## Solution

## RELATIONS AND FUNCTIONS

## Class 11 - Mathematics

1. **(a)** not anti symmetric

   **Explanation:** A relation R on a non empty set A is said to be reflexive if xRx for all x $\in$ R , Therefore , R is not reflexive.
   A relation R on a non empty set A is said to be symmetric if xRy $\Leftrightarrow$ yRx, for all x , y $\in$ R. Therefore, R is not symmetric.
   A relation R on a non empty set A is said to be antisymmetric if xRy and yRx $\Rightarrow$ x = y , for all x , y $\in$ R. Therefore, R is not antisymmetric.

2. **(d)** Domain = R - {4}, Range = {-1}

   **Explanation:** We have, f(x) $= \frac{4-x}{x-4}$ = -1, for x$\neq$4

3. **(c)** R – {-2, 3}

   **Explanation:** We have, f(x) $= \frac{x^2+2x+1}{x^2-x-6}$

   f(x) is not defined, if $x^2$ - x - 6 = 0
   $\Rightarrow$ (x - 3)(x + 2) = 0
   $\therefore$ x = -2, 3
   $\therefore$ Domain of f = R - {-2, 3}

4. **(d)** 3 f(x)

   **Explanation:** $\mathrm{f(g(x))} = \log\left(\frac{1+g(\mathrm{x})}{1-\mathrm{g(x)}}\right)$

   $= \log\left(\frac{1+\frac{3x+x^2}{1+3x^2}}{1-\frac{3x+x^2}{1+3x^2}}\right)$

   $= \log\left(\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right)$

   $= \log\left(\frac{1+x}{1-x}\right)^3 = 3\log\left(\frac{1+x}{1-x}\right)$

   f(g)(x) = 3f(x)

5. **(b)** $R_1$ = {( 2,2 ) , ( 3,3 ) , ( 6,6 )}

   **Explanation:** $R_1$ is a reflexive on A, because ( a,a ) $\in R_1$ for each a $\in$ A

6. **(d)** $-\sqrt{2}$

   **Explanation:** Let $f(x) = \sin x + cos\ x$
   $\therefore f'(x) = \cos x - \sin x$
   $\Rightarrow f''(x) = -\sin x - \cos x$
   Now, $f'(x) = 0$
   $\Rightarrow \cos x - \sin x = 0 \Rightarrow \sin x = \cos x \Rightarrow \tan x = 1$
   $\Rightarrow x = n\pi + \frac{\pi}{4}$, n $\in$ z
   At x $= \pi + \frac{\pi}{4}$,
   f'' (x) $= -\sin\left(\pi + \frac{\pi}{4}\right) - \cos\left(\pi + \frac{\pi}{4}\right)$
   $= \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} > 0$
   $\therefore$ x $= \pi + \frac{\pi}{4}$ is point of minimum
   Minimum value $= \sin\left(\pi + \frac{\pi}{4}\right) + \cos\left(\pi + \frac{\pi}{4}\right)$
   $= -\sin\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right)$
   $= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

7. **(d)** reflexive

   **Explanation:** Any relation R is reflexive if xRx for all x $\in$ R. Here, (a, a), (b, b), (c, c)$\in$ R. Therefore, R is reflexive.

8. **(b)** transitive but not symmetric

   **Explanation:** Consider the non – empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is not symmetric, because aRb means a is brother of b, then, it is not necessary that b is also brother of a , it can be the sister of a. Therefore, bRa is not true. Therefore, the relation is not symmetric . Again, if aRb and bRc is true, then aRc is also true. Therefore, R is transitive only.

9. **(d)** R - {0}
**Explanation:** Since log x is defined for x$\geq 0$, therefore domain of log |x| is R - {0}

10. **(b)** $f(\beta) = -10$
**Explanation:** 3x + $\frac{1}{x}$ = 2
Now, f(x) = $\left(3x+\frac{1}{x}\right)^3 - 3(3x)\left(\frac{1}{x}\right)\left(3x+\frac{1}{x}\right)$
Since, $a, \beta$ are roots of $3x+\frac{1}{x}$= 2
So, $f(\alpha) = f(\beta)$
$= (2)^3 - 9(2)$
= 8 -18
= -10

11. **(b)** $[-3, -2] \cup [2, 3]$
**Explanation:** $5|x| - x^2 - 6 \geq 0$
$x^2 - 5|x| + 6 \leq 0$
$(|x| - 2)(|x| - 3) \leq 0$
So, $|x| \in [2, 3]$
Therefore, $x \in [-3, -2] \cup [2, 3]$

12. **(a)** many-one and into
**Explanation:** f: R $\rightarrow$ R : f(x) = $x^2$
One-One function
Let p, q be two arbitrary elements in R
Then, f(p) = f(q)
$\Rightarrow$ $p^2$ = $q^2$
$\Rightarrow$ p = q and -q
Thus f(x) is many one function.
Onto function
Let v be an arbitrary element of R(co-domain)
Then, f(x) = v
$x^2$ = v
$\Rightarrow$ x = $\sqrt{v}$
Since v $\in$ R
If v = 2, $\sqrt{v}$ = 1.1414, which is not possible as x $\in$ R
Thus, f(x) is not onto function. It is into function.

13. **(c)** none of these
**Explanation:** Given set A = {1, 2, 3, 4, 5} and relation R ={(a, b):| $a^2$-$b^2$| < 16}
According to the condition |$a^2$-$b^2$|<16:
$\Rightarrow$ R = {(1, 1),(1,2), (2, 1), (1,3), (3, 1), (1,4), (4, 1), (2, 3),(2, 2), (3, 2), (4, 2), (2, 4),(3, 3), (4, 3), (5, 4), (3, 4), (4,4), (5,5)}.Which is the required solution.

