Solution

QUESTION BANK (SETS)

Class 11 - Mathematics

1. **(a)** 2^n

   Explanation: 2^n

   The no. of subsets containing n elements is 2^n.

2. **(d)** 6, 3

   Explanation: Since, let A and B be such sets, i.e., n (A) = m, and n(B) = n

   Thus, n (P(A)) = 2^m , n (P(B)) = 2^n

   Therefore, n (P(A)) – n (P(B)) = 56, i.e., $2^m - 2^n = 56$

   $\Rightarrow 2^n (2^{m-n} - 1) = 2^3\ 7$

   $\Rightarrow$ n = 3, $2^{m-n} - 1 = 7$

   $\Rightarrow$ m = 6

3. **(b)** { }

   Explanation: Here value of x is not possible so A is a null set.

4. **(d)** $B \subseteq A$

   Explanation: $B \subseteq A$

   Because B is contained in A..So the union of these two will be A

5. **(d)** A

   Explanation: Common between set A and $(A \cup B)$is set A itself

6. **(d)** $F_2 \cup F_3 \cup F_4 \cup F_1$

   Explanation: We know that

   Every rectangle, square and rhombus is a parallelogram

   But, no trapezium is a paralleogrm

   Thus, $F_1 = F_2 \cup F_3 \cup F_4 \cup F_1$

7. **(d)** $\{x : x \in R,\ 4 \leq x < 5\}$

   Explanation: Set A represents the elements which are greater or equals to 4 and the elements are real no. A$[4, \infty)$

   Set B represents the elements which are less than 5 and are real no. B$(-\infty, 5)$

   So if we represent these two in number line we can see the common region is between 4(included) and 5(excluded).

8. **(c)** A ∩ B = ϕ

   Explanation: We have,A = $\left\{(x, y) | y = \frac{1}{x}, 0 \neq x \in \mathbf{R}\right\}$ and B = {(x, y) | y = -x, x ∈ R}

   For any element of A ∩ B, A and B will have same value of y

   $\Rightarrow -x^2 = 1$

   $\Rightarrow x^2 = -1$

   Square of any value cannot be negative

   Thus, there is no value of x for which A and B will have same value of y

   $\Rightarrow$ A ∩ B = ϕ

9. **(d)** $B^c \subset A^c$

   Explanation: Let A ⊂ B

   To prove $B^c \subset A^c$, it is enough to show that $x \in B^c \Rightarrow x \in A^c$

   Let $x \in B^c$

   $\Rightarrow x \notin B$

   $\Rightarrow x \notin A$ since A ⊂ B

   $\Rightarrow x \in A^c$

   Hence $B^c \subset A^c$

10. **(c)** 45

Explanation: Now to find value of n

Since elements are not repeating, number of elements in $A_1 \cup A_2 \cup A_3 \cup \dots\dots\dots \cup A_{30}$ is 30×5

But each element is used 10 times

Thus, $10 \times S = 30 \times 5$

$\Rightarrow 10 \times S = 150$

$\Rightarrow S = 15$

Since elements are not repeating, number of elements in $B_1 \cup B_2 \cup B_3 \cup \dots\dots\dots \cup B_n$ is $3 \times n$

But each element is used 9 times

Thus, $9 \times S = 3 \times n$

$\Rightarrow 9 \times S = 3n$

$\Rightarrow S = \frac{n}{3}$

$\Rightarrow \frac{n}{3} = 15$

$\Rightarrow n = 45$

Therefore, the value of n is 45

11. **(d)** four points

Explanation: From A, $x^2 + y^2 = 25$ and from B, $x^2 + 9y^2 = 144$

$\therefore$ From B, $(x^2 + y^2) + 8y^2 = 144$

$\Rightarrow 25 + 8y^2 = 144$

$\Rightarrow 8y^2 = 119$

$\Rightarrow y = \pm\sqrt{\frac{119}{8}}$

$\therefore x^2 + y^2 = 25 \Rightarrow x^2 = 25 - y^2 = 25 - \frac{119}{8} = \frac{81}{8}$

$\Rightarrow x = \pm\sqrt{\frac{81}{8}}$

Since we solved equations simultaneously, therefore $A \cap B$ has four points A has 2 elements & B has 2 elements.

