LOGARITHS

POINTS TO REMEMBER:

1. $y = a^x$ is called exponential function where $a > 0$ and $a \neq 1$. This function is defined for all $x \in R$. The exponential function has following properties:

(i) $a^0 = 1$ (ii) $a^x = a^y \Leftrightarrow x = y$ (iii) $a^m \times a^n = a^{m+n}$

(iv) $\frac{a^m}{a^n} = a^{m-n}$ (v) $(a^m)^n = a^{mn}$ (vi) $(ab)^n = a^n \times b^n$

2. if $a^y = x$ then we say $y = \log_a x$, where $a > 0$ & $a \neq 1$. a is called base of the logarithm.
3. $y = \log_a x$ is defined for $x \in (0,\infty)$.
4. $\log_a a = 1$

   Proof: Let $\log_a a = x \Rightarrow a^x = a \Rightarrow x = 1$
5. $\log_a 1 = 0$

   Proof: Let $\log_a 1 = x \Rightarrow a^x = 1 \Rightarrow a^x = a^0 \Rightarrow x = 0$
6. $\log_a (mn) = \log_a m + \log_a n$, $(m > 0, n > 0)$

   Proof:

   Let $\log_a m = x$ & $\log_a n = y \Rightarrow a^x = m$ & $a^y = n \Rightarrow a^x \times a^y = m \times n$

   $\Rightarrow a^{x+y} = mn \Rightarrow \log_a (mn) = x + y \Rightarrow \log_a (mn) = \log_a m + \log_a n$

7. $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$, $(m > 0 , n > 0)$

   Proof: Let $\log_a m = x$ & $\log_a n = y \Rightarrow a^x = m$ & $a^y = n$

   $\Rightarrow \frac{a^x}{a^y} = \frac{m}{n} \Rightarrow a^{x-y} = \frac{m}{n} \Rightarrow \log_a \left(\frac{m}{n}\right) = x - y$

   $\Rightarrow \log_a \left(\frac{m}{n}\right) \log_a m - \log_a n$

8. $\log_a m^n = n \log_a m$, $(m > 0)$

   Proof: Let $\log_a m^n = x \Rightarrow a^x = m^n$

   $\Rightarrow (a^x)^{1/n} = (m^n)^{1/n} \Rightarrow a^{x/n} = m \Rightarrow \log_a m = \frac{x}{n} \Rightarrow n \log_a m = x \Rightarrow n \log_a m = \log_a m^n$
9. $\log_{a^n} m = \frac{1}{n} \log_a m$, $(m > 0)$

   Proof: Let $log_{a^n} m = x \Rightarrow (a^n)^x = m \Rightarrow a^{nx} = m \Rightarrow \log_a m = nx$

   $\Rightarrow \frac{1}{n} \log_a m = x \Rightarrow \frac{1}{n} \log_a m = log_{a^n} m$
10. Combining the properties $\log_a m^n = n \log_a m$ & $\log_{a^n} m = \frac{1}{n} \log_a m$ we get:

    $log_{a^p} m^p = \frac{p}{q} \log_a m$, $(m > 0)$
11. Base change rule: $\log_a b = \left(\frac{log_c b}{log_c a}\right)$ $(b > 0, c > 0$ & $c \neq 1)$.

Proof:

Let $\log_a b = x \Rightarrow a^x = b \Rightarrow \log_c a^x = \log_c b \Rightarrow x \log_c a = \log_c b$

$\Rightarrow x = \frac{log_c b}{log_c a} \Rightarrow \log_a b = \frac{log_c b}{log_c a}$

12. $\log_b a = \frac{1}{log_a b}$, $(b > 0, b \neq 1, a > 0, a \neq 1)$.

Proof:

Changing the base to a $\log_b a$:

$\log_b a = \frac{\log_a a}{\log_a b}$ $\Rightarrow$ $\log_b a = \frac{1}{\log_a b}$

13. $x = a^{\log_a x}$, (a > 0, a ≠ 1, x > 0).

Proof: Let $\log_a x = y \Rightarrow a^y = x \Rightarrow a^{\log_a x} = x$.