14. **(b)** $[-1, 2) \cup [3, \infty)$
**Explanation:** Here $\frac{(x+1)(x-3)}{(x-2)} \geq 0$
But $x \neq 2$
so, $x \in [-1, 2) \cup [3, \infty)$

15. **(a)** {-1, 1}
**Explanation:** We know that
$|x| = -x \operatorname{in}(-\infty, 0)$ and $|x| = x$ in $[0, \infty)$
So, $f(x) = \frac{x}{-x} = -1$ in $(-\infty, 0)$
And $f(x) = \frac{x}{x} = 1$ in $(0, \infty)$
As clearly shown above f(x) has only two values 1 and -1
So, range of f (x) = {-1, 1}

16. **(c)** $A \times (B \cup C)$
**Explanation:** A × (B ∪ C) = (A × B) ∪ A × C
= {a, b} × {c, d} ∪ {a, b} × {d, c}
= {(a, c), (a, d), (b, c), (b, d)} ∪ {(a, d), (a, c), (b, d), (b, c)}
= {(a, c), (a, d), (a, c), (b, c), (b, d), (b, e)}

17. **(a)** two points
**Explanation:** From A, $x^2 + y^2 = 5$ and from B, 2x = 5y
Now, 2x = 5y ⇒ x = $\frac{5}{2y}$
∴ $x^2 + y^2 = 5 \Rightarrow \left(\frac{5}{2y}\right)^2 + y^2 = 5$
⇒ $29y^2 = 20 \Rightarrow y = \pm\sqrt{\frac{20}{29}}$
⇒ $29y^2 = 20 \Rightarrow y = \pm\sqrt{\frac{20}{29}}$
∴ x = $\frac{5}{2}(\pm\sqrt{\frac{20}{29}})$
∴ Possible ordered pairs = four
But two ordered pair in which c is positive and y is negative will be rejected as it will not be satisifed by the equation in B.
Therefore,
A ∩ B contains 2 elements.

18. **(a)** an equivalence relation
**Explanation:** Given Relation R = {(1, 1), (2, 2), (3, 3)}
**Reflexive:** If a relation has {(a, b)} as its element, then it should also have {(a, a), (b, b)} as its elements too.
**Symmetric:** If a relation has (a, b) as its element, then it should also have {(b, a)} as its element too.
**Transitive:** If a relation has {(a, b), (b, c)} as its elements, then it should also have {(a,c)} as its element too.
Now, the given relation satisfies all these three properties.
Therefore, its an equivalence relation.

19. **(d)** Symmetric but neither reflexive nor transitive.
**Explanation:** The relation R is symmetric only , because if $L_1$ is perpendicular to $L_2$ ,then $L_2$ is also perpendicular to $L_1$,but no other cases that is reflexive and transitive is not possible.

20. **(c)** $(-\infty, -1) \cup (1, 4]$
**Explanation:** We have, f(x)= $\sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$
f(x) is defined if $4 - x \geq 0$ and $x^2 - 1 > 0$
⇒ $x - 4 \leq 0$ and (x + 1)(x - 1) > 0
⇒ $x \leq 4$ and (x < -1 or x > 1)
∴ Domain of f $= (-\infty, -1) \cup (1, 4]$

21. Here we have, f(x) = $\sqrt{x-1}$ and g(x) = $\sqrt{x+1}$
Now, {x : g(x) = 0} = {x : $\sqrt{x+1}$ = 0} = {x : x + 1 = 0} = {-1}
∴ dom (f) ∩ dom (g) = {x : g(x) = 0} = [1, ∞)∩[-1, ∞) - {-1} = [1, ∞)
∴ $\frac{f}{g}$ → [1, ∞) → R is given by:
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x-1}}{\sqrt{x+1}}$

22. $f_3$ = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}
Every element of set X has an ordered pair in the relation $f_3$.
However, two ordered pairs (2, 9) and (2, 11) have the same first component but different second components.
Hence, the given relation $f_3$ is not a function.

23. We observe that f(x) = 0 for any $x \in R - \{-4\}$ Therefore, $\frac{1}{f} : R - \{-4\} \rightarrow R$ is given by
$\left(\frac{1}{f}\right)(x) = \frac{1}{f(x)} = \frac{1}{1/(x+4)}$ = (x + 4)

24. R = {(x, $x^3$) : x is a prime number less than 10}
Putting x = 2, 3, 5 , 7
R = {(2, 8), (3, 27), (5, 125), (7, 343)}

25. $R_4$ is not a relation from A to B, because (q, a) and (s, b) are elements of $R_4$ but (q, a) and (s, b) are not in A $\times$ B. As such $R_4 \subsetneq$ A $\times$ B.

26. Here we have, R = {(x, y)y = 2x + 7, where x$\in$R and $-5 \leq x \leq 5$ )}

Domain of $R_1 = \{-5 \leq x \leq 5, x \in R\} = [-5, 5]$

$x \in [-5, 5]$

$\Rightarrow 2x \in [-10, 10]$

$\Rightarrow 2x + 7 \in [-3, 17]$

Range is [-3, 17].

27. When $0 \leq x \leq 1,$ f(x) = x

Therefore,we have,

f $\left(\frac{1}{2}\right) = \frac{1}{2}$

28. To find: $\left(\frac{f}{g}\right)(x)$

$\left(\frac{f}{g}\right)(x) = \left(\frac{f(x)}{g(x)}\right)$

= $\left(\frac{x^3+1}{x+1}\right)$

= $\left(\frac{x^3+1^3}{x+1}\right)$

= $\left(\frac{(x+1)(x^2-x+1)}{x+1}\right)$(Because $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$)

= $x^2 - x + 1$

29. Here we need to express {(x, y) : $x^2 + y^2 = 25$, where x, y $\in$ W} as a set of ordered pairs.

It is easy to verify that each of the following ordered pairs of whole numbers satisfies the given relation $x^2 + y^2 = 25$:

(5, 0), (0, 5), (3, 4) and (4, 3).

Hence, the set of required ordered pairs is

{(5, 0), (0, 5), (3, 4), (4, 3)}.

30. Clearly, 2R2, 2R4, 2R6, 2R8, 4R4, and 4R8.

$\therefore$ R = {(2, 2), (2, 4 ), (2, 6), (2, 8), (4, 4), (4, 8)}.

31. Here we have A = [1, 3, 5] and B = [2, 3]

We have

$B \times A = [2, 3] \times \{1, 3, 5\}$

= {(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)}.

32. Given: f(x) = $\frac{x}{c}$

To find:(cf) (x)

(cf)(x) = c $\cdot$ f(x)

= c $\cdot$ ($\frac{x}{c}$)

= x

33. We have given that, relation R is defined on Z of integers

And R = {(x, y) : x, y $\in$ Z, $x^2 + y^2 \leq 4$}

= {(-2, 0), (-1, 0), (0, 0), (1, 0), (2, 0), (0, -2), (0, -1), (0, 1), (0, 2), (1, 1), (-1, -1), (1, -1), (-1, 1)}

Now we know,Domain is the set which consist all first elements of ordered pairs in relation R.