12. **(d)** {1, 2, 3, 4}

Explanation: Given A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}

$B \cap C = \{4\}$

$A \cup (B \cap C) = \{1, 2, 3, 4\}$

13. **(d)** (4, 5)

Explanation: We have,$A = \{x : x \in R, x > 4\}$ and $B = \{x \in R : x < 5\}$

$A \cap B = (4, 5)$

14. **(b)** {3, 6, 9, 12, 18, 21, 24, 27}

Explanation: Since set B represent multiple of 5 so from Set A common multiple of 3 and 5 are excluded.

15. **(c)** {3, 5, 9}

Explanation: The union of two sets A and B is the set of elements in A, or B, or both.

So smallest set A = {3, 5, 9}

16. **(a)** A and the complement of B are always non-disjoint

Explanation: Let $x \in A$, then $x \notin B$ as A is not a subset of B

$\therefore x \in A$ and $x \notin B$

$\Rightarrow x \in A$ and $x \in B'$

$\Rightarrow x \in A \cap B'$

$\Rightarrow$ A and B' are non - disjoint.

17. **(c)** 7, 4

Explanation: Now to find value of m and n

The number of subsets of a set containing x elements is given by 2^x

According to question: $2^m - 2^n = 112$

$\Rightarrow 2^n (2^{m-n} - 1) = 16 \times 7$

$\Rightarrow 2^n (2^{m-n} - 1) = 2^4 \times 7$

On comparing on both sides $2^n = 2^4$ and $2^{m-n} - 1 = 7$

$\Rightarrow$ n = 4 and $2^{m-n} = 8$

$\Rightarrow 2^{m-n} = 2^3$

$\Rightarrow$ m – n = 3

$\Rightarrow$ m – 4 = 3

$\Rightarrow$ m = 7

Therefore,the value of m and n is 7 and 4 respectively

18. **(b)** 20

Explanation: The correct answer is (B)

Since, $n(X_r) = 5$, $\bigcup_{r=1}^{20} X_r$ = S, we obtain n(S) = 100

But each element of S belong to exactly 10 of the X 's

Thus, $\frac{100}{10}$ = 10 are the number of distinct elements in S.

Also each element of S belong to exactly 4 of the Y_r's and each $Y_{r's}$contain 2 elements. If S has n number of Y_r in it.

Then $\frac{2n}{4}$ = 10

which gives n = 20

19. **(d)** (x: x $\neq$x).

Explanation: (x: x $\neq$x). x is not equal to x is null set as it refers to there is no element in the set.And it also representing the set builder form pattern

20. **(a)** an infinite set

Explanation: Set A = {2, 3, 5, 7,...} so it is infinite.

21. We know that all factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

$\therefore$ A = {1, 2, 3, 4, 6, 8, 12, 24}

22. Here A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15} and D = {15, 17}

$B \cap D = \{7, 9, 11, 13\} \cap \{15, 17\} = \phi$

23. Here, x + 3 = 3

x = 0

So, {x : x + 3 = 3} = {0}

It is not ϕ

$\therefore$ The correct form would be

{x : x + 3 = 3} = {0}

24. The answer is D = [-5, 2]

D = {x : x $\in$ R, –5 $\leq$ x $\leq$ 2} is a closed interval from -5 to 2 and contains the end points.

25. We have

C = {x : x $\in$ R, -2 $\leq$ x < 0} = [-2, 0).

length (C) = 0 - (-2) = 2.

26. The answer is A $\subset$ B

Explanation: we have, A ={ set of real numbers }and B ={ set of complex numbers}, a combination of the real and imaginary number in the form of a+ib, where a and b are real, and i is imaginary.

Since, any real number can be expressed as complex number, A $\subset$ B.

27. The interval [6, 12] can be written in set builder form as $\{x : x \in R, 6 \leqslant x \leqslant 12\}$

28. C = {x : x is a two-digit natural number such that the sum of its digit is 8}

$\therefore$ C = {17, 26, 35, 44, 53, 62, 71, 80}

29. Note that $x \in X \cap Y \Rightarrow x \in X \, and \, x \in Y$

Thus $X \cap Y \subset X$

Also, since $X \subset Y$,

$x \in X \Rightarrow x \in Y \Rightarrow x \in X \cap Y$

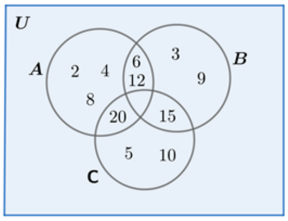
so that $X \subset X \cap Y$

Therefore the result X = X ∩ Y follows.

