16. **(b)** 4

Explanation: x = $\sqrt{5}$+ 2, then equals

$\frac{1}{x}$ = $\frac{1}{\sqrt{5}+2}$

=$\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$

= $\frac{\sqrt{5}-2}{5-4}$

= $\sqrt{5}-2$

now,

x - $\frac{1}{x}$ = $\sqrt{5}$+ 2 - ($\sqrt{5}-2$)

= $\sqrt{5}$ +2- $\sqrt{5}$ + 2

=4

17. **(d)** $\frac{7}{9}$

Explanation: $0.\overline{3}$ + $0.\overline{4}$

=$0.\overline{7}$= $\frac{7}{9}$

18. **(b)** 0.4472

Explanation: $\sqrt{5}$ = 2.236

So, $\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ = $\frac{2.236}{5}$

= 0.4472

19. **(c)** $\frac{33}{2}$

Explanation: g = $t^{\frac{2}{3}} + 4t^{\frac{-1}{2}}$

= $t^{\frac{2}{3}}$ + 4 $\times \frac{1}{t^{\frac{1}{2}}}$

= $(64)^{\frac{2}{3}}$ + 4 $\times \frac{1}{64^{\frac{1}{2}}}$

= $\left(4^3\right)^{\frac{2}{3}}$ + 4 $\times \frac{1}{(8^2)^{\frac{1}{2}}}$

= $4^{\frac{2}{3}\times 3}$ + 4 $\times \frac{1}{8^{2\times\frac{1}{2}}}$

= 4^2 + $\frac{4}{8}$

= 16 + $\frac{1}{2}$

= $\frac{33}{2}$

20. **(c)** $\frac{0}{4}$

Explanation: Since 0 is rational number it is in the form of $\frac{p}{q}$, and where q $\neq$ 0 as $\frac{0}{1}$

21. **(a)** 7

Explanation: $x = \frac{2}{3+\sqrt{7}}$

= $\frac{2}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}}$

= $\frac{2(3-\sqrt{7})}{(3)^2-(\sqrt{7})^2}$

= $\frac{2(3-\sqrt{7})}{9-7}$

= $\frac{2(3-\sqrt{7})}{2}$

= 3 - $\sqrt{7}$

Now $(x - 3)^2$ = $(3 - \sqrt{7} -3)^2$

= $(-\sqrt{7})^2$

=7

22. **(d)** 3

Explanation: $\left[(81)^{\frac{1}{2}}\right]^{\frac{1}{2}}$

= $\left(3^4\right)^{\frac{1}{2}\times\frac{1}{2}}$

= $\left(3^4\right)^{\frac{1}{4}}$

= 3

23. **(b)** $\sqrt{3}-\sqrt{5}$

Explanation: The simplest rationalising factor of $\sqrt{3}+\sqrt{5}$ is $\sqrt{3}-\sqrt{5}$

24. **(d)** 3

Explanation: $(243)^{1/5}$

$=(3^5)^{1/5}$
$=3$

25. **(b)** 125

Explanation: $\{2 - 3(2 - 3)^3\}^3$
$= \{2 - 3(-1)^3\}^3$
$= \{2 - 3 \times (-1)\}^3$
$= (2 + 3)^3$
$= (5)^3$
$= 125$

26. -1
Explanation:
$2^{x-3} = 4^{x-1}$
$\Rightarrow 2^{x-3} = (2^2)^{x-1}$
$\Rightarrow 2^{x-3} = 2^{2x-2}$
$\Rightarrow x - 3 = 2x - 2$ [Equating the exponent]
$\Rightarrow x - 2x = -2 + 3$
$\Rightarrow -x = +1$
$\Rightarrow x = -1$

27. 16
Explanation:
$(x - 1)^3 = 8$
$(x + 1)^3 = 2^3$
Comparing both sides, we get
$x - 1 = 2 \Rightarrow x = 2 + 1 = 3$
$\therefore x = 3$
Now $(x + 1)^2 = (3 + 1)^2 = (4)^2 = 16$