So, Domain(R) = {-2, -1, 0, 1, 2}

34. Given, A = {1, 2}, B = {2, 3 ,4 } and C = {4, 5}

B $\cap$ C = {2, 3, 4} $\cap$ {4, 5} = {4}

$\therefore$ A $\times$ (B $\cap$ C) = {1, 2}$\times$ {4}

= {(1, 4), (2, 4)}

35. **Read the text carefully and answer the questions:**

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever.

ONE – NATION
ONE – ELECTION
FESTIVAL OF
DEMOCRACY
GENERAL ELECTION –
2019


Let I be the set of all citizens of India who were eligible to exercise their voting right in the general election held in 2019. A relation 'R' is defined on I as follows:

R = {($v_1$, $v_2$) : $v_1$, $v_2$ ∈ I and both use their voting right in general election – 2019}

(i) **(d)** (X, Y) ∉ R

**Explanation:** (X, Y) ∉ R

36. According to the question , we can state,

Let (x, y) be an arbitrary element of (A ∩ B) × C.
⇒ (x, y) ∈ (A ∩ B) × C
Since, (x, y) are elements of Cartesian product of (A ∩ B) × C
⇒ x ∈ (A ∩ B) and y ∈ C
⇒ (x ∈ A and x ∈ B) and y ∈ C
⇒ (x ∈ A and y ∈ C) and (x ∈ B and y ∈ C)
⇒ (x, y) ∈ A × C and (x, y) ∈ B × C
⇒ (x, y) ∈ (A × C) ∩ (B × C) …1
Let (x, y) be an arbitrary element of (A × C) ∩ (B × C).
⇒ (x, y) ∈ (A × C) ∩ (B × C)
⇒ (x, y) ∈ (A × C) and (x, y) ∈ (B × C)
⇒ (x ∈ A and y ∈ C) and (x ∈ B and y ∈ C)
⇒ (x ∈ A and x ∈ B) and y ∈ C
⇒ x ∈ (A ∩ B) and y ∈ C
⇒ (x, y) ∈ (A ∩ B) × C …2
From 1 and 2, we get: (A ∩ B) × C = (A × C) ∩ (B × C)

37. Here it is given that A = {2, 3, 5} and R = {(2, 3), (2, 5), (3, 3), (3, 5)} and we need to show that R is a binary relation on A.

Now, $A \times A$ = {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)}
Since, R is a subset of $A \times A$. it's a binary relation on A.
Therefore, the domain of R is the set of first coordinates of R
Dom (R) = {2, 3}
The range of R is the set of second coordinates of R
Range (R) = {3, 5}

38. Here we have, $\left(\frac{a}{3}+1, b-\frac{1}{3}\right)=\left(\frac{5}{3}, \frac{2}{3}\right)$

∵ Given ordered pair are equal, So, corresponding elements are also equal.
$\therefore \frac{a}{3}+1=\frac{5}{3}$ ............(i)
After solving Eq. (i), we get
$\frac{a}{3}+1=\frac{5}{3} \Rightarrow a=3\left(\frac{5}{3}-1\right) \Rightarrow a=5-3 \Rightarrow a=2$
Now, $b-\frac{1}{3}=\frac{2}{3}$ .......(ii)
After solving Eq. (ii), we get
$b-\frac{1}{3}=\frac{2}{3}$
$b=\frac{2}{3}+\frac{1}{3} \Rightarrow b=1$
Therefore, the value of a = 2 and b = 1.

39. Here we have A = {-2, - 1, 0, 1, 2 }, B = { 0, 1, 4, 9 } and R = {(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)}

i. Since $R \subset A \times B$, so R is a relation from A to B Note that -2R4, -1R1, 0R0, 1R1 and 2R4.
ii. Dom (R) = set of first coordinates of elements of R
= {-2 ,-1 ,0 ,1 ,2 }.
Range (R) = set of second coordinates of elements of R

= {0, 1, 4}.
Co-domain of R = {0, 1, 4, 9} = B.

40. Here we have, f(x) = $\frac{1}{\sqrt{x^2-1}}$

we need to find where the function is defined

The condition for the function to be defined

$x^2 - 1 > 0$

$\Rightarrow x^2 > 1$

$\Rightarrow x > 1$

So, the domain of the function is the set of all the real numbers greater than 1

The domain of the function, $D_{\{f(x)\}} = (1, \infty)$

Now put any value of x within the domain set we get the value of the function always a fraction whose denominator is not equalled to 0

The range of the function, $R_{f(x)} = (0, 1)$.

41. Given a and b are integers, i.e. a, b $\in$ Z.

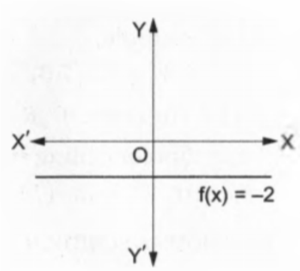
$\therefore$ Domain of R = Set of all first elements in the relation.

= Values of 'a' which are in the relation.

Since, $a^2 + b^2 = 25$ and a, b are integers;

⇒ R= {(5, 0), (0, 5), (-5, 0), (0, -5), (3, 4), (4, 3), (-3, -4), (-4, -3), (-3, 4), (4, -3), (-4, 3), (3, -4)}

⇒ Domain of R= {-5, -4, -3, 0, 3, 4, 5}

42. 


The graph of the function f(x) = -2 is the line y = -2. f is a constant function drawn parallel to the x-axis at a distance of 2 units below the x-axis.

43. Given: f(x) = $\sqrt{x+1}$ and $g(x) = \sqrt{9-x^2}$

We know $\left(\frac{1}{g}\right)(x) = \frac{1}{g(x)}$ and (cg)(x) = cg(x)

$\therefore \left(\frac{5}{g}\right)(x) = \frac{5}{\sqrt{9-x^2}}$

Domain of $f\frac{5}{g} =$ Domain of g = [–3, 3]

However, $\frac{5}{g} = (x)$ is defined for all real values of x $\in$ [–3, 3], except for the case when $9 - x^2 = 0$ or x = ±3

When x = ±3, $\frac{5}{g} = (x)$ will be undefined as the division result will be indeterminate.

