QUESTION BANK (RELATIONS AND FUNCTIONS)

Class 12 - Mathematics

1. Let A = {1, 2, 3} and let R = {(1,1), (2, 2), (3, 3), (1, 3), (3, 2), (1, 2)}. Then, R is **[1]**

a) symmetric and transitive but not reflexive
b) reflexive and transitive but not symmetric
c) reflexive and symmetric but not transitive
d) an equivalence relation

2. Which of the following functions from Z into Z are bijections? **[1]**

a) $f(x) = x^3$
b) f (x) = 2x + 1
c) f (x) = x + 2
d) $f(x) = x^2 + 1$

3. If A = {a, b, c, d}, then a relation R = {(a, b), (b, a), (a, a)} on A is **[1]**

a) none of these
b) symmetric only
c) symmetric and transitive only
d) reflexive and transitive only

4. let A = R - (3) and B = R - 1 Then, f : A $\rightarrow B : f(x) = \frac{(x-2)}{(x-3)}$ is **[1]**

a) many-one and onto
b) many-one and into
c) one-one and onto
d) one-one and into

5. Which of the following is not an equivalence relation on Z? **[1]**

a) a R b $\Leftrightarrow$ a - b is an even integer
b) a R b $\Leftrightarrow$ a + b is an even integer
c) a R b $\Leftrightarrow$ a < b
d) a R b $\Leftrightarrow$ a = b

6. For real numbers x and y, define xRy if and only if x – y + $\sqrt{2}$ is an irrational number. Then the relation R is **[1]**

a) none of these
b) reflexive
c) transitive
d) symmetric

7. Let f : $N \rightarrow N : f(n) = \begin{cases} \frac{1}{2}(n+1), & \text{when } n \text{ is odd} \\ \frac{n}{2}, & \text{when } n \text{ is even.} \end{cases}$ **[1]**

then, f is

a) many-one and onto
b) one-one and into
c) many-one and into
d) one-one and onto

8. S is a relation over the set R of all real numbers and its is given by (a, b) $\in$ S $\Leftrightarrow$ ab $\geq$ 0. Then, S is **[1]**

a) an equivalence relation
b) reflexive and symmetric only
c) symmetric and transitive only
d) antisymmetric relation

9. Let A = {1, 2, 3} and let R = {(1,1), (2, 2), (3, 3), (1, 2), (2,1), (2, 3), (3, 2)}. Then, R is **[1]**

a) reflexive and transitive but not symmetric
b) an equivalence relation

c) symmetric and transitive but not reflexive
d) reflexive and symmetric but not transitive

10. Let f : Z $\rightarrow$ Z be given by $f(x) = \begin{cases} \frac{x}{2}, \text{ if } x \text{ is even} \\ 0, \text{ if } x \text{ is odd} \end{cases}$. Then f is **[1]**

a) onto but not one-one
b) neither one-one nor onto
c) one-one but not onto
d) one-one and onto

11. Let A = {1, 2, 3,… n} and B = {a, b}. Then the number of surjections from A to B is **[1]**

a) ${}^nP_2$
b) none of these
c) $2^n - 2$
d) $2^n - 1$

12. Let $f : R \rightarrow R$ be defined as f (x) = 3x. Choose the correct answer. **[1]**

a) many – one onto
b) neither one – one nor onto
c) one – one but not onto
d) one – one onto

13. The function f : A $\rightarrow$ B defined by f(x) = $-x^2 + 6x - 8$ is a bijection, if **[1]**

a) $A = (-\infty, 3]$ and $B = (-\infty, 1]$
b) $A = [-3, \infty)$ and $B = (-\infty, 1]$
c) $A = (-\infty, 3]$ and $B = [1, \infty)$
d) $A = [3, \infty)$ and $B = |1, \infty)$

14. The relation S defined on the set R of all real number by the rule a S b if a $\geq$ b is **[1]**

a) neither transitive nor reflexive but symmetric
b) symmetric, transitive but not reflexive
c) an equivalence relation
d) reflexive, transitive but not symmetric