28. 14
Explanation:
It is given that , $x = 2 + \sqrt{3}$
$\therefore \frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$
Now, $x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2$
$\Rightarrow x^2 + \frac{1}{x^2} = (2 + \sqrt{3} + 2 - \sqrt{3})^2 - 2 = 4^2 - 2 = 16 - 2 = 14.$

29. 4
Explanation:
We have,
$(64)^{\frac{1}{3}} = \left(4^3\right)^{\frac{1}{3}} = 4^{3\times\frac{1}{3}}$
$= 4^1 = 4$ [$\because (a^m)^n = a^{mn}$]

30. 5
Explanation:
We have,
$5^{x-3} \times 3^{2x-8} = 225$
$\Rightarrow 5^{x-3} \times 3^{2x-8} = 25 \times 9$
$\Rightarrow 5^{x-3} \times 3^{2x-8} = 5^2 \times 3^2$
$\Rightarrow x - 3 = 2$ and $2x - 8 = 2$ [Comparing and equating the exponents on both sides]

$\Rightarrow$ x = 5
Thus, required value of x is 5.

31. 2
Explanation:
We know that: $\sqrt[n]{a} \times \sqrt[n]{b}$ = $\sqrt[n]{ab}$
$\therefore$ $\sqrt[5]{16} \times \sqrt[5]{2}$ = $\sqrt[5]{16 \times 2}$ = $\sqrt[5]{32}$
= $\sqrt[5]{2^5}$ = $\left(2^5\right)^{\frac{1}{5}}$
= 2^1 = 2

32. 6
Explanation:
Given,
$\frac{(2\sqrt{45}+3\sqrt{20})}{2\sqrt{5}} = \frac{(2\sqrt{45}+3\sqrt{20})}{2\sqrt{5}} \times \frac{2\sqrt{5}}{2\sqrt{5}}$ (Rationlising the denominator)
$= \frac{2\sqrt{5}(2\sqrt{45}+3\sqrt{20})}{20}$
$= \frac{4\sqrt{45\times5}+6\sqrt{20\times5}}{20}$
$= \frac{4\sqrt{3^2\times5^2}+6\sqrt{2^2\times5^2}}{20}$
$= \frac{4(3\times5)+6(2\times5)}{20}$
$= \frac{60+60}{20}$
= 6.

33. 46
Explanation:
(8 + $3\sqrt{2}$)(8 - $3\sqrt{2}$)
= $(8)^2 - \left(3\sqrt{2}\right)^2$ [$\because$ (a + b)(a - b) = a^2 - b^2]
= 64 - 18 = 46

34. 3
Explanation:
We have,
$3^{3/4} \times 3^{1/4} = 3^{(3/4 + 1/4)}$
= 3^1 = 3 [$\because$ $a^m \times a^n = a^{(m+n)}$]

35. 18
Explanation:
It is given that,
(5 + $\sqrt{7}$)(5 - $\sqrt{7}$)
= $(5)^2$ - $(\sqrt{7})^2$ [Using Identity, (a + b)(a - b) = a^2 - b^2]
= 25 - 7
= 18
Thus, (5 + $\sqrt{7}$)(5 - $\sqrt{7}$) = 18.

36. Given, $\frac{\sqrt{25}}{\sqrt[3]{64}} + \left(\frac{256}{625}\right)^{-1/4} + \frac{1}{\left(\frac{64}{125}\right)^{2/3}}$
$= \frac{\sqrt{5\times5}}{\sqrt[3]{4\times4\times4}} + \left(\frac{625}{256}\right)^{1/4} + \left(\frac{125}{64}\right)^{2/3}$
$= \frac{5}{4} + \left(\frac{5^4}{4^4}\right)^{1/4} + \left(\frac{5^3}{4^3}\right)^{2/3}$
$= \frac{5}{4} + \left(\frac{5}{4}\right)^{4\times\frac{1}{4}} + \left(\frac{5}{4}\right)^{3\times\frac{2}{3}}$