Domain of $\frac{5}{g} =$ = [–3, 3] – {–3, 3}

$\therefore$ Domain of $\frac{5}{g} =$ = (–3, 3)

Thus, $\frac{5}{g} =$ (–3, 3)

R is given by $\left(\frac{5}{g}\right)(x) = \frac{5}{\sqrt{9-x^2}}$

44. Here we have, f(x) = $\sqrt{\frac{x-5}{3-x}}$

We need to find where the function is defined.

The condition for the function to be defined,

3 - x > 0 or x < 3

So, the domain of the function is the set of all the real numbers lesser than 3

The domain of the function, $D_{\{f(x)\}} = (\infty, 3)$

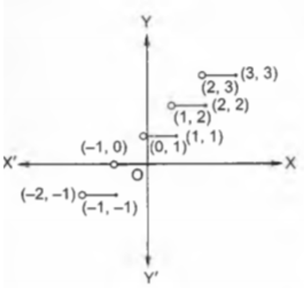
The condition for the range of the function to be defined,

x - 5 $\geq$ 0 & 3 - x > 0

x $\geq$ 5 & x < 3

Both the conditions can't be satisfied simultaneously, it means there is no range for the function f(x).

45. F(x ) = [x]




As the definition of the function suggests,

for all x such that $-3 < x \leq -2$ , we have f(x ) = - 2;

for all x such that -2 < x $\leq$ -1 , we have f(x ) = -1 ;

for all x such that -1 < x $\leq$ 0, we have f(x ) = 0;

for all x such that 0 < x $\leq$ 1, we have /(x ) = 1; and so on.

i.e., f(x ) = [x] $= \begin{cases} -2 \text{ when } x \in (-3,-2] \\ -1 \text{ when } x \in (-2,-1] \\ 0 \text{ when } x \in (-1,0] \\ 1 \text{ when } x \in (0,1] \\ 2 \text{when } x \in (1,2] \\ 3 \text{ when } x \in (2,3] \\ \text{and so on.} \end{cases}$

Plotting these points, we can get the required graph. The function jumps at the points (-2, -1), (-1, 0), (0, 1), (1, 2), etc., or is discontinuous at each integral value of x. Required graph of function shown in figure.

46. Given that R = {x, y} : x, y $\in$ Z and $x^2 + y^2 = 25$} and we need to express R and $R^{-1}$ as sets of ordered pairs.

Now, $x^2 + y^2 = 25$

Put x = 0, y = 5, $0^2 + 5^2 = 25$

Put x = 3, y = 4, $3^2 + 4^2 = 25$

R = {(0, 5), (0, -5), (5, 0), (3, 4), (-3, 4), (-3, -4), (3, -4)}

Since, x and y get interchanged in the ordered pairs, R and $R^{-1}$ are same.

47. We are given the function,

$f(x) = 4x - x^2$

f(a + 1) - f(a - 1) = $[4(a + 1) - (a + 1)^2] - [4(a - 1) - (a - 1)^2]$

= $4[(a + 1) - (a - 1)] - [(a + 1)^2 - (a + 1)^2]$

= 4(2) - [(a + 1 + a - 1) (a + 1 - a + 1)]

Using: $a^2 - b^2 = (a + b)(a - b)$

f(a + 1) - (a - 1) = 4(2) - 2a (2)

= 4(2 - a)

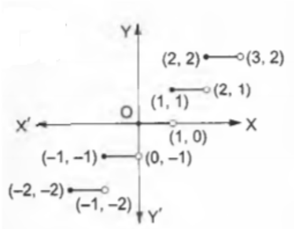
48. Given, $|x-2| = \begin{cases} x-2, & x \geq 2 \\ -(x-2), & x < 2 \end{cases}$ and $|x+2| = \begin{cases} (x+2) & x \geq -2 \\ -(x+2), & x < -2 \end{cases}$

$\Rightarrow \quad f(x) = \begin{cases} (x-2)+(2+x), & 2 \leq x \leq 3 \\ -(x-2)+(x+2), & -2 \leq x < 2 \\ -(x-2)-(x+2), & -3 \leq x < -2 \end{cases}$

$= \begin{cases} x-2+2+x, & 2 \leq x \leq 3 \\ -x+2+x+2, & -2 \leq x < 2 \\ -x+2-x-2, & -3 \leq x < -2 \end{cases}$

$= \begin{cases} 2x, & 2 \leq x \leq 3 \\ 4, -2 \leq x < 2 \\ -2x, -3 \leq x < -2 \end{cases}$

49. f(x) = [x].




As the definition of the function indicates,
for all x such that -2 $\leq$ x < - 1, we have f(x) = - 2 ;
for all x such that -1 $\leq$ x < 0, we have f(x) = - 1;
for all x such that 0 $\leq$ x < 1, we have/(x) = 0;
for all x such that 1 $\leq$ x <2, we have f(x) = 1,

and so on, f(x) = [x] = $\begin{cases} -2 \text{ when } x \in [-2,-1) \\ -1 \text{ when } x \in [-1,0) \\ 0 \text{ when } x \in [0,1) \\ 1 \text{ when } x \in [1,2) \\ \text{and so on.} \end{cases}$

Clearly, the function jumps at the points (-1 , -2), (0, -1), (1, 0), (2,1), etc.
In other words, the given function is discontinuous at each integral value of x.

50. Here f (x) $= \frac{1}{x+2}$
f(x) assume real values for all real values of x except for x + 2 = 0 i.e. x = - 2.
Thus domain of f (x) =R - {-2}.