15. Let S be the set of all straight lines in a plane. Let R be a relation on S defined by $LRM \Leftrightarrow L \perp M$ Then R is **[1]**

a) transitive but neither reflexive nor symmetric
b) an equivalence relation
c) reflexive but neither symmetric nor transitive
d) symmetric but neither reflexive nor transitive

16. Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is **[1]**

a) 4
b) 2
c) 1
d) 3

17. A relation R on the set N of natural numbers is defined as R = {(a, b): a + b is even, $\forall$ a, b $\in$ N}, then R is **[1]**

a) a reflexive relation but not symmetric
b) an equivalence relation
c) symmetric but not transitive
d) not an equivalence relation

18. If the set A contains 5 elements and the set B contains 6 elements, then the number of one – one and onto mappings from A to B is **[1]**

a) none of these
b) 720
c) 120
d) 0

19. Let R be a relation on the set N of natural numbers defined by nRm if n divides m. Then R is **[1]**

a) Transitive and symmetric
b) Reflexive and symmetric

c) Reflexive, transitive but not symmetric d) Equivalence

20. The relation R in N × N such that (a, b) R (c, d) ⇔ a + d = b + c is **[1]**

a) reflexive and transitive but not symmetric b) an equivalence relation

c) reflexive but symmetric d) none of these

21. Give an example of a function which is one-one but not onto. **[1]**

22. Give an example of a function which is neither one-one nor onto. **[1]**

23. Determine whether the below relation is reflexive, symmetric and transitive: **[1]**

Relation R in the set A of human beings in a town at a particular time given by

R = {(x, y) : x and y work at the same place}

24. State the reason for the relation R on the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive. **[1]**

25. Given A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of a mapping from A to B which is not injective. **[1]**

26. Let A = {0, 1, 2, 3} and define a relation R on A as follows: **[1]**

R = {(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)}

Is R reflexive? symmetric? transitive?

27. Find whether function f defined as below is one-one or not: **[1]**

f : R → R given by f(x) = x^3 + 2 for all x ∈ R.

28. Give an example of a function which is one-one and onto. **[1]**

29. Determine whether the relation is reflexive, symmetric and transitive: **[1]**

Relation R in the set A of human beings in a town at a particular time given by

R = {(x, y) : x is wife of y}

30. Let R be the equivalence relation in the set Z of integers given by R = {(a, b) : 2 divides a– b}. Write the equivalence class[0]. **[1]**

31. Let A be the set of all students of a boys school. Show that the relation R in A given by R = {(a, b) : a is a sister of b} is the empty relation and R′ = {(a, b) : the difference between heights of a and b is less than 3 meters} is the universal relation. **[1]**

32. If A = { 1, 2, 3}, B = { 4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one. **[1]**

33. Show that the function f : R → R: f(x) = x^3 is one-one and onto **[1]**

34. Let the function f : R → R be defined by f(x) = 4x - 1, ∀ x ∈ R. Then, show that f is one-one. **[1]**

35. Show that the Signum function f :R →R, given by **[1]**

$$f(x)=\begin{cases}1, if\, x>0\\ 0, if\, x=0\\ -1, if\, x<0\end{cases}$$

is neither one-one nor onto.

36. Show that the function f : R → R: f(x) = 1 + x^2 is many-one into. **[1]**

37. Let A be the set of all 50 students of Class X in a school. Let f : $A \rightarrow N$ be function defined by f(x) = roll number of the student x. Show that f is one-one but not onto. **[1]**

38. Write the smallest reflexive relation of set A = {1, 2, 3, 4} **[1]**

39. Let S = {a, b, c} and T = {1, 2, 3}. Find F^{-1} of the function F from S to T, if it exists. **[1]**

F = {(a, 3), (b, 2), (c, 1)}

40. If A = {a, b, c, d} and f = {a, b), (b, d), (c, a), (d, c)}, show that f is one-one from A onto A. Find f^{-1} **[1]**

41. State whether the function is one-one, onto or bijective. Justify your answer. f: R $\rightarrow$ R defined by f(x) = 1+ x^2 **[2]**

42. Show that the number of equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) is two. **[2]**

43. Let A = {1, 2, 3}. Then show that the number of relations containing (1, 2) and (2, 3) which are reflexive and transitive but not symmetric is three. **[2]**