14. $a^{\log_x b} = b^{\log_x a}$, (a > 0, a ≠ 1, x > 0, x ≠ 1)

Proof: $a^{\log_x b} = b^{\log_b a \log_x b}$, (using $x = a^{\log_a x}$)

$= b^{\log_x a}$

Alternative:

$a^{\log_x b} = a^{\frac{\log b}{\log x}} = a^{\frac{\log b}{\log a} \times \frac{\log a}{\log x}}$

$= a^{\log_a b . \log_x a} = (a^{\log_a b})^{\log_x a} = b^{\log_x a}$

SECTION A:

1. Write the following in logarithmic form:

(i) $3^5 = 243$ (ii) $8^{-2} = \frac{1}{64}$ (iii) $10^{-5} = 0.00001$

(iv) $9^{-5/2} = \frac{1}{243}$ (v) $(125)^{-1/3} = 0.2$ (vi) $4^{3/2} = 8$

2. Express each of the following in exponential form :

(i) $\log_3 243 = 5$ (ii) $\log_2 \left(\frac{1}{8}\right) = -3$ (iii) $\log_5 25 = 2$

(iv) $\log_2 64 = 6$ (v) $\log_{10} 1000 = 3$ (vi) $\log_{10} 0.0001 = -4$

(vii) $\log_{10} .01 = -2$

3. Find the value of each of the following:

(i) $\log_6 216$ (ii) $\log_5 125$ (iii) $\log_4 128$

(iv) $\log_{10} (.001)$ (v) $\log_{3\sqrt{2}} 324$ (vi) $\log_{\sqrt{5}} 625$

(vii) $\log_9 (\frac{1}{243})$ (viii) $\log_{10} 1000000$ (xi) $\log_2 0.015625$

(x) $\log_\pi \tan(0.25\pi)$ (xi) $\log_2 (\log_3 81)$

4. Find x, if

(i) $\log_2 x = 3$ (ii) $\log_x 0.001 = -3$ (iii) $\log_{0.4} 0.0256 = x$ (iv) $\log_x 9 = 2$

(v) $\log_2 (\frac{1}{16}) = x$ (vi) $\log_{\sqrt{2}} x = 4$ (vii) $\log_{\frac{1}{9}} 27\sqrt{3}$ (viii) $\log_{0.2} x = 3$

5. Which of following is a valid logarithmic form:

(i) $\log_3 2$ (ii) $\log_2 (-8)$ (iii) $\log_1 25$ (iv) $\log_{25} 1$

(v) $\log_{-10} 100$ (vi) $\log_{10} 0.0001$ (vii) $\log_0 0$ (viii) $\log_{-5} (-125)$

6. If the logarithm of 81 is 3, find the base of the logarithm.
7. If $\log_{2\sqrt{2}} 512 = a$, find $\log_a 216$.
8. if $\log_a 32 = 10$, find $\log_8 a$.

9. If $\log_{10} y = x$, find the value of 10^{2x} in terms of y

10. If $\log_3 y = x$ and $\log_2 z = x$, find 72^x in terms of y and z.

11. Show that $\log_2 [\log_2 (\log_2 16)] = 1$.

12. If $\log_2 (x^2 - 4x + 5) = 0$, then show that x = 2.

13. Prove that $\log_2 3$ is an irrational number.

14. Give $\log_{10} x = a$, and $\log_{10} y = b$,

(i) Write down 10^{a-1} in terms of x. (ii) Write down 10^{2b} in terms of y.

(iii) if log P = 2a – b , express P in terms of x and y

15. Prove the followings:

(i) $\frac{1}{2}\log 25 - 2\log 3 + \log 18 = 1$ (ii) $\log \frac{75}{28} = 2\log 5 + \log 3 - 2\log 2 - \log 7$

(iii) $\log 2 + 16\log \frac{16}{15} + 12\log \frac{25}{24} + 7\log \frac{81}{80} = 1$ (iv) $\log \frac{35}{78} = \log 7 + \log 5 - \log 2 - \log 3 - \log 13$

(v) $2\log \frac{11}{13} + \log \frac{130}{77} - \log \frac{55}{91} = \log 2$ (vi) $\log \frac{42}{55} = \log 2 + \log 3 + \log 7 - \log 5 - \log 11$

