DETERMINANTS QUESTION BANK

Class 12 - Mathematics

1. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then value of x is **[1]**

   a) 3 b) ± 3

   c) 6 d) ± 6

2. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists if. **[1]**

   a) $\lambda = 2$ b) $\lambda \neq -2$

   c) None of these d) $\lambda \neq 2$

3. Matrices A and B are inverses of each other only when **[1]**

   a) AB = BA = O b) AB = O, BA = I

   c) AB = BA= I d) AB = BA

4. If A is a 3-rowed square matrix and IAI = 4 then adj (adj A) = ? **[1]**

   a) None of these b) 64A

   c) 4A d) 16A

5. If I_3 is the identity matrix of order 3, then I_3^{-1} is **[1]**

   a) 0 b) $3I_3$

   c) I_3 d) None of these

6. If f(x) = $\begin{vmatrix} 2\cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$ then, f $(\frac{\pi}{3})$ = **[1]**

   a) 0 b) 1

   c) –1 d) 2

7. Let X = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, A = $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$. If AX = B, then X is equal to **[1]**

   a) $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$ b) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

   c) $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$ d) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

8. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then **[1]**

a) none of these

b) a = cos 2θ, b = sin 2θ

c) a = 1, b = 1

d) a = sin 2θ, b = cos 2θ

9. If A and B are any 2 $\times$ 2 matrices, then det. (A+B) = 0 implies **[1]**

a) detA + det B = 0

b) det A = 0 or det B = 0

c) None of these

d) det A = 0 and det B = 0

10. If a matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$, then A^{-1} is equal to **[1]**

a) $3A^2 - 2A - 5$

b) none of these

c) $3A^2 + 2A + 5$

d) $-(3A^2 + 2A + 5)$

11. If A is a non singular matrix and A' denotes the transpose of A, then **[1]**

a) $|AA'| \neq |A^2|$

b) None of these

c) $|A| + |A'| \neq 0$

d) $|A| \neq |A'|$

12. The system of equations **[1]**

x + y + z = 2,

3 x - y + 2z = 6

3x + y + z = -18

has:

a) zero solution as the only solution

b) an infinite number of solutions

c) a unique solution

d) no solution

13. The existence of the unique solution of the system of equations: **[1]**

x + y + z= λ

5x - y + μ z = 10

2x+ 3y - z = 6 depends on

a) λ and μ both

b) λ only

c) neither λ nor μ

d) μ only

14. Let a, b, c be positive real numbers.What type of solutions do the following system of equations in x, y and z has? **[1]**

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

a) finitely many solutions

b) no solution

c) unique solution

d) infinitely many solutions

15. If A is a square matrix of order 3, such that A (adj A) = 10 I, then |adj A| is equal to: **[1]**

a) 100

b) 101

c) 1

d) 10

16. $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$ = ? **[1]**

a) None of these

b) $\frac{\sqrt{3}}{2}$

c) $\frac{1}{2}$

d) 1

17. If A^2 - A + I = O, then the inverse of A is **[1]**

a) A + I

b) A^{-2}

c) A - I

d) I - A

18. For non-singular square matrix A,B and C of the same order $(AB^{-1}C)^{-1}$ = **[1]**

a) $C^{-1}BA^{-1}$

b) CBA^{-1}

c) $C^{-1}\ B^{-1}A^{-1}$

d) $A^{-1}BC^{-1}$

19. If A and B are invertible matrices of the same order then $(AB)^{-1}$ = ? **[1]**

a) $(A^{-1} \times B)$

b) $(A \times B^{-1})$

c) $(A^{-1} \times B^{-1})$

d) $(B^{-1} \times A^{-1})$

20. If $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ and B is square matrix of order 2 such that AB = I then B =? **[1]**

a) $\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

c) None of these

d) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

21. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32}.A_{32}$. **[1]**

22. If A is a square matrix such that A (adj A) = 5I, where I denotes the identity matrix of the same order. Then, find the value of |A| **[1]**

23. Evaluate : $\begin{vmatrix} \sqrt{6} & \sqrt{5} \\ \sqrt{20} & \sqrt{24} \end{vmatrix}$. **[1]**

24. Evaluate : $\Delta = \begin{vmatrix} 3 & 4 & 5 \\ -6 & 2 & -3 \\ 8 & 1 & 7 \end{vmatrix}$ **[1]**

25. Let A be a square matrix of order 3, write the value of |2A|, where |A| = 4 **[1]**

26. Evaluate: $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ **[1]**

27. Evaluate $\begin{vmatrix} x^2 - x + 1 & x + 1 \\ x + 1 & x + 1 \end{vmatrix}$ **[1]**

28. If $A = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$, then find |adj A|. **[1]**

29. If A is a square matrix of order 3, with |A| = 9, then write the value of |2.adjA|. **[1]**

30. For what value of x, the matrix $\begin{bmatrix} 2x + 4 & 4 \\ x + 5 & 3 \end{bmatrix}$ is a singular matrix? **[1]**

31. If for any 2×2 square matrix A, A(adj A)= $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of |A|. **[1]**

32. If A is a non-singular square matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then find $(A^T)^{-1}$ **[1]**

33. Let A be a square matrix of order 3 $\times$3. Write the value of $|2A|$, where $|A|$= 4. **[1]**

[1]

34. If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 5 & 3 & 8 \end{vmatrix}$, write the minor of element a_{22}.

35. Find minors and cofactors of all the elements of the determinant $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$ **[1]**

36. Using co-factors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$ **[2]**

37. Using matrix method, solve the system of equations **[2]**

x - 2y + z = 0;

y - z = 2;

2x - 3z = 10.