51. (i) Let $(x,y) \in R. Now, (x,y) \in R \Rightarrow x-y$ is divisible by n
$\Rightarrow x-y=kn$ for some k$\in Z$
$\Rightarrow y-x=(-k)n$
$\Rightarrow y-x$ is divisible by n
$\Rightarrow (y,x) \in R$ .
(ii) Let $(x,y) \in R, (y,z) \in R$
$Now\ (x,y) \in R \Rightarrow x-y$ is divisible by n $\Rightarrow x-y=kn$ for some $k \in z.\ldots.(1)$
$Now, (y,z) \in R \Rightarrow\ y-z$ is divisible by n $\Rightarrow y-z=mn$ for some $m \in z.\ldots..(2)$
$\Rightarrow$ Adding (1) & (2), we get,
$\Rightarrow$ (x - z) = n(k + m)
$\Rightarrow$ x - z is divisible by n
$\Rightarrow (x,z) \in R$

52. Here $f(x) = \frac{x^2}{1+x^2}$
Put $y = \frac{x^2}{1+x^2} \Rightarrow y + yx^2 = x^2 \Rightarrow x^2(1-y) = y$
$\Rightarrow x^2 = \frac{y}{1-y} \Rightarrow x = \pm\sqrt{\frac{y}{1-y}}$
$\frac{y}{1-y} \geqslant 0$
$\Rightarrow \frac{y}{y-1} \leqslant 0$
$\Rightarrow 0 \leqslant y < 1$
$\Rightarrow y \in [0,1)$
$\therefore$ Range of f(x) = [0, 1)

53. Suppose $(x,y) \in A \times A$
$\Rightarrow \quad x,y \in A$
$\Rightarrow \quad x,y \in B$ [ Given: $A \subseteq B$ ]
Now, $x \in A, y \in B \Rightarrow (x,y) \in A \times B$
and $x \in B, y \in A$
$\Rightarrow \quad (x,y) \in B \times A$
$\Rightarrow \quad (x,y) \in (A \times B) \cap (B \times A)$
$\therefore \quad A \times A \subseteq (A \times B) \cap (B \times A)$ ...... (i)
Suppose $(x,y) \in (A \times B) \cap (B \times A)$
$\Rightarrow (x,y) \in A \times B$ and $(x,y) \in B \times A$
$\Rightarrow$ ($x \in A$ and $y \in B$) and ($x \in B$ and $y \in A$)

$\Rightarrow \quad x, y \in A \Rightarrow (x, y) \in A \times A$

$\therefore \quad (A \times B) \cap (B \times A) \subseteq (A \times A)$ ....... (ii)

From Eqs. (i) and (ii), we get

$A \times A = (A \times B) \cap (B \times A)$

Hence proved.

54. Here we have, f(x) = $\frac{x^2+1}{x^2-1}$

Clearly, f(x) is defined for all real values of x except that at which $x^2 - 1 = 0$, i.e., x = $\pm 1$

$\therefore$ dom(f) = R - {-1, 1}

Let y = f(x). Then, we have

y = $\frac{x^2+1}{x^2-1} \Rightarrow x^2y - y = x^2 + 1 \Rightarrow x^2(y - 1) = (y + 1)$

$\Rightarrow x^2 = \frac{y+1}{y-1} \Rightarrow x = \pm\sqrt{\frac{y+1}{y-1}}$ .........(i)

It is clear from equation (i) that x is not defined when y - 1 = 0 or when $\frac{y+1}{y-1} < 0$

Now, y - 1 = 0 $\Rightarrow$ y = 1 ..............(ii)

And $\frac{y+1}{y-1} < 0 \Rightarrow$ (y + 1 > 0 and y - 1 < 0) or (y + 1 < 0 and y - 1 > 0)

$\Rightarrow$ (y > -1 and y < 1) or (y < -1 and y > 1)

$\Rightarrow$ -1 < y < 1 .............(iii)

[$\because$ y < -1 and y > 1 is not possible]

Thus, x is not defined when -1 < y $\leq$ 1. [using (ii) and (iii)]

$\therefore$ range (f) = R - (-1, 1]

Hence, dom(f) = R- {-1, 1} and range (f) = R - (-1, 1].

55. Given, $f(x) = \frac{3}{2-x^2}$

We know that, f(x) is not defined when $(2 - x^2) = 0$

i.e., $x = \pm\sqrt{2}$

$\therefore$ Domain of $f = R - \{-\sqrt{2}, \sqrt{2}\}$

Also let, $y = \frac{3}{2-x^2} \Rightarrow 2 - x^2 = \frac{3}{y}$

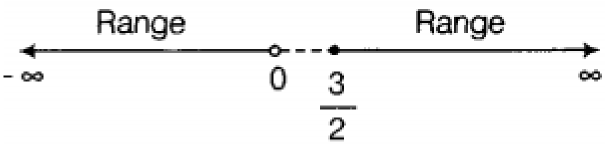
$\Rightarrow \quad x^2 = 2 - \frac{3}{y} \quad \Rightarrow x = \pm\sqrt{2 - \frac{3}{y}} = \pm\sqrt{\frac{2y-3}{y}}$

x is defined, if $\frac{2y-3}{y} \geq 0$ and $y \neq 0,$ i.e.

$(2y - 3) \geq 0, y < 0$ and $y \neq 0$

$\Rightarrow -\infty < y < 0$ and $\frac{3}{2} \leq y < \infty$

$\therefore$ Range of $f = (-\infty, 0) \cup \left[\frac{3}{2}, \infty\right)$




56. Here R = {(x, x + 5) : $x \in$ (0, 1, 2, 3, 4, 5)}

= {(a, b): a = 0, 1, 2, 3, 4, 5}

Now a = x and b = x + 5

Putting a = 0, 1, 2, 3, 4, 5 we get b = 5, 6, 7, 8, 9, 10

$\therefore$ Domain of R = {0, 1, 2, 3, 4, 5}

Range of R = {5, 6, 7, 8, 9, 10}

57. We have, f :R $\rightarrow$ R: f(x) = $x^3$ for all x $\in$ R

dom (f) = R and range (f) = R.