(vii) $5\log \frac{2}{7} + 4\log \frac{7}{3} + 5\log \frac{11}{2} + \log \frac{7}{11} + 4\log \frac{3}{11} = 0$ (viii) $\log \frac{9}{14} + \log \frac{35}{24} - \log \frac{15}{16} = 0$

16. Simplify the following:

(i) $\frac{1}{2}\log 9 + 2\log 6 + \frac{1}{4}\log 81 - \log 12$ (ii) $\log \frac{75}{16} - 2\log \frac{5}{9} + \log \frac{32}{243}$

(iii) $\frac{1}{2}\log 4 + \frac{1}{4}\log 81 + 3\log 2 - \log 6 + 2$ (iv) $7\log \frac{16}{15} + 5\log \frac{25}{24} + 3\log \frac{81}{80}$

17. Evaluate the followings:

(i) $2\log 5 + \log 8 - \frac{1}{2}\log 4$ (ii) $2\log 15 + \log 36 - \log 9 - 2\log 3$

(iii) $\log 15 + 2\log 0.5 + 3\log 2 - \log 3 - \log 5$ (iv) $\log \frac{10}{\sqrt[3]{10}}$

(v) $\log 21 + \log 4 + 2\log 5 - \log 3 - \log 7$ (vi) $\log_4 \log_3 81$.

18. Solve the followings:

(i) $2\log x - \log(3x-2) = 0$ (ii) $\log(3x+2) + \log(3x-2) = 5\log 2$

(iii) $\log_x 3 + \log_x 9 + \log_x 729 = 9$ (iv) $\log_3 (3+x) + \log_3 (8-x) - \log_3 (9x-8) = 2 - \log_3 9$

(v) $\log(2x+1) - \log(2x-1) = 1$ (vi) $3^{\log x} - 2^{\log x} = 2^{(\log x)+1} - 3^{(\log x)-1}$

19. Find x if $\frac{1}{2}\log_{10}(11 + 4\sqrt{7}) = \log_{10}(2+x)$.

20. If $x = \log_{2a} a$, $y = \log_{3a} 2a$, $z = \log_{4a} 3a$, show that $xyz + 1 = 2yz$.

21. If $a^3 + b^3 = ab(8-3a-3b)$, show that

$\log \frac{a+b}{2} = \frac{1}{3}(\log a + \log b)$.

22. Given log 2 = 0.3010, log 3 = 0.4771, find the value of:

(i) $\log \frac{\sqrt{2}}{27}$ (ii) $\log 0.5$

23. If $a = 1 + \log_{10} 2 - \log_{10} 5$, $b = 2 \log_{10} 3$ and $c = \log_{10} m - \log_5 5$, find the value of m if $a + b = 2c$.

24. If a, b, c, d are any four positive numbers, prove that $\log_b a \times \log_c b \times \log_d c = \log_d a$

25. If $\frac{\log_e x}{b-c} = \frac{\log_e y}{c-a} = \frac{\log_e z}{a-b}$, show that

(i) $xyz = 1$ (ii) $x^a y^b z^c = 1$ (iii) $x^{b+c} y^{c+a} z^{a+b} = 1$

26. Show that $x^{\log y - \log z} y^{\log z - \log x} z^{\log x - \log y} = 1$

27. Solve: $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$

28. If a, b, c, d are three consecutive natural numbers, then show that $\log(1+ac) = 2 \log b$.

29. Prove that

(i) $\log_3 11 \times \log_{11} 13 \times \log_{13} 15 \times \log_{15} 27 = 3$ (ii) $\frac{1}{\log_{xy} xyz} + \frac{1}{\log_{yz} xyz} + \frac{1}{\log_{zx} xyz} = 2$

30. If $\log\left(\frac{x+y}{2}\right) = \frac{1}{2}(\log x + \log y)$, show that $x = y$

31. If $\log(m+n) = \log m + \log n$, show that $m = \frac{n}{n-1}$

32. Given that $\log_a x = \frac{1}{\alpha}$, $\log_b x = \frac{1}{\beta}$, $\log_c x = \frac{1}{\gamma}$ Find $\log_{abc} x$.