44. Prove that the function f : $R \rightarrow R : f(x) = 2x$ is one-one and onto. **[2]**

45. Let f : R $\rightarrow$ R be the Signum Function defined as $\begin{cases} 1, \ x > 0 \\ 0, \ x = 0 \\ -1, \ x < 0 \end{cases}$ and g : R $\rightarrow$ R be the Greatest Function given by g(x) = [x], where [x] is greatest integer less than or equal to x. Then, does fog and gof coincide in (0, 1)? **[2]**

46. Show that the function f : N $\rightarrow$ N : f(x) = x^3 is one-one into. **[2]**

47. Let C be the set of complex numbers. Prove that the mapping f : C $\rightarrow$ R given by f(z) = |z|, $\forall$z $\in$ C, is neither one-one nor onto. **[2]**

48. Show that the function f : N $\rightarrow$ N : f(x) = x^2 is one one into **[2]**

49. Show that the function f : $N \rightarrow N : f(x) = x^2$ is one-one and into. **[2]**

50. Show that the relation is congruent to on the set of all triangles in a plane is an equivalence relation. **[2]**

51. Let Z be the set of all integers and let R be a relation on Z defined by R = (a, b) : (a - b) is even) Show that R is an equivalence relation in Z. **[2]**

52. Classify the function f : N $\rightarrow$ N given by f (x) = x^3 as injection, surjection or bijection. **[2]**

53. Let A = R – {3} and B = R- {1}. Consider the function of f: A $\rightarrow$ B defined by $f(x) = \frac{x-2}{x-3}$ . .Is f one – one and onto. **[2]**

54. Check the injectivity and surjectivity of the below function: **[2]**

    f : R $\rightarrow$ R given by f(x) = x^2

55. Show that the function $f : R \rightarrow R : f(x) = x^4$ is neither one-one nor onto. **[2]**

56. State whether the function is one – one, onto or bijective f: R to R defined by f(x) = 3 – 4x **[3]**

57. Let R = (a,b) : a, b $\in$ Z and (a - b) is divisible by 5 show that R is an equivalence relation on Z. **[3]**

58. Let R = {(a, b): a, b $\in$ N, a > b}. **[3]**

    Show that R is a binary relation which is neither reflexive, nor symmetric. Show that R is transitive.

59. If N denotes the set of all natural numbers and R be the relation on N $\times$ N defined by (a, b) R (c, d), if ad(b + c) = bc(a + d). Show that R is an equivalence relation. **[3]**

60. Let L be the set of all lines in plane and R be the relation in L define if R = {(l_1, L_2): L_1 is$\perp$ to L_2 } .Show that R is symmetric but neither reflexive nor transitive. **[3]**

61. On the set Z of all integers, consider the relation R = {{a, b) : (a - b) is divisible by 3}. Show that R is an equivalence relation on Z Also find the partitioning of Z into mutually disjoint equivalence classes. **[3]**

62. In the set of natural numbers N, define a relation R as follows: **[3]**

    $\forall$n, m $\in$ N, nRm if on division by 5 each of the integers n and m leaves the remainder less than 5, i.e. one of the numbers 0, 1, 2, 3 and 4. Show that R is equivalence relation. Also, obtain the pairwise disjoint subsets determined by R.

63. Let X be a nonempty set and let S be the collection of all subsets of X. Let R be a relation in S, defined by $R = \{(A,B) : A \subset B\}$ Show that R is transitive but neither reflexive nor symmetric **[3]**

64. If R is a relation defined on the set of natural numbers N as follows: R={(x,y) : x $\in$ N, y $\in$ N and 2x+ y = 24}, then find the domain and range of the relation R. Also, find whether R is an equivalence relation or not. **[3]**

65. Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2) : T_1 is similar to T_2}, is **[3]**

equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

66. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R. **[5]**

67. Let f : W → W be defined as f(n) = n - 1, if n is odd and f(n) = n + 1, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers. **[5]**

68. Let R be relation defined on the set of natural number N as follows: **[5]**
R = {(x, y): x ∈ N, y ∈ N, 2x + y = 41}. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

69. Let A = R – {3} and B = R – {1}. Consider the function f: A → B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer. **[5]**

70. Give an example of a map **[5]**
    i. which is one-one but not onto
    ii. which is not one-one but onto
    iii. which is neither one-one nor onto.