We have,

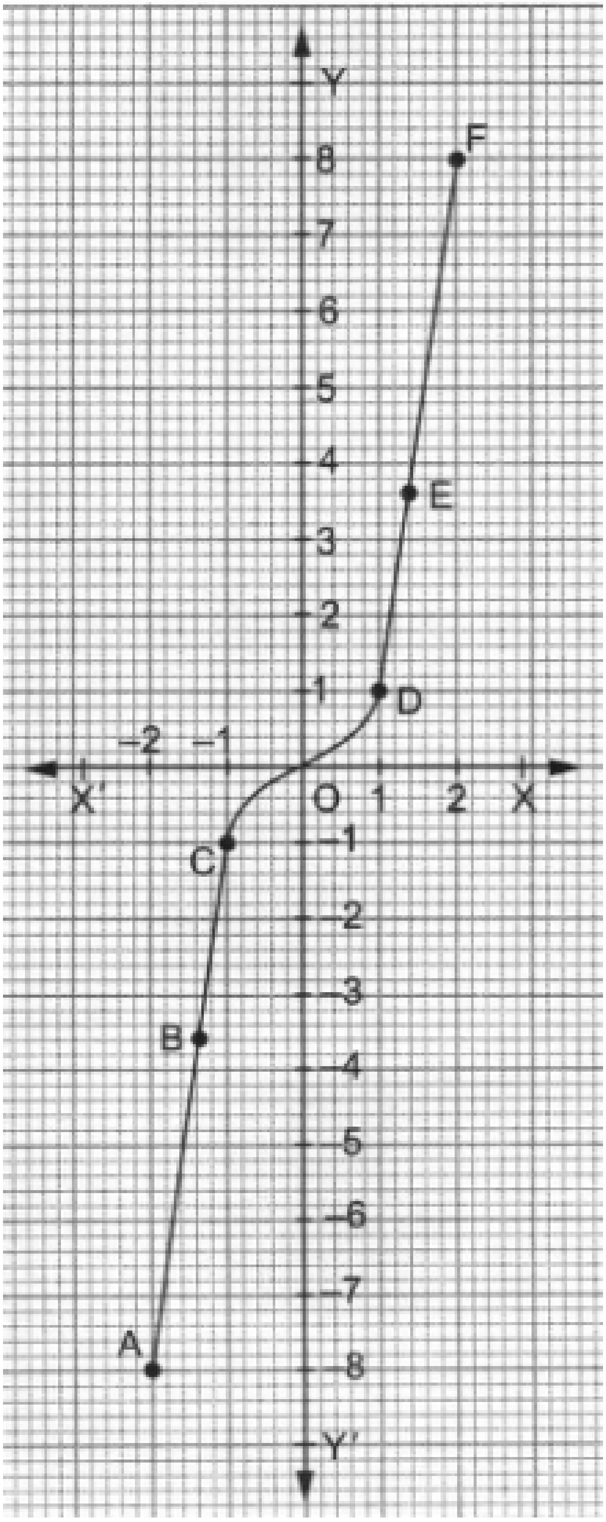
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | -2 | -1.5 | - 1 | 0 | 1 | 1.5 | 2 |
| f(x) = $x^3$ | -8 | -3.375 | - 1 | 0 | 1 | 3.375 | 8 |

On a graph paper, we draw X' OX and YOY' as the x-axis and the y-axis respectively.

We take the scale as 5 small divisions = 1 unit.

Now, we plot the points A(-2, - 8), B(-1.5, -3.375), C(-1, -1), 0(0, 0), D(l, 1), E(1.5, 3.375) and F(2, 8).

We join these points freehand successively to obtain the required curve shown in the figure below.




58. Here f(x) = 2x - 5

  i. Putting x = 0

  $\therefore$ f (0) = $2 \times 0 - 5 = -5$

  ii. Putting x = 7

  $\therefore f(7) = 2 \times 7 - 5 = 9$

  iii. Putting x = -3

  $\therefore f(-3) = 2 \times -3 - 5 = -11$

59. We have,

f(x) = $x^2$ -2x - 3

Now,

f(-2) = $(-2)^2$ - 2(-2) - 3

= 4 + 4 - 3

= 5

f (-1) = $(-1)^2$ -2(-1)-3

= 1 + 2 - 3

= 0

f (0) = $(0)^2$ - 2 $\times$0 - 3

= -3

f(1) = $(1)^2 - 2\times1 - 3$

= 1 - 2 - 3

= - 4

f(2) = $(2)^2$ - 2 $\times$ 2 - 3

= 4 - 4 - 3

= -3

Range(f) = {-4, -3, 0, 5}

60. Here (x, 1) $\in$ A $\times$ B $\Rightarrow$ x $\in$ A and 1 $\in$ B

    (y, 2) $\in$ A $\times$ B $\Rightarrow$ y $\in$ A and 2 $\in$ B

    (z, 1) $\in$ A $\times$ B $\Rightarrow$ z $\in$ A and 1

    It is given that n(A) = 3 and n(B) = 2

    $\therefore$ A = {x, y, z}

    and B = {1, 2}

61. i. To show: F is a relation from X to Y

       First elements in F = 1, 2, 3, 4

       All the first elements are in Set X

       So, the first element is from set x

       Second elements in F = 5, 9, 1, 11

       All the second elements are in Set Y

       So, the second element is from set Y

       since the first element is from set X and the second element is from set Y

       Hence,F is a relation from X to Y

    ii. To show: F is a function from X to Y

       Function:

       i. all elements of the first set are associated with the elements of the second set.
       ii. An element of the first set has a unique image in the second set. F = {(1, 5),(2, 9),(3, 1),(4, 5),(2, 11)}

           Here, 2 is coming twice.

           Hence, it does not have a unique (one) image.

           So, it is not a function.

62. Given, f(x) = $\begin{cases} 1+2x & x<0 \\ 3+5x, & x \geq 0 \end{cases}$

    Here, f(x) = 1 + 2x, x < 0, this gives

    f(-4) =1 + 2(-4) = -7

    f(-3) = 1 + 2(-3) = -5

    f(-2) = 1 + 2(-2) = -3

    f(-1) = 1 + 2(-1) = -1

    f(x) = 3 + 5x, x $\geq$0

    f(0) = 3 + 5(0) = 3

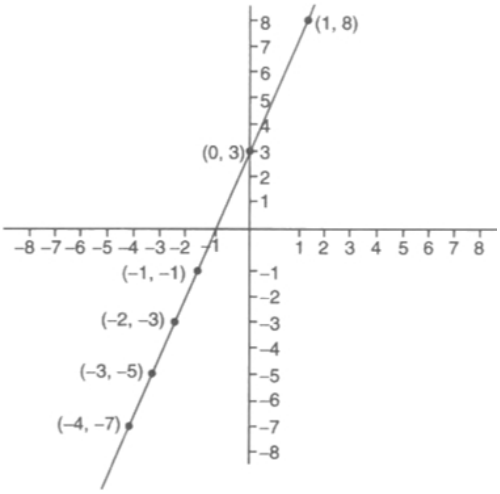
    f(1) = 3 + 5(1) = 8

    f(2) = 3 + 5(2) = 13

    f(3) = 3 + 5(3) = 18

    f(4) = 3 + 5(4) = 23

    Now the graph of f is as shown in following figure




**Range:** Let $y_1$ = f(x), x < 0

$\therefore y_1 = 1 + 2x, x < 0$

$\therefore x = \frac{y_1 - 1}{2}, x < 0$

$\because x < 0 \Rightarrow y_1 - 1 < 0 \Rightarrow y_1 < 1$

Let $y_2$ = f(x), $x \geq 0$

$\Rightarrow y_2 = 3 + 5x, x \geq 0$

$\Rightarrow x = \frac{y_2 - 3}{5}, x \geq 0$

$\because x \geq 0 \Rightarrow y_2 - 3 \geq 0 \Rightarrow y_2 \geq 3$

Therefore, range of f$(-\infty, 1), \cup[3, \infty)$

63. Here we need to find the sum and the difference of the identity function and the modulus function.

Let f :R $\rightarrow$ R: f(x) = x be the identity function.