$= \frac{5}{4} + \frac{5}{4} + \left(\frac{5}{4}\right)^2 = \frac{5}{4} + \frac{5}{4} + \frac{25}{16}$

$= \frac{20+20+25}{16} = \frac{65}{16}$

37. Given,

$\sqrt{2}$ = 1.4142

Now, $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}}$ [by rationalising]

$= \sqrt{\frac{(\sqrt{2}-1)^2}{2-1}} = \frac{\sqrt{(\sqrt{2}-1)^2}}{1}$ [∵(a + b)(a - b) = a^2 - b^2]

= $\sqrt{2}$ - 1 = 1.4142 - 1 [∵ $\sqrt{2}$ = 1.4142]

= 0.4142

38. $x = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

= $\frac{(\sqrt{2}+1)}{(\sqrt{2}-1)} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$

= $= \frac{(\sqrt{2})^2+(1)^2+2\times\sqrt{2}\times1}{(\sqrt{2})^2-(1)^2}$

= $\frac{2+1+2\sqrt{2}}{(2-1)}$

= $3 + 2\sqrt{2}$

$y = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

= $\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}$

= $\frac{(\sqrt{2})^2+(1)^2-2\times\sqrt{2}\times1}{(\sqrt{2})^2-(1)^2}$

= $\frac{2+1-2\sqrt{2}}{(2-1)}$

= $3 - 2\sqrt{2}$

∴$x^2 + y^2 + xy = (x + y)^2 - xy$

$= [(3+\sqrt{2}) + (3-\sqrt{2})]^2 - (3+2\sqrt{2})(3-2\sqrt{2})$

= $(6)^2 - [(3)^2 - (2\sqrt{2})^2]$ = 36 - (9 - 8)

= 36 - 1

= 35

39. $(256)^{-\left(4^{-\frac{3}{2}}\right)} = \left(2^8\right)^{-\left(4^{-\frac{3}{2}}\right)} = \left(2^8\right)^{-\left(2^{2\times-\frac{3}{2}}\right)} = \left(2^8\right)^{-\left(2^{-3}\right)}$

$= \left(2^8\right)^{-\left(\frac{1}{8}\right)} = 2^{8\times\left(-\frac{1}{8}\right)} = 2^{-1} = \frac{1}{2}$

40. We have $a = 2 + \sqrt{3}$

$\therefore\ \frac{1}{a} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$= \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2 - \sqrt{3}$

$\therefore\ a - \frac{1}{a} = 2 + \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3}$

41. We have $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$

$= 4(216)^{\frac{2}{3}} + (256)^{\frac{3}{4}} + 2(243)^{\frac{1}{5}}$

$= 4(6^3)^{\frac{2}{3}} + (4^4)^{\frac{3}{4}} + 2(3^5)^{\frac{1}{5}}$

$= 4 \times 6^{3\times\frac{2}{3}} + 4^{4\times\frac{3}{4}} + 2 \times 3^{5\times\frac{1}{5}}$

$= 4 \times 6^2 + 4^3 + 2 \times 3$

= 144 + 64 + 6 = 214

42. Given, $\frac{1}{(\sqrt{7}+\sqrt{3})-\sqrt{2}}$

$= \frac{1}{(\sqrt{7}+\sqrt{3})-\sqrt{2}} \times \frac{(\sqrt{7}+\sqrt{3})+\sqrt{2}}{(\sqrt{7}+\sqrt{3})+\sqrt{2}}$

$= \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{(\sqrt{7}+\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{(\sqrt{7})^2+(\sqrt{3})^2+2\sqrt{21}-2}$

$= \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{7+3+2\sqrt{21}-2}$

$= \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{8+2\sqrt{21}}$

$= \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{2(4+\sqrt{21})} \times \frac{4-\sqrt{21}}{4-\sqrt{21}}$