30. Therefore, (-7, 0) = {x : x $\in$ R and -7 < x < 0}

31. Here X = {a, b, c, d} and Y = {f, b, d, g}

Y - X = {f, b, d, g} - {a, b, c, d}
= {f , g}
32. Here A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15} and D = {15, 17}
$A \cap B = \{3, 5, 6, 7, 11\} \cap \{7, 9, 11, 13\} = \{7, 9, 11\}$
33. C = {x : x is an integer, $X^2 \leqslant 4$
$\therefore X^2 \leqslant 4 \Rightarrow x \leqslant \pm 2 \Rightarrow -2 \leqslant x \leqslant 2 \quad \therefore C = (-2, -1, 0, 1, 2)$
34. Since there is no natural number between 4 and 5. Therefore ,C is an empty set.
35. Therefore, {x : x ∈ R, -12 < x < -10} = (-12, -10).
Length = -1 0 - (-12) = 2
36. We know that, AΔB represents the symmetric difference of sets A and B.
That is,
AΔB = (A - B) ∪ (B - A)
According to the question,
A = {1, 3, 5, 7, 9}, and B = {2, 3, 5, 7, 11}
Then, A - B = {1, 3, 5, 7, 9 } - { 2, 3, 5, 7, 11} = {1, 9}
and B - A = {2, 3, 5, 7, 11} - {1, 3, 5, 7, 9} = {2, 11}
Hence,
AΔB = (A - B) ∪ (B - A)
= {1, 9}∪{2, 11}
= { 1, 2, 9 , 11}
37. Venn diagram showing relation of U, A, B and C sets in below.
We have, U is a universal set
The intersection of two sets A and B, consists of all elements that are both in A and B.
For example: {1, 2} ∩ {2, 3} = {2}
Thus,A = {2, 4, 6, 8, 12, 20}, B = {3, 6, 9, 12, 15} and C = {5, 10, 15, 20}
⇒ A ∩ B = {6, 12}, B ∩ C = {15}, A ∩ C = {20}, A ∩ B ∩ C = {ϕ}
The venn diagram showing relation of given sets is as follows:



38. Here, we have,
A contains two elements, namely 1 and {2, 3}
{2, 3} = B, then A = {1, B}
∴ P(A) = {ϕ, {1}, {B}, {1, B}}
⇒ P(A) = {ϕ , ,{1}, {{2, 3}}, {1, {2, 3}}}.
39. Let H be the set of students who know Hindi and E be the set of students who know English.
Here n(H) = 100, n(E) = 50 and $n(H \cap E) = 25$
We know that $n(H \cup E) = n(H) + n(E) - n(H \cap E)$
= 100 + 50 - 25 = 125.
40. Here, B - C = represents all elements in B that are not in C
B ∩ C = {e, g}
A - (B ∩ C) = {a, b, c, d}.....(1)
(A - B) = {b, d}
(A - C) = {a, c, d}
(A - B) ∩ (A - C) = {a, b, c, d}.....(2),
From (1) and (2)
⇒ A - (B ∩ C) = (A - B) ∩ (A - C)
Hence proved.
41. Suppose B = {x | x ∈ X and x + 5 = 8}
We get, B = {3}

as x = 3 ∈ X and 3 + 5 = 8 and there is no other element belonging to X such that x + 5 = 8.

42. We know that, for any given set having m elements, the number of subsets can be represented as 2^m
According to the given condition,
$2^m = 112 + 2^n$
⇒ $2^m - 2^n = 112 = 128 - 16$
⇒ $2^m - 2^n = 2^7 - 2^4$ (as 2^7=128 and 2^4 = 16)
On comparing both the sides,
$2^m = 2^7$ and $2^n = 2^4$
∴ m = 7 and n = 4

43. The given set T is not an empty set.
Justification
$\because \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}$
$\Rightarrow \frac{x+5}{x-7} - \frac{5}{1} = \frac{4x-40}{13-x}$
$\Rightarrow \frac{x+5-5x+35}{x-7} = \frac{4x-40}{13-x}$
$\Rightarrow \frac{40-4x}{x-7} = \frac{4x-40}{13-x}$
$\Rightarrow \frac{-(4x-40)}{x-7} - \frac{4x-40}{13-x} = 0$
$\Rightarrow \quad (4x-40)\left[\frac{6}{(13-x)(x-7)}\right] = 0$
$\Rightarrow 4x - 40 = 0$
$\therefore x = 10$
⇒ T is not an empty set.