33. If $a^{2x-3} b^{2x} = a^{6-x} b^{5x}$, Prove that $3 \log a = x \log \frac{a}{b}$

34. Find the value of x if $(\log_e 2)(\log_x 625) = (\log_{10} 16)(\log_e 10)$

35. Find the value of

(i) $[\log_{10}(5 \log_{10} 100)]^2$ (ii) $\log_2 [\log_2 \log_2 \log_2 (65536)]$ (iii) $\log_4 \{\log_2 [(\log_2 (\log_3 81))]\}$

36. Find the value of $\frac{\log_7 49}{\log_6 17} \div \frac{\log_{2401} 49}{\log_{36} 17}$

37. If $\log_a bc = x$, $\log_b ca = y$, $\log_c ab = z$, Prove that $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1$

38. If $x = 1 + \log_a bc$, $y = 1 + \log_b ca$ and $z = 1 + \log_c ab$ Prove that $xyz = xy + yz + zx$.

39. Value of $\log_{10} \tan 40^0 . \log_{10} \tan 41^0 . \log_{10} \tan 42^0 .... \log_{10} \tan 50^0$

(a) 0 (b) 1 (c) An irrational quantity (d) None of these

40. If $\log_{12} 18 = \alpha$ & $\log_{24} 54 = \beta$ then value of $\alpha\beta + 5(\alpha - \beta)$ is equal to

(a) 0 (b) -1 (c) 2 (d) 1

41. If $\frac{\log_a x . \log_b x}{\log_a z + \log_b x} = (\log_x p)q$ then (p,q) is

(a) (-1, ab) (b) (ab, -1) (c) (a+b, -1) (d) $(\frac{a}{b}, -1)$

42. $a^{\log b} = b^{\log a}$ (a) True (b) False

(c) True only if both log have same base (d) True only if both log have different base

43. The number $\log_2 7$ is

(a) an integer (b) a rational number (c) an irrational number (d) a prime number

44. $\log_a x . \log_b y = \log_b x . \log_a y$. The statement is

(a) True (b) False (c) True if a = b (d) True if x = y

45. If y = $a^{1/(1-\log_a x)}$, & z = $a^{1/(1-\log_a y)}$ then X = $a^{1/(1-\log_a z)}$. The statement is

(a) True (b) False

46. Logarithm of $32\sqrt[5]{4}$ to the base $2\sqrt{2}$ is

(a) 3.6 (b) 5 (c) 5.6 (d) None of these

47. If $\log_4 5 = a$ & $\log_5 6 = b$, then $\log_3 2$ is equal to

(a) $\frac{1}{2a+1}$ (b) $\frac{1}{2b+1}$ (c) 2ab+1 (d) $\frac{1}{2ab-1}$

48. If $a^2 + 4b^2 = 12ab$, then log (a + 2b) is

(a) $\frac{1}{2}$ [log a + log b – log 2] (b) log $\frac{a}{2}$ + log $\frac{b}{2}$ + log 2

(c) $\frac{1}{2}$ [log a + log b + 4 log 2] (d) $\frac{1}{2}$ [log a – log b + 4 log 2]

49. If $\log_{10} x = y$, then $\log_{1000} x^2$ is equal to

(a) y^2 (b) 2y (c) $\frac{3y}{2}$ (d) $\frac{2y}{3}$

50. If x = $\log_a (bc)$, y = $\log_b (ca)$, z = $\log_c (ab)$, Then which of the following is equal to 1

(a) x + y + z (b) $(1 + x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$

(c) xyz (d) None of these

51. If $\log_{12} 27 = a$, then $\log_6 16$ =

(a) 3. $\frac{3-a}{3+a}$ (b) 3. $\frac{3-a}{3+a}$ (c) 4. $\frac{3-a}{3+a}$ (d) None of these

52. If $\log_{\sqrt{3}} 5 = a$ and $\log_{\sqrt{3}} 2 = b$, then $\log_{\sqrt{3}} 300$ =

(a) 2(a+b) (b) 2(a+b+1) (c) 2(a+b+2) (d) a+b+4

53. The value of $\log_p$

$$\left(\log_p \sqrt[P]{\sqrt[P]{\sqrt[P]{\ldots\ldots\ldots\ldots \sqrt[P]{\sqrt[P]{P}}}}} \right)$$