$= \frac{4\sqrt{7}+4\sqrt{3}+4\sqrt{2}-7\sqrt{3}-3\sqrt{7}-\sqrt{42}}{2(16-21)}$

$= \frac{\sqrt{7}-3\sqrt{3}+4\sqrt{2}-\sqrt{42}}{-10}$

$= \frac{3\sqrt{3}-4\sqrt{2}+\sqrt{42}-\sqrt{7}}{10}$

43. $\frac{1}{\sqrt{7}-2}$

$= \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$

$= \frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2} = \frac{\sqrt{7}+2}{7-4}$

$= \frac{\sqrt{7}+2}{3}$

44. Given, $a = \frac{2+\sqrt{5}}{2-\sqrt{5}}$

$\Rightarrow a = \frac{2+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}}$

$= \frac{(2+\sqrt{5})^2}{2^2-(\sqrt{5})^2}$

$= \frac{4+5+4\cdot\sqrt{5}}{4-5}$

$= -(9+4\sqrt{5})$

Also, $b = \frac{2-\sqrt{5}}{2+\sqrt{5}}$

$= \frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$

$= \frac{(2-\sqrt{5})^2}{2^2-(\sqrt{5})^2}$

$= \frac{2^2+(\sqrt{5})^2-2.2\sqrt{5}}{4-5}$

$= \frac{4+5-4\sqrt{5}}{-1}$

= -(9-4$\sqrt{5}$)

= 4$\sqrt{5}$-9

We know, $a^2 - b^2 = (a + b)(a - b)$

Here, a + b = -9 -4 $\sqrt{5}$ + 4$\sqrt{5}$-9= - 18

a - b = -9 -4$\sqrt{5}$ - (4$\sqrt{5}$ -9) = -8$\sqrt{5}$

Hence, $a^2 - b^2$ = -18 (-8$\sqrt{5}$)

= 144$\sqrt{5}$

45. $\frac{\sqrt{5}-2}{\sqrt{5}+2} - \frac{\sqrt{5}+2}{\sqrt{5}-2}$

$= \frac{(\sqrt{5}-2)(\sqrt{5}-2)-(\sqrt{5}+2)(\sqrt{5}+2)}{(\sqrt{5}+2)(\sqrt{5}-2)}$

$= \frac{(\sqrt{5}-2)^2-(\sqrt{5}+2)^2}{(\sqrt{5})^2-(2)^2}$

$= \frac{\left\{(\sqrt{5})^2-2(\sqrt{5})(2)+(2)^2\right\}-\left\{(\sqrt{5})^2+2(\sqrt{5})(2)+(2)^2\right\}}{5-4}$

$= \frac{(5-4\sqrt{5}+4)-(5+4\sqrt{5}+4)}{1} = \frac{-8\sqrt{5}}{1}$

46. $3\sqrt[3]{250} + 7\sqrt[3]{16} - 4\sqrt[3]{54}$

$3\sqrt[3]{5 \times 5 \times 5 \times 2} + 7\sqrt[3]{2 \times 2 \times 2 \times 2} - 4\sqrt[3]{3 \times 3 \times 3 \times 2}$