44. Let A = {1, 2, 3, 4} , B = {2, 3, 4, 5, 6}, C = {2, 3, 4, 9, 10}
∴ $A \cap B$= {1, 2, 3, 4} ∩ {2, 3, 4, 5, 6}
= {2,3, 4}
$A \cap C$= {1, 2, 3, 4}, B = {2, 3, 4, 5, 6}, C = {2, 3, 4, 9, 10}
= {2, 3, 4}
$A \cap C$= {1, 2, 3, 4} ∩ {2, 3, 4, 9, 10}
= {2, 3, 4}
Now we have $A \cap B = A \cap C$
But $B \neq C$

45. Here, we have X ⊂ A and X ⊄ B
⇒ X is a subset of A but X is not a subset of B
⇒ X ∈ P (A) but X ∉ P (B),we get
⇒ X = {d}, {a, b, d}, {b, c, d}, {a, c, d}, {a, d}, {b, d}, {c, d}, {a, b, c, d}.

46. To prove: A ∪ C = C ∪ A
Since the element of set C is not provided,
Supposex be any element of C
L.H.S = A ∪ C
= {a, b, c, d, e} ∪ { x |x ∈ C}
= { a, b, c, d, e, x}
= {x, a, b, c, d, e }
= { x |x ∈ C} ∪ { a, b, c, d, e}
= C ∪ A
= R.H.S
Hence proved.

47. Suppose x ∈ A ∩ B
⇒ x ∈ A and x ∈ B
⇒ x ∈ A
⇒ (A ∩ B) ⊂ A Hence Proved

48. L.H.S , A ∩ (B - C) = (A ∩ (B ∩ C') [∵ B - C = B ∩ C']
= (A ∩ B) ∩ C'
= ϕ∪ ((A ∩ B) ∩ c')

= ((A ∩ B) ∩ A') ∪ ((A ∩ B) ∩ C') [∵ (A ∩ B) ∩ A' = ϕ]
= (A ∩ B) ∩ (A' ∪ C')
= (A ∩ B) ∩ (A ∩ C)'
= (A ∩ B) - (A ∩ C)
=R.H.S
hence proved

49. We know that,Natural numbers = 1, 2, 3, 4, 5, 6,…
If x = 1, then 2x + 3 = 2(1) + 3 = 2 + 3 = 5 $\neq$ 4
∴ no elements in the set B because the given equationequation 2x + 3 = 4 is not satisfied for any natural number of x.
Hence, It is a null set.

50. We are given the following tree sets :
L= {1, 2, 3, 4} , M = {3, 4, 5, 6} and N = {1, 3, 5}.
We are to verify the following :
L - (M ∪ N) = (L - M) ∩ (L - N).
We have
M ∪ N = {3, 4, 5, 6} ∪ {1, 3, 5} = {1, 3, 4, 5, 6}
So, L.H.S. = L - (M ∪ N) = {1, 2, 3, 4} - {1, 3, 4, 5, 6} = {2}.
Now,
L - M = {1, 2, 3, 4} - {3, 4, 5, 6} = {1, 2}
L - N = {1, 2, 3, 4} - {1, 3, 5} = {2, 4}.
So, R.H.S. = (L - M) ∩ (L - N) = {1, 2} ∩ {2, 4} = {2}.
Therefore, we get
L - (M ∪ N) = (L - M) ∩ (L - N).
Hence verified.

51. **Given:** $A = B$
To Prove: $A \subseteq B$ and $B \subseteq A$
Proof: As we know that every element of A is in B and every element of B is in A in equal sets
∴ $A \subseteq B$ and $B \subseteq A$
∴ $A = B \Rightarrow A \subseteq B$ and $B \subseteq A$
Now, Suppose $A \subseteq B$ and $B \subseteq A$
By the definition of a subset, if $A \subseteq B$ then it follows that every element of A is in B and if $B \subseteq A$ then it follows that every element of B is in A.
∴ $A = B$
∴ $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$
Hence Proved.

52. Natural numbers start from
A = {1, 2, 3, 4, 5, 6, 7}
B = {2, 3, 5, 7}
C = {1, 3, 5, 7, 9}
B ∪ C = {1, 2, 3, 5, 7, 9}
A ∩ (B ∪ C) = {1, 2, 3, 5, 7}...(1)
A ∩ B = {2, 3, 5, 7}
A ∩ C = {1, 3, 5, 7}
(A ∩ B) ∪ (A ∩ C) = {1, 2, 3, 5, 7}....(2)
Using (1) and (2) ,
$\Rightarrow$ A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)
Hence proved.