And, let g : R $\rightarrow$ R : g(x) = |x| be the modulus function.

Then, dom (f) = R and dom (g) = R.

$\therefore$ dom (f) $\cap$ dom (g) = R$\cap$R = R.

i. dom (f + g) = dom (f) $\cap$ dom (g) = R.

Now, (f + g ): R $\rightarrow$ R is given by

(f + g)(x) = f(x) + g(x)

= x + |x| = x + $\begin{cases} x, \text{ when } x \geq 0 \\ -x, \text{ when } x < 0 \end{cases}$

= $\begin{cases} x + x, \text{ when } x \geq 0 \\ x - x, \text{ when } x < 0 \end{cases} = \begin{cases} 2x, \text{ when } x \geq 0 \\ 0, \text{ when } x < 0 \end{cases}$

Hence, (f + g)(x) = $\begin{cases} 2x, \text{ when } x \geq 0 \\ 0, \text{ when } x < 0 \end{cases}$

ii. dom (f - g) = dom (f) $\cap$ dom (g) = R.

$\therefore$ (f - g)(x) = f(x) - g(x)

= x - |x| = x - $\begin{cases} x, \text{ when } x \geq 0 \\ -x, \text{ when } x < 0 \end{cases}$

= $\begin{cases} x - x, \text{ when } x \geq 0 \\ x + x, \text{ when } x < 0 \end{cases} = \begin{cases} 0, \text{ when } x \geq 0 \\ 2x, \text{ when } x < 0 \end{cases}$

$\therefore$ (f - g)(x) = $\begin{cases} 0, \text{ when } x \geq 0 \\ 2x, \text{ when } x < 0 \end{cases}$

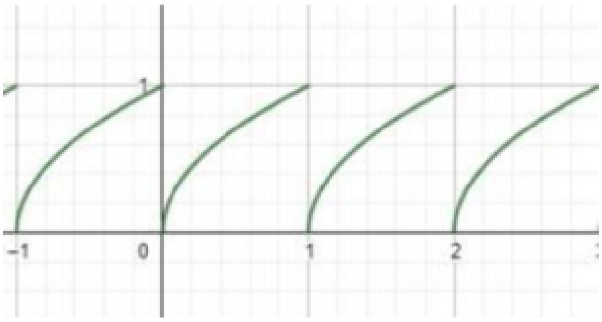
64. Here we are given that, f(x) = $\sqrt{x - [x]}$

Where [x] is the Greatest integer Function of x.

f(x) = $\sqrt{\{x\}}$

Where {x} is fractional part of x

The graph of f(x) is:




i. **Domain:**

Domain of {x} is R.

The value of the fractional part of x is always either positive or zero.

Hence domain of x is R

ii. **Range:**

Range of {x} is [0, 1)

As the root value [0, 1) between interval lies between [0, 1)

Hence range of f(x) is [0, 1).

65. Here we are given that A, B and C three sets.

To prove: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let us consider, $(x, y) \in A \times (B \cap C)$

$\Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow (x \in A$ and $y \in B)$ $(x \in A$ and $y \in C)$

$\Rightarrow (x, y) \in (A \times B)$ and $(x, y) \in (A \times C)$

$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

From above, we can say that,

$\Rightarrow A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ ...............(i)

Let us consider again, $(a, b) \in (A \times B) \cap (A \times C)$

$\Rightarrow (a, b) \in (A \times B)$ and $(a, b) \in (A \times C)$

$\Rightarrow (a \in A$ and $b \in B)$ and $(a \in A$ and $b \in C)$

$\Rightarrow a \in A$ and $(b \in B$ and $b \in C)$

$\Rightarrow a \in A$ and $b \in (B \cap \mathrm{C})$

$\Rightarrow (a, b) \in A \times (B \cap C)$

From above, we can say that,

$\Rightarrow (A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ ...............(ii)

From (i) and (ii).

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hence proved.

66. i. Given: A = {2, 3, 5} and B = {5, 7}

To find $A \times B$

As we know that According to the definition of the Cartesian product,

Given two non-empty sets X and Y. The Cartesian product X × Y is the set of all ordered pairs of elements from X and Y, i.e,

$\mathrm{X} \times \mathrm{Y} = \{(\mathrm{X}, \mathrm{Y}) : \mathrm{x} \in \mathrm{X}, \mathrm{y} \in \mathrm{Y}\}$

Here, A = {2, 3, 5} and B = {5, 7}. So, A × B = {2, 3, 5} × {5, 7}

A × B = {(2, 5), (3, 5), (5, 5), (2, 7), (3, 7), (5, 7)}

ii. Given: A = {2, 3, 5} and B = {5, 7}

To find: B × A

As we know that According to the definition of the Cartesian product,

Given two non-empty sets X and Y. The Cartesian product X × Y is the set of all ordered pairs of elements from X and Y, i.e.,

$\mathrm{X} \times \mathrm{Y} = \{(\mathrm{X}, \mathrm{Y}) : \ \mathrm{x} \in \mathrm{X}, \mathrm{y} \in \mathrm{Y}\}$

Here, A = {2, 3, 5} and B = {5, 7}. So, B × A = (5, 7) × (2, 3, 5)

B × A = {(5, 2), (5, 3), (5, 5), (7, 2), (7, 3), (7, 5)}

iii. Given: A = {2, 3, 5} and B = {2, 3, 5}

To find: $A \times A$

As we know that According to the definition of the Cartesian product,

Given two non-empty sets X and Y. The Cartesian product X $\times$ Y is the set of all ordered pairs of elements from X and Y, i.e,