$3 \times 5\sqrt[3]{2} + 7 \times 2\sqrt[3]{2} - 4 \times 3\sqrt[3]{2}$

$15\sqrt[3]{2} + 14\sqrt[3]{2} - 12\sqrt[3]{2}$

$(15 + 14 - 12)\sqrt[3]{2}$

$= 17\sqrt[3]{2}$

47. $3^{2x+4} + 1 = 2 \times 3^{x+2}$

$\Rightarrow 3^{2x} \cdot 3^4 + 1 = 2 \cdot 3^x \cdot 3^2$

$\Rightarrow (3^x)^2 \times 81 + 1 = 18.3^x$

$\Rightarrow 81(3^x)^2 - 18.3^x + 1 = 0$

Let $3^x = a$

Thus, we have $81a^2 - 18a + 1 = 0$

$\Rightarrow 81a^2 - 9a - 9a + 1 = 0$

$\Rightarrow (9a)^2 - 2(9a)(1) + (1)^2 = 0$

$\Rightarrow (9a - 1)^2 = 0$

$\Rightarrow 9a - 1 = 0$

$\Rightarrow 9a = 1$

$\Rightarrow a = \frac{1}{9}$

$\Rightarrow 3^x = \frac{1}{9}$

$\Rightarrow 3^x = \frac{1}{3^2} = 3^{-2}$

$\Rightarrow x = -2$

48. LHS $= \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

$= \frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2}$

$= \frac{6+2\sqrt{6}+3\sqrt{6}+6}{18-12}$

$= \frac{12+5\sqrt{6}}{6} = 2 + \frac{5\sqrt{6}}{6}$

Now, $a - b\sqrt{6} = 2 + \frac{5}{6}\sqrt{6}$

a = 2

b = $-\frac{5}{6}$

49. $\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}$

$= \frac{(4+\sqrt{5})(4+\sqrt{5})+(4-\sqrt{5})(4-\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})}$

$= \frac{(4+\sqrt{5})^2+(4-\sqrt{5})^2}{(4-\sqrt{5})(4+\sqrt{5})}$

$= \frac{\left\{(4)^2+2(4)(\sqrt{5})+(\sqrt{5})^2\right\}+\left\{(4)^2-2(4)(\sqrt{5})+(\sqrt{5})^2\right\}}{(4)^2-(\sqrt{5})^2}$

$= \frac{(16+8\sqrt{5}+5)+(16-8\sqrt{5}+5)}{16-5} = \frac{42}{11}$

50. Let x $= 0.\overline{38}$

i.e. x = 0.3838…. ….(i)

Multiply eq. (i) by 100 we get,

$\Rightarrow$ 100x = 38.3838…. ….(ii)

On subtracting eq. (i) from (ii), we get

100x - x = 38.3838... - 0.3838...

99x = 38

$\Rightarrow x = \frac{38}{99}$

Let y $= 1.\overline{27}$

i.e. y = 1.2727….(iii)

Multiply eq. (i) by 100 we get,

$\Rightarrow$ 100y = 127.2727…(iv)

On subtracting (iii) from (iv), we get

100y-y = 127.2727.... - 1.2727....

99y = 126

$\Rightarrow y = \frac{126}{99}$

$\therefore x + y = 0.\overline{38} + 1.\overline{27}$

$= \frac{38}{99} + \frac{126}{99}$

$= \frac{38+126}{99}$
$= \frac{164}{99}$

51. Given

$\sqrt[3]{2} = 2^{\frac{1}{3}}$; $\sqrt{3} = 3^{\frac{1}{2}}$; $\sqrt[6]{5} = 5^{\frac{1}{6}}$

LCM of 3,2 and 6 = 6

$\therefore \quad 2^{\frac{1}{3}} = 2^{\left(\frac{1}{3}\times\frac{2}{2}\right)} = 2^{\frac{2}{6}} = \left(2^2\right)^{\frac{1}{6}} = 4^{\frac{1}{6}}$

$3^{\frac{1}{2}} = 3^{\left(\frac{1}{2}\times\frac{3}{3}\right)} = 3^{\frac{3}{6}} = \left(3^3\right)^{\frac{1}{6}} = (27)^{\frac{1}{6}}$

Clearly, $(27)^{\frac{1}{6}} > 5^{\frac{1}{6}} > 4^{\frac{1}{6}}$

So, $3^{\frac{1}{2}} > 5^{\frac{1}{6}} > 2^{\frac{1}{3}}$

or $\sqrt{3} > \sqrt[6]{5} > \sqrt[3]{2}$

Hence, the correct descending order is $\sqrt{3}$, $\sqrt[6]{5}$ and $\sqrt[3]{2}$.