53. Given, $a_1 = 1$ and $a_{n+1} = 3a_n$, for all n∈N
Putting n = 1 in $a_{n+1} = 3a_n$, we get
$a_2 = 3a_1 = 3 \times 1 = 3$ [∵ $a_1 = 1$]
Putting n = 2 in $a_{n+1} = 3a_n$, we get
$a_3 = 3a_2 = 3 \times 3 = 3^2$ [∵ $a_2 = 3$]

Putting n = 3 in $a_{n+1} = 3a_n$, we get

$a_4 = 3a_3 = 3 \times 3^2 = 3^3$ [$\because a_3 = 3$]

Similarly, we obtain

$a_5 = 3a_4 = 3 \times 3^3 = 3^4$,

$a_6 = 3a_5 = 3 \times 3^4 = 3^5$ and so on.

Hence, A = {a_1, a_2, a_3, a_4, ...} = {1, 3, 3^2, 3^3, 3^4, 3^5,.....}

54. A = {2, 3} and B = {x : x is solution of $x^2 + 5x + 6 = 0$}

Now $x^2 + 5x + 6 = 0 \Rightarrow x^2 + 3x + 2x + 6 = 0$

$\Rightarrow (x + 3)(x + 2) = 0 \Rightarrow x = -3, -2$

$\therefore$ B = {-2, -3}

Hence A and B are not equal sets.

55. We have,the set of integers = {…, -4, -3, -2, -1, 0, 1, 2, 3, 4, …}

x = -4, $x^2 = (-4)^2 = 16 > 9$

x = -3, $x^2 = (-3)^2 = 9$

x = -2, $x^2 = (-2)^2 = 4$

x = -1, $x^2 = (-1)^2 = 1$

x = 0, $x^2 = (0)^2 = 0$

x = 1, $x^2 = (1)^2 = 1$

x = 2, $x^2 = (2)^2 = 4$

x = 3, $x^2 = (3)^2 = 9$

x = 4, $x^2 = (4)^2 = 16$

The elements of this set are -3, -2, -1, 0, 1, 2, 3

Therefore, D = {-3, -2, -1, 0, 1, 2, 3}

56. Here $A - (B - C) = A - (B \cap C)\,[\because A - B = A \cap B']$

$= A \cap (B \cap C')'$

$= A \cap (B' \cap C)\,[\because (A \cap B)' = A' \cup B']$

$= (A \cap B') \cup (A \cap C)$

$= (A - B') \cup (A \cap C)$

57. A = {x : x is an odd natural number}

$\therefore$A = {1, 3, 5, 7, . . . }

58.

Suppose,x $\in$ (A – B) $\cap$ (C – B)

$\Rightarrow$ x $\in$ A – B and x $\in$ C – B

$\Rightarrow$ (x $\in$ A and x $\notin$ B) and (x $\in$ C and x $\notin$ B)

$\Rightarrow$ (x $\in$ A and x $\in$ C) and x $\notin$ B

$\Rightarrow$ (x $\in$ A $\cap$ C) and x $\notin$ B

$\Rightarrow$ x $\in$ (A $\cap$ C) – B

Thus, (A – B) $\cap$ (C – B) $\subset$ (A $\cap$ C) – B ... (1)

Now, conversely

Suppose, y $\in$ (A $\cap$ C) – B

$\Rightarrow$ y $\in$ (A $\cap$ C) and y $\notin$ B

$\Rightarrow$ (y $\in$ A and y $\in$ C) and (y $\notin$ B)

$\Rightarrow$ (y $\in$ A and y $\notin$ B) and (y $\in$ C and y $\notin$ B)

$\Rightarrow$ y $\in$ (A – B) and y $\in$ (C – B)

$\Rightarrow$ y $\in$ (A – B) $\cap$ (C – B)

Therefore, (A $\cap$ C) – B $\subset$ (A – B) $\cap$ (C – B) ... (2)