$\mathrm{X} \times \mathrm{Y} = \{(\mathrm{X}, \mathrm{Y}) : \mathrm{x} \in \mathrm{X}, \mathrm{y} \in \mathrm{Y}\}$

Here, A = {2, 3, 5} and A = {2, 3, 5}. So, A $\times$ A = (2, 3, 5) $\times$ (2, 3, 5)

A $\times$ A = {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)}

iv. Given: B = {5, 7}

To find: $B \times B$

As we know that According to the definition of the Cartesian product,

Given two non-empty sets X and Y. The Cartesian product X $\times$ Y is the set of all ordered pairs of elements from X and Y, i.e.,

$\mathrm{X} \times \mathrm{Y} = \{(\mathrm{X}, \mathrm{Y}) : \mathrm{x} \in \mathrm{X}, \mathrm{y} \in \mathrm{Y}\}$

Here, B = {(5, 7) and B = {5, 7}. So, B $\times$ B = (5, 7) $\times$ (5, 7)}

B $\times$ B = {(5, 5), (5, 7), (7, 5), (7, 7)}

67. Given that,

R = {(1, 39), (2, 37), (3, 35) .... (19, 3), (20, 1)}

Domain = {1,2,3,.......,20}

Range = {1,3,5,7......,39}

R is not reflexive as (2, 2) $\notin$ R as

$2 \times 2 + 2 \neq 41$

R is not symmetric

as (1, 39) $\in$ R but (39, 1) $\notin$ R

R is not transitive

as (11, 19) $\in$ R, (19, 3) $\in$ R

But (11, 3) $\notin$ R

Hence, R is neither reflexive, nor symmetric and nor transitive.

68. i. **To determine A $\times$ (B $\cup$ C)**

B $\cup$ C = {b, c, e} $\cup$ {b, c, f} = {b, c, e, f}

$\therefore$ A$\times$(B $\cup$ C) = {a, d} $\times$ {b, c, e, f}

= {(a, b), (a, c), (a, e), (a, f), (d, b), (d, c), (d, e), (d, f)} ...(i)

**To determine (A $\times$ B) $\cup$ (A $\times$ C)**

A $\times$ B = {a, d} $\times$ {b, c, e}

= {(a, b), (a, c), (a, e), (d, b), (d, c), (d, e)}

A $\times$ C = {a, d} $\times$ {b, c, f}

= {(a, b), (a, c), (a, f), (d, b), (d, c), (d, f)}

$\therefore$ (A$\times$B) $\cup$ (A$\times$C)

= {(a, b), (a, c), (a, e), (a, f), (d, b), (d,c), (d,e),(d,f)} ...(ii)

From Eqs. (i) and (ii), we get

A$\times$(B$\cup$C) = (A$\times$B)$\cup$(A$\times$C)

**Hence verified.**

ii. **To determine A $\times$ (B $\cap$ C)**

(B $\cap$ C) = {b, c, e} $\cap$ {b, c, f} = {b, c}

$\therefore$ A $\times$ (B $\cap$ C) = {a, d} $\times$ {b, c}

= {(a, b), (a, c), (d, b), (d, c)} ...(iii)

**To determine (A$\times$B)$\cap$(A$\times$C)**

A $\times$ B = {(a, b), (a, c), (a, e), (d, b), (d, c), (d, e)}

A $\times$ C = {(a, b), (a, c), (a, f), (d, b), (d, c), (d, f)}

$\therefore$ (A$\times$B)$\cap$(A$\times$C) = {(a, b), (a, c), (d, b), (d, c)} ...(iv)

From Eqs. (iii) and (iv), we get

A$\times$(B$\cap$C) = (A$\times$B)$\cap$(A$\times$C)

**Hence verified.**

69. For all $x_1\ x_2 \in$ A

if $f(x_1) = f(x_2)$ implies $x_1 = x_2$ then f is one one

Now $f(x_1) = f(x_2)$

$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

Cross multiplying and solving, we get

$x_1 = x_2$

f is one – one

$y = \frac{(x-2)}{(x-3)}$

$(x-3)y = x-2$

$xy - 3y = x - 2$

$xy - x = 3y - 2$

$x = \frac{(3y-2)}{(y-1)}$

$f\left(\frac{3y-2}{y-1}\right) = y$

Hence f is onto.

70. i. a. Let $(x, y) \in$ R

$\Rightarrow x - y$ is divisible by n.

$\Rightarrow x - y = kn$ for some k $\in$ Z

$\Rightarrow y - x = (-k)n$

$\Rightarrow y - x$ is divisible by n.

$\Rightarrow (y, x) \in R$

b. Let $(x, y) \in R$ and (y, z)$\in$ R

Now, $(x, y) \in R \Rightarrow x - y$ is divisible by n.

$\Rightarrow x - y = kn$ for some k $\in$ Z

Also, $(y, z) \in R \Rightarrow y - z$ is divisible by n.

$\Rightarrow y - z = mn$ for some m $\in$ Z.

$\Rightarrow (x-y) + (y-z) = kn + mn$

$\Rightarrow x - z = (k+m)n$

$\Rightarrow$ x - z is divisible by n.

$\Rightarrow$ (x, z) $\in R$

ii. Here, $f(x) = \frac{x^2-9}{x-3}$

f(x) assume all real values of x except for x - 3 = 0

i.e., x = 3.

Thus, domain of f(x) = R - {3}.

Let f(x) = y

$\therefore \quad y = \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{(x-3)}$

$\Rightarrow y = x + 3$

Since y takes all real values except 6.

Thus, range of f(x) = R - {6}.

iii. Here, $f(x) = \frac{x^2+3x+5}{x^2+x-6}$

$= \frac{x^2+3x+5}{(x+3)(x-2)}$

The function f(x) is defined for all values of x except for $x + 3 = 0$ and $x - 2 = 0$ i.e., $x = -3$ and $x = 2$.

Thus, domain of f(x) = R - {-3, 2}.

71. State True or False:

(i) **(a)** True

**Explanation:** True

(ii) **(a)** True

**Explanation:** True

(iii) **(b)** False

**Explanation:** False

(iv) **(b)** False
**Explanation:** False

(v) **(a)** True
**Explanation:** True