52. We have $a = \frac{3+\sqrt{5}}{2}$

$\Rightarrow \ a^2 = \frac{(3+\sqrt{5})^2}{4}$

$= \frac{9+5+6\sqrt{5}}{4} = \frac{14+6\sqrt{5}}{4} = \frac{7+3\sqrt{5}}{2}$

Now, $\frac{1}{a^2} = \frac{2}{7+3\sqrt{5}} = \frac{2}{7+3\sqrt{5}} \times \frac{7-3\sqrt{5}}{7-3\sqrt{5}} = \frac{2(7-3\sqrt{5})}{(7)^2-(3\sqrt{5})^2}$

$= \frac{2(7-3\sqrt{5})}{49-45} = \frac{2(7-3\sqrt{5})}{4} = \frac{7-3\sqrt{5}}{2}$

$\therefore a^2 + \frac{1}{a^2} = \frac{7+3\sqrt{5}}{2} + \frac{7-3\sqrt{5}}{2}$

$= \frac{7+3\sqrt{5}+7-3\sqrt{5}}{2} = \frac{14}{2} = 7$

53. We know that

$a = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$.

$= \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$= \frac{(\sqrt{5}+\sqrt{2})^2}{\sqrt{5}^2-\sqrt{2}^2}$

$= \frac{5+2\sqrt{10}+2}{5-2}$

$= \frac{7+2\sqrt{10}}{3}$

We know that

$\mathrm{b} = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$= \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{(\sqrt{5}-\sqrt{2})^2}{\sqrt{5}^2-\sqrt{2}^2}$

$= \frac{5-2\sqrt{10}+2}{5-2}$

$= \frac{7-2\sqrt{10}}{3}$

$a^2 = \left(\frac{7+2\sqrt{10}}{3}\right)^2 = \frac{49+28\sqrt{10}+40}{9} = \frac{89+28\sqrt{10}}{9}$

$\mathrm{b}^2 = \left(\frac{7-2\sqrt{10}}{3}\right)^2 = \frac{49-28\sqrt{10}+40}{9} = \frac{89-28\sqrt{10}}{9}$

LHS = $3a^2 + 4ab - 3b^2$

Substituting the values, we get

$= 3 \times \frac{89+28\sqrt{10}}{9} + 4 \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} - 3 \times \frac{89-28\sqrt{10}}{9}$

$= \frac{89+28\sqrt{10}}{3} + 4 - \frac{89-28\sqrt{10}}{3}$

$= 4 + \frac{89+28\sqrt{10}-89+28\sqrt{10}}{3}$

$= 4 + \frac{56}{3}\sqrt{10}$

= RHS

54. $a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{(\sqrt{3}-\sqrt{2})^2}{\sqrt{3}^2-\sqrt{2}^2}$

$= \frac{3+2-2\sqrt{6}}{3-2}$
$= 5 - 2\sqrt{6}$
$b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$
$= \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$
$= \frac{(\sqrt{3}+\sqrt{2})^2}{\sqrt{3}^2-\sqrt{2}^2}$
$= \frac{3+2+2\sqrt{6}}{3-2}$
$= 5 + 2\sqrt{6}$
$a^2 = (5-2\sqrt{6})^2 = 25 - 20\sqrt{6} + 24 = 49 - 20\sqrt{6}$
$b^2 = (5+2\sqrt{6})^2 = 25 + 20\sqrt{6} + 24 = 49 + 20\sqrt{6}$

So $a^2 + b^2 - 5ab$

Substituting the values we get

$= (49 - 20\sqrt{6}) + (49 + 20\sqrt{6}) - 5 \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