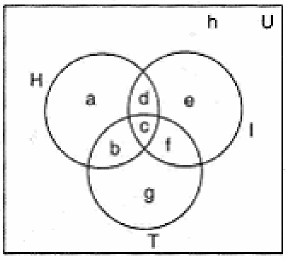
From (1) and (2),we get

(A – B) $\cap$ (C – B) = (A $\cap$ C) – B

59. Let x$\in A \cup (B - A)$

$\Rightarrow x \in A$ or $x \in (B - A)$

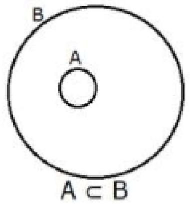
$\Rightarrow x \in A$ or $x \in B$ and $x \notin A$
$\Rightarrow x \in B$
$\Rightarrow x \in (A \cup B)[\because B \subset (A \cup B)]$
This is true for all $x \in A \cup (B - A)$
$\therefore A \cup (B - A) \subset (A \cup B)$ ....(i)
Conversely,
Let $x \in (A \cup B)$
$\Rightarrow x \in A$ or $x \in B$
$\Rightarrow x \in A$ or $x \in (B - A)[\because B \subset (B - A)]$
$\Rightarrow x \in A \cup (B - A)$
$\therefore (A \cup B) \subset A \cup (B - A)$ ....(ii)
From (i) and (ii), we get
$A \cup (B - A) = (A \cup B)$

60. Here
n(U) = a + b + c + d + e + f + g + h = 60 ....(i)
n(H) = a + b + c +d = 25 ....(ii)
n(T) = b + c + f + g = 26 .....(iii)
n(I) = c + d + e + f = 26 ....(iv)
$n(H \cap I)$ = c + d = 9 ....(v)
$n(H \cap T)$= b + c = 11 .....(vi)
$n(T \cap I)$ = c + f = 8 .....(vii)
$n(H \cap T \cap I)$ = c = 3 ....(viii)


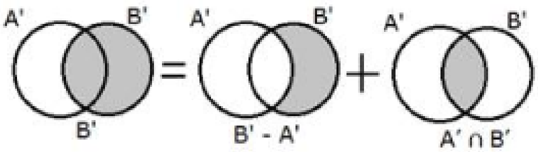
Putting value of c in (vii),
3 + f = 8 $\Rightarrow$ f = 5
Putting value of c in (vi),
3 + b = 11 $\Rightarrow$ b = 8
Putting values of c in (v),
3 + d = 9 $\Rightarrow$ d = 6
Putting value of c, d, f in (iv),
3 + 6 + e + 5 = 26 $\Rightarrow$ e = 26 - 14= 12
Putting value of b, c, f in (iii),
8 + 3 + 5 + g = 26 $\Rightarrow$ g = 26 - 16= 10
Putting value of b, c, d in (ii)
a + 8 + 3 + 6 = 25 $\Rightarrow$ a = 25 - 17 = 8
Number of people who read exactly one newspapers
= a + e + g
= 8 + 12 + 10 = 30

61. Here$A \cup X = B \cup X$ for some set X
$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$
$\Rightarrow A = (A \cap B) \cup (A \cap X)\ [\because A \cap (A \cup X) = A]$
$\Rightarrow A = (A \cap B) \cup \phi\ [\therefore A \cap (A \cup X) = A]$
$\Rightarrow A = A \cap B$
$\Rightarrow A \subset B...(1)$
Also $A \cup X = B \cup X$
$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$
$\Rightarrow (B \cap A) \cup (B \cap X) = B\ [\because B \cap (B \cap X) = B]$

$\Rightarrow (B \cap A) \cup \phi = B\ [\therefore B \cap X = \phi]$
$\Rightarrow B \cap A = B$
. . . (ii)$\Rightarrow B \subset A. . . (ii)$
From (i) and (ii), we have
A=B
62. As A ⊂ B the set A is inside set B



Therefore, A ∪ B = B
Taking compliment
⇒ (A ∪ B)' = B'
Using De-Morgan's law (A ∪ B)' = A'∩ B'
⇒ A'∩ B' = B' …(i)
Now we know that
B' = (B' – A') + (A'∩ B')