38. Find minors and cofactors of the elements of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ and verify that $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$. **[2]**

39. Find the value of k if the area of Δ is 35 square cms with vertices (k, 4), (2, - 6) and (5, 4). **[2]**

40. Using matrix method, solve the system of equations **[2]**

2x - 3y + 1 = 0;

x + 4y + 3 = 0.

41. Solve the system of linear equation, using matrix method 2x - y = - 2; 3x + 4y = 3 **[2]**

42. For the matrices $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1} A^{-1}$ **[2]**

43. Using determinants, show that the following system of linear equation is inconsistent: **[2]**

x - 3y + 5z = 4

2x - 6y + 10z = 11

3x - 9y + 15z = 12

44. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular? **[2]**

45. Find the value of x, if: **[2]**

i. $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

ii. $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

46. Show that the system of linear equations is inconsistent: **[2]**

x + 2y = 9;

2x + 4y = 7.

47. Solve the system of equations using Cramer's rule: **[2]**

5x - 7y + z =11, 6x - 8y - z = 15 and 3x + 2y - 6z =7.

48. Solve the system of homogeneous linear equations by matirix method: **[2]**

3x + y - 2z = 0

x + y + z = 0

x - 2y + z = 0

49. Write the cofactor of a_{12} in the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ **[2]**

[2]

50. If A = $\begin{bmatrix} 0 & i \\ i & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, find the value of |A| + |B|.

51. Show that $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ satisfies the equation A^2 + 4A - 42I = 0. Hence, find A^{-1} **[3]**

52. Find A^{-1}, if A = $\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$ Hence, solve the following system of linear equations: **[3]**

x + 2y + 5z = 10, x - y - z = -2 , 2x + 3y - z = -11

53. Find the inverse of matrix: $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ **[3]**

54. If A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations **[3]**

x + 2y + z = 4,

- x + y + z = 0,

x - 3y + z = 2.

55. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award ₹ x each ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with total award money of ₹ 16,00. School B wants to spend ₹ 2,300 to award its 4 ,1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for awards. **[3]**

56. A shopkeeper has 3 varieties of pens A, B and C. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jean purchased 4 pens of A variety, 3 pens of B variety and 2 pens of C variety for ₹ 60. While Shikha purchased 6 pens of A variety, 2 pens of B variety and 3 pens of C variety for ₹ 70. Using matrix method find the cost of each pen. **[3]**

57. A typist charges Rs145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are Rs 180. Using matrices, find the charges of typing 1 English and 1 Hindi page separately. However, typist charged only Rs 2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy? Which values are reflected in this problem? **[3]**

58. Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation A^2 – 4A + I = 0. where I is 2 $\times$ 2 identity matrix and O is 2 $\times$ 2 zero matrix. Using this equation, find A^{-1} **[3]**

59. Solve the system of linear equations by Cramer's rule: **[3]**

x + 2y = 1

3x + y = 4

60. If A = $\begin{bmatrix} -1 & -1 \\ 2 & -2 \end{bmatrix}$, show that A^2 + 3A + 4I_2 = O and hence find A^{-1}. **[3]**

61. Solve the system of the linear equations by Cramer's rule: **[5]**

3x + y + z = 2

2x - 4y + 3z = -1

4x + y - 3z = -11

62. Verify A (adj. A) = (adj. A) A = |A|I: **[5]**

$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

63. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = 0$. **[5]**

64. Solve the following system of the linear equations by Cramer's rule: **[5]**

x + y + z + 1 = 0

ax + by + cz + d = 0

$a^2x + b^2y + c^2z + d^2 = 0$

65. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}, using A^{-1} solve the system of equations **[5]**

2x - 3y + 5z = 11;

3x + 2y - 4z = -5;

x + y - 2z = -3.

66. Two schools P and Q want to award their selected students on the values of Tolerance, Kindness, and Leadership. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with total award money of Rs2200. **[5]**

School Q wants to spend Rs 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is Rs1200, using matrices, find the award money for each value.

67. Using properties of determinant, prove the $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix}$ = (1 + pxyz) (x - y) (y - z) (z - x), where p is any scalar. **[5]**

68. Solve the system of the linear equations by Cramer's rule: **[5]**

x + y = 1

x + z = -6

x - y - 2z = 3

69. The cost of 4kg onion, 3kg wheat and 2kg rice is Rs. 60. The cost of 2kg onion, 4kg wheat and 6kg rice is Rs. 90. The cost of 6kg onion 2kg wheat and 3kg rice is Rs. 70. Find the cost of each item per kg by matrix method. **[5]**

70. If a, b, c are positive and unequal, show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative? **[5]**

71. Using matrix method, solve the following system of equations **[5]**

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$

and $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2, where\ x, y, z \neq 0$.

72. Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of Rs.x, Rs.y, and Rs.z, respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with total prize money of Rs.37000 and the second institution decided to award respectively 5, 3 and 4 employees with total prize money of, Rs.47000. If all the three prizes per person together amount to Rs.12000, then using a matrix method, find the values of x, y, and z. What values are described in this question? **[5]**

73. Find the inverse of the matrix (if it exists) given $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

74. By using determinants, solve the following system of equations: **[5]**

x + y + z = 1

x + 2y + 3z = 4

x + 3y + 5z = 7

75. A total amount of Rs 7000 is deposited in three different savings bank accounts with annual interest rates of 5%, 8% and $8\frac{1}{2}\%$,respectively. The total annual 2% interest from these three accounts is Rs 550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices. **[5]**