= 98 - 5

= 93

55. We know that,

$x = \frac{5-\sqrt{3}}{5+\sqrt{3}}$
$= \frac{5-\sqrt{3}}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$
$= \frac{(5-\sqrt{3})^2}{5^2-\sqrt{3}^2}$
$= \frac{25-10\sqrt{3}+3}{25-3}$
$= \frac{28-10\sqrt{3}}{22}$
$= \frac{14-5\sqrt{3}}{11}$

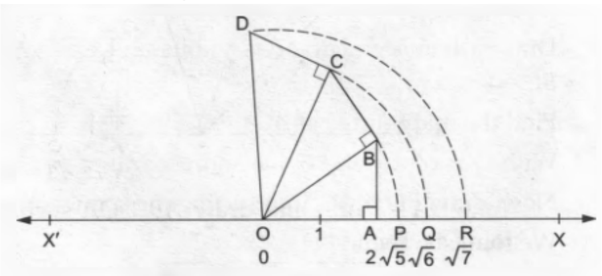
Again,

$y = \frac{5+\sqrt{3}}{5-\sqrt{3}}$
$= \frac{5+\sqrt{3}}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}}$
$= \frac{(5+\sqrt{3})^2}{5^2-\sqrt{3}^2}$
$= \frac{25+10\sqrt{3}+3}{25-3}$
$= \frac{28+10\sqrt{3}}{22}$
$= \frac{14+5\sqrt{3}}{11}$

x-y $= \frac{14-5\sqrt{3}}{11} - \frac{14+5\sqrt{3}}{11}$
$= \frac{14-5\sqrt{3}-14-5\sqrt{3}}{11}$
$= \frac{-10\sqrt{3}}{11}$

56. Draw a horizontal line X'OX, taken as the x-axis.

Take O as the origin to represent 0.



Let OA =2 units and let AB $\perp$OA such that AB = 1 unit

Join OB. Then, by Pythagoras Theorem

$OB = \sqrt{OA^2 + AB^2}$
$=\sqrt{2^2 + 1^2}$
$= \sqrt{5}$

With O as centre and OB as radius, draw an arc, meeting OX at P.

Then, OP = OB = $\sqrt{5}$

Thus, P represents $\sqrt{5}$ or the real line.

Now, draw BC $\perp$OB and set off BC = 1 unit.

Join OC. Then, by Pythagoras Theorem

$OC = \sqrt{OB^2 + BC^2} = \sqrt{(\sqrt{5})^2 + 1^2} = \sqrt{6}$

With O as centre and OC as radius, draw an arc, meeting OX at Q

Then, OQ = OC = $\sqrt{6}$

Thus, Q represents $\sqrt{6}$ on the real line.

Now, draw CD $\perp$ OC and set off CD = 1 unit.

Join OD. Then,by Pythagoras Theorem

$OD = \sqrt{OC^2 + CD^2} = \sqrt{(\sqrt{6})^2 + 1^2} = \sqrt{7}$

With O as centre and OD as radius, draw an arc, meeting OX at R. Then

OR = OD =$\sqrt{7}$

Thus, the points P, Q, R represent the real numbers $\sqrt{5}$, $\sqrt{6}$ and $\sqrt{7}$ respectively

57. Given.

$\left(\frac{x^b}{x^c}\right)^{b+c-a} \cdot \left(\frac{x^c}{x^a}\right)^{c+a-b} \cdot \left(\frac{x^a}{x^b}\right)^{a+b-c}$

$= \left(\frac{x^{b^2+bc-ab}}{x^{bc+c^2-ac}}\right) \cdot \left(\frac{x^{c^2+ac-bc}}{x^{ac+a^2-ab}}\right) \cdot \left(\frac{x^{a^2+ab-ac}}{x^{ab+b^2-bc}}\right)$

$= \left(x^{b^2+bc-ab-bc-c^2+ac}\right)\left(x^{c^2+ac-bc-ac-a^2+ab}\right)\left(x^{a^2+ab-ac-ab-b^2+bc}\right)$