(a) P^n (b) P^{-n} (c) n (d) –n

54. If $\frac{3}{4}(\log_2 x)^2 + (\log_2 x) - \frac{5}{4} = \sqrt{2}$ then

(a) x = 2, $\frac{1}{4}$, $2^{1/3}$ (b) x = 2, $\frac{1}{8}$, $2^{-1/3}$ (c) x = 4, $\frac{1}{4}$, $2^{1/3}$ (d) x = 2, $\frac{1}{4}$, $2^{-1/3}$

55. $log_{\sqrt{5}}$ x + $log_{5^{1/3}}$ x + $log_{5^{1/4}}$ x + ... up to 7 terms = 35, then the value of x is equal to

(a) 5 (b) 5^2 (c) 5^3 (d) 5^4

56. If $\frac{\log a}{y-z} = \frac{\log b}{z-x} = \frac{\log c}{z-y}$, then a^{y^2} + yz + b^{z^2} + zx + x^2 c^{x^2} + xy + y^2 =

(a) 0 (b) 1 (c) -1 (d) abc

57. Evaluate the following logarithms without using log table.

(a) $log_{2\sqrt{2}}\left(32 \times \sqrt[5]{4}\right)$ (b) $log_{a^{-1/2}}\left(\sqrt[3]{a^{\frac{-15}{2}}}\right)$, ab ≠ 1

(c) $log_{2\sqrt{3}}$ 1728 (d) $\log_{ab}\left[\sqrt{a\sqrt[3]{b}} \times \sqrt{b\sqrt[3]{a}}\right]$, ab ≠ 1.

58. Find the value of $\frac{\log_2 24}{\log_{96} 2}$ - $\frac{\log_2 192}{\log_{12} 2}$, without using log table.

59. If a, b, c are three consecutive positive Integers then the simplified value of log $log_{\sqrt{b}}$ (1+ac) is

(a) 0 (b) 2 (c) 4 (d) 8

60. Find the least integer n such that $7^n > 10^5$, given that $\log_{10} 343 = 2.5353$.

61. If $a^x = b^y = c^z$ where x,y,z are the GP then b show that $\log_b a = \log_c b$.

62. Solution set of $\log_x 2.\log_{2x} 2 = \log_{4x} 2$ is.

(a) $\{2^{-\sqrt{2}}, 2^{\sqrt{2}}\}$ (b) $\left\{\frac{1}{2}\ ,\ 2\right\}$ (c) $\left\{\frac{1}{4}\ ,2^2\right\}$ (d) $\left\{\frac{1}{4}\ ,3\right\}$