71. Read the case study carefully and answer the questions that follow: **[5]**
Students of Grade 9, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line y = x − 4. Let L be the set of all lines which are parallel on the ground and R be a relation on L.

Based on the information given above, answer the following questions:

i. Let relation R be defined by R = {(L_1, L_2) : $L_1 \| L_2$ where L_1, L_2 ∈ L} then R is ________ relation.
   a. Equivalence
   b. Only reflexive
   c. Not reflexive
   d. Symmetric but not transitive

ii. Let R = {(L_1, L_2) ∶ $L_1 \perp L_2$ where L_1, L_2 ∈ L} which of the following is true?
   a. R is Symmetric but neither reflexive nor transitive
   b. R is Reflexive and transitive but not symmetric
   c. R is Reflexive but neither symmetric nor transitive
   d. R is an Equivalence relation

iii. The function f : R → R defined by f(x) = x − 4 is ________
   a. Bijective
   b. Surjective but not injective
   c. Injective but not Surjective
   d. Neither Surjective nor Injective

iv. Let f : R → R be defined by f(x) = x − 4. Then the range of f(x) is ________.

a. R

b. Z

c. W

d. Q

v. Let R = {(L_1, L_2) : L_1 is parallel to L_2 and L_1 : y = x – 4} then which of the following can be taken as L_2?

a. 2x - 2y + 5 = 0

b. 2x + y = 5

c. 2x + 2y + 7 = 0

d. x + y = 7

72. Read the case study carefully and answer the questions that follow: **[5]**

Raji visited the Exhibition along with her family. The Exhibition had a huge swing, which attracted many children. Raji found that the swing traced the path of a Parabola as given by $y = x^2$.

Based on the information given above, answer the following questions:

i. Let f : R → R be defined by $f(x) = x^2$ is ________.

a. Neither Surjective nor Injective

b. Surjective

c. Injective

d. Bijective

ii. Let f : N → N be defined by $f(x) = x^2$ is ________.

a. Surjective but not Injective

b. Surjective

c. Injective

d. Bijective

iii. Let f : {1, 2, 3, ... } → {1, 4, 9, ...} be defined by $f(x) = x^2$ is ________.

a. Bijective

b. Surjective but not Injective

c. Injective but Surjective

d. Neither Surjective nor Injective

iv. Let : N → R be defined by $f(x) = x^2$. Range of the function among the following is ________.

a. {1, 4, 9, 16, …}

b. {1, 4, 8, 9, 10, …}

c. {1, 4, 9, 15, 16, …}

d. {1, 4, 8, 16, …}

v. The function f : Z → Z defined by $f(x) = x^2$ is ________.

a. Neither Injective nor Surjective

b. Injective

c. Surjective

d. Bijective

73. Consider the mapping f : A $\rightarrow$ B is defined by f(x) = $\frac{x-1}{x-2}$ such that f is a bijection. **[5]**

Based on the above information, answer the following questions,

i. Domain of f is

a. R - {2}

b. R

c. R - {1, 2}

d. R - {0}

ii. Range of f is

a. R

b. R - {1}

c. R - {0}

d. R - {1, 2}

iii. If g : R - {2} $\rightarrow$ R - {1} is defined by g(x) = 2f{x} - 1, then g(x) in terms of x is

a. $\frac{x+2}{x}$

b. $\frac{x+1}{x-2}$

c. $\frac{x-2}{x}$

d. $\frac{x}{x-2}$

iv. The function g defined above, is

a. One-one

b. Many-one

c. into

d. None of these

v. A function f(x) is said to be one-one iff

a. $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

b. $f(-x_1) = f(-x_2) \Rightarrow -x_1 - x_2$

c. $f\{x_1) = f\{x_2) \Rightarrow x_1 = x_2$

d. None of these

74. **Assertion (A):** Every function is invertible. **[1]**

Reason (R): Only bijective functions are invertible.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

75. Let A = {1, 2, 3, 4} and B = {a, b} be two sets. Write total number of onto functions from A to B. **[2]**