$= \left(x^{b^2-ab-c^2+ac}\right)\left(x^{c^2-bc-a^2+ab}\right)\left(x^{a^2-ac-b^2+bc}\right)$

$= x^{b^2-ab-c^2+ac+c^2-bc-a^2+ab+a^2-ac-b^2+bc}$

= x^0

= 1

58. Given, $a = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $b = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

Here, $a = \frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2})^2-1^2}$

$= \frac{(\sqrt{2})^2+1+2\sqrt{2}}{2-1} = \frac{2+1+2\sqrt{2}}{1} = 3 + 2\sqrt{2}$

$\therefore \quad a = 3 + 2\sqrt{2}$ ...(i)

$b = \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-1}$

$= \frac{(\sqrt{2})^2+1^2-2\sqrt{2}}{2-1} = \frac{2+1-2\sqrt{2}}{1} = 3 - 2\sqrt{2}$

$\therefore \quad b = 3 - 2\sqrt{2}$ ...(ii)

From equation (i) and (ii)

$a + b = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$

ab = $(3 + 2\sqrt{2})(3 - 2\sqrt{2}) = 3^2 - (2\sqrt{2})^2$

$= 9 - 4 \times 2 = 9 - 8 = 1$

$\therefore a^2 + b^2 - 4ab = a^2 + b^2 + 2ab - 6ab$

$= (a + b)^2 - 6ab$

$= 6^2 - 6$

$= 36 - 6 = 30$

59. We know that

$\frac{9^n \times 3^2 \times \left(3^{-\frac{n}{2}}\right)^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{(3^2)^n \times 3^2 \times \left(3^{-\frac{n}{2}}\right)^{-2} - (3^3)^n}{3^{3m} \times 2^3} = \frac{1}{3^3}$

$\Rightarrow \frac{(3)^{2n} \times 3^2 \times 3^{\frac{n}{2} \times 2} - (3)^{3n}}{3^{3m} \times 2^3} = \frac{1}{3^3}$

$\Rightarrow \frac{(3)^{2n+2} \times 3^n - (3)^{3n}}{3^{3m} \times 2^3} = \frac{1}{3^3}$

$\Rightarrow \frac{(3)^{2n+2+n}-(3)^{3n}}{3^{3m}\times 2^3} = \frac{1}{3^3}$

$\Rightarrow \frac{(3)^{3n+2}-(3)^{3n}}{3^{3m}\times 2^3} = \frac{1}{3^3}$

$\Rightarrow 3^3 \times [(3)^{3n+2} - (3)^{3n}] = 3^{3m} \times 2^3$

$\Rightarrow 3^{3+3n} \times [(3)^2 - 1] = 3^{3m} \times 2^3$

$\Rightarrow 3^{3+3n} \times [8] = 3^{3m} \times 2^3$

$\Rightarrow 3^{3+3n} \times 2^3 = 3^{3m} \times 2^3$

$\Rightarrow 3^{3+3n} = 3^{3m}$

$\Rightarrow$ 3+3n = 3m

$\Rightarrow$ 3m - 3n = 3

$\Rightarrow$ m - n = 1

60. Given, $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

$= \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$

$= \frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{(\sqrt{10})^2-(\sqrt{3})^2} - \frac{2\sqrt{30}-2\times 5}{(\sqrt{6})^2-(\sqrt{5})^2} - \frac{3\sqrt{30}-18}{(\sqrt{15})^2-(3\sqrt{2})^2}$

$= \frac{7(\sqrt{30}-3)}{10-3} - \frac{(2\sqrt{30}-10)}{6-5} - \frac{3\sqrt{30}-18}{15-18}$

$= \sqrt{30} - 3 - (2\sqrt{30} - 10) - (6 - \sqrt{30})$

$= \sqrt{30} - 3 - 2\sqrt{30} + 10 - 6 + \sqrt{30}$

$= 10 - 9 + 2\sqrt{30} - 2\sqrt{30} = 1$