63. Value of $\sqrt{(log_x 2)(log_{2^{1/2}} x^2)(log_{x^{1/3}} 2^3) \dots\dots\dots (log_{2^{1/n}} x^n)}$

Where n is an even Positive integer and x > 1 , is

(a) n! (b) n! Log_2 x (c) n! $\log_x 2$ (d) None of these

64. The number of solution of $\log_4 (x-1) = \log_2 (x-3)$ is

(a) 5 (b) 1 (c) 2 (d) 0

65. Value of $a^{(log_b b.log_b c.\ log_c d)}$ is:

(a) a (b) abcd (c) d (d) None of these

66. The simplified value of log $\left(\frac{16\times (256)^{1/3}}{6 \div (448)^{1/4}}\right)$

(a) $\frac{1}{6}$ log 2 (b) $\frac{43}{6}$ log 2 (c) 3 log 2 (d) None of these

67. The value of x satisfying the equation

$4^{log_9 3} + 9^{log_2 4} = 10^{log_x 83}$ is: (a) 83 (b) 2 (c) 5

(d) None of these

68. If logarithm of $\frac{1}{3}$ is - $\frac{1}{3}$ then base of the log is:

(a) 3 (b) 9 (c) 18 (d) 27

69. If $7^{log_7(x^2-4x+5)}$ = x – 1 then x may be values:

(a) 2, 3 (b) -2, -3 (c) 0, 1 (d) None of these

70. If $\log_a x = 0.3$ and $\log_a 3 = 0.4$, then $\log_3 x$ is:

(a) 0.12 (b) 0.7 (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

71. If $\log_7 2 = m$, then the value of $\log_{49} 28$ is:

(a) 2(1+2m) (b) $\frac{1}{2}$(1+2m) (c) $\frac{2}{1+2m}$ (d) 1+m

72. If 2 log(x+1) – log(x^2 -1) = log 2, then x =

(a) 1 (b) 0 (c) 2 (d) 3

ANSWER SHEET

1. (i) $\log_3 243 = 5$ (ii) $\log_8 \left(\frac{1}{64}\right) = -2$ (iii) $\log_{10} 0.00001$

   (iv) $\log_9 \left(\frac{1}{243}\right) = \left(-\frac{5}{2}\right)$

3. (i) 3 (ii) 3 (iii) $\frac{7}{2}$ (iv) $\log_9 \left(\frac{1}{243}\right) = -\frac{5}{2}$ (v) $\log_{10} 0.001 = -3$

   (vi) $\log_{3\sqrt{2}} 324 = 4$ (vii) $\log_{\sqrt{5}} = 625 = 8$ (viii) $\log_{10} 1000000 = 6$

   (ix) $\log_2 0.015625$ (x) $\log_{\pi} \tan (0.25) = 0$ (xi) 2

4. (i) 8 (ii) 10 (iii) 4 (iv) 3 (v) -4 (vi) 5 (vii) $-\frac{7}{4}$ (viii) .008

5. (i) valid (ii) invalid (iii) invalid (iv) valid

   (v) invalid (vi) valid (vii) invalid (viii) invalid

6. $3^{4/3}$ 7. 3 8. $\frac{1}{6}$ 9. y^2

10. $y^2 z^3$ 11. To Prove 12. To Prove 13. To Prove

14. (i) $\frac{x}{10}$ (ii) y^2 (iii) $\frac{x^2}{y}$

15. (i) To Prove (ii) To Prove (iii) To Prove (v) $\log_{125} (0.2) = -\frac{1}{3}$

    (vi) $\log_4 (8) = \frac{3}{2}$

2. (i) $3^5 = 243$ (ii) $2^{-3} = \frac{1}{8}$

   (v) To Prove (vi) To Prove (vii) To Prove (viii) To Prove

16.(i) 3 log 3 (ii) log 2 (iii) log 800 (iv) log (2)

17. (i) 2 (ii) 2 (iii) log 2 (iv) $\frac{2}{3}$

    (v) 2 (vi) 1

18.(i) x = 2 , or x = 1 (ii) 2 (iii) 3 (iv) 4 (v) $\frac{11}{18}$

(vi) 100

19. $\sqrt{7}$ 20. 2yz 21. $\frac{1}{3}$(log a + log b) 22. (i) - 1.2806
(ii) - 0.3010

23. 60 24. To Prove 25. (i) 1 (ii) 1 (iii) 1

26. 1 27. 8 28. To Prove 29. (i) 3 (ii) 2

30. x = y

31. $\frac{n}{n-1}$ 32. $\frac{1}{\alpha+\beta+\gamma}$ 33. To Prove (iii) $5^2 = 25$ (iv) $2^6 = 64$
(v) $10^3 = 1000$ (vi) $10^{-4} = 0.0001$ (vii) $10^{-2} = 0.01$

34. 5 35. (i) 1 (ii) 1 (iii) 0 36. 2 37. 1

38. To Prove

39. (a) 40. (d) 41. (b) 42. (c) 43. (c) 44. (a)

45. (a) 46. (a) 47. (d) 48. (c) 49. (d) 50. (b)

51. (c) 52. (b) 53. (d) 54. (d) 55. (a) 56. (b)

57. (i) $\frac{18}{5}$ (ii) 5 (iii) 6 (iv) $\frac{2}{3}$

58. 3 59. (c) 60. 6 61. To Prove 62. (a)

63. (a) 64. (b) 65. (c) 66. (d) 67. (d)

68. (d) 69. (a) 70. (c)