Using (i),we get
⇒ B' = (B' - A') + B'
⇒ (B' - A') = B' – B'
⇒ (B' - A') = ϕ
Hence proved.
63. The set of lines which are parallel to the x-axis is an infinite set because we can draw infinite number of lines parallel to x-axis.
64. i. We have
A = (A ∩ U)
⇒ A = A ∩ (B ∪ B') [∵ B ∪ B' = U]
⇒ A = (A ∩ B) ∪ (A ∩ B') [∵ ∩ is distributive over union], therefore we get
⇒ A = (A ∩ B) ∪ ϕ [∵ A ∩ B' = ϕ]
⇒ A = A ∩ B
∴ A ⊂ B
ii. From (i), we have
A ∩ B' = ϕ
⇔ (A ∩ B')' = ϕ'
⇔ A' ∪ (B')' = U [∵ ϕ' = U]
⇔ A' ∪ B = U [∵ (B')' = B]
Therefore ,A ∩ B' = ϕ ⇔ A' ∪ B = U and, A ∩ B' = ϕ ⇒ A ⊂ B
∴ A' ∪ B = U ⇒ A ⊂ B
65. Here, x ∈ Z and |x| $\leq$ 2
Z is a set of integers ,the theref
Integers are …-3, -2 , -1, 0, 1, 2, 3, …
Now, if we take x = -3 then we have to check that it satisfies the given condition |x| $\leq$ 2
|-3| = 3 > 2
Therefore, -3 ∉ H
If x = -2 then |-2| = 2 [satisfying |x| $\leq$ 2]
⇒ -2 ∈ H
If x = -1 then |-1| = 1 [satisfying |x| $\leq$ 2]⇒-1 ∈ H
If x = 0 then |0| = 0 [satisfying |x| $\leq$ 2]

$\Rightarrow 0 \in$ H

If x = 1 then |1| = 1 [satisfying $|x| \leq 2$]

$\Rightarrow 1 \in$ H

If x = 2 then |2| = 2 [satisfying $|x| \leq 2$]

$\Rightarrow 2 \in$ H

If x = 3 then |3| = 3 > 2 [satisfying $|x| \leq 2$]

$\Rightarrow 3 \notin$ H

Therefore, H = {-2, -1, 0, 1, 2}

66. (i) Let A= {2, 3, 4, 5} and B = {3, 6}

Now $A \cap B = \{2, 3, 4, 5\} \cap \{3, 6\}$

= {3}

Hence A and B are not disjoint sets. So the statements is true.

67. We have, $(A \cap B) = \{x : x \in A \text{ and } x \in B\}$

= {7}

$(A \cap B)'$ means Complement of $(A \cap B)$ with respect to universal set U.

Therefore, $(A \cap B)' = U - (A \cap B)$

$U - (A \cap B)'$ is defined as $\{x \in U : x \notin (A \cap B)'\}$

U = {2, 3, 5, 7, 9}

$(A \cap B)'$ = {7}

$U - (A \cap B)'$ = {2, 3, 5, 9}

A' means Complement of A with respect to universal set U.

Therefore, A' = U - A

U - A is defined as $\{x \in U : x \notin A\}$

U = {2, 3, 5, 7, 9}

A = {3, 7}

A' = {2, 5, 9}

B' means Complement of B with respect to universal set U.

Therefore, B' = U - B

U - B is defined as $\{x \in U : x \notin B\}$

U = {2, 3, 5, 7, 9}

B = {2, 5, 7, 9}.

B' = {3}

$A' \cup B' = \{x: x \in A \text{ or } x \in B\}$

= {2, 3, 5, 9}

Hence verified.

68. Let x $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in (B \cup C)$

$\Rightarrow x \in A$ and $(\mathrm{X} \in \mathrm{B} \text{ or } \mathrm{X} \in \mathrm{C})$

$\Rightarrow (x \in A \text{ and } x \in B)$ or $(\mathrm{x} \in \mathrm{A} \text{ and } \mathrm{x} \in \mathrm{C})$

$\Rightarrow (x \in A \cap B)$ or $(x \in A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow A \cap (B \cup C) \subset (A \cap B) \cup (A \cap C)$ ..... (i)

Now let $y \in (A \cap B) \cup (A \cap C)$

$\Rightarrow \mathrm{y} \in (\mathrm{A} \cap \mathrm{B})$ or $\mathrm{y} \in (\mathrm{A} \cap \mathrm{C})$

$\Rightarrow (\mathrm{y} \in \mathrm{A}$ and $y \in B)$ or (y ∈ A and y ∈ C)

$\Rightarrow \mathrm{y} \in \mathrm{A}$ and $(y \in B \text{ or } y \in C)$

$\Rightarrow \mathrm{y} \in \mathrm{A}$ and $y \in (B \cup C)$

$\Rightarrow \mathrm{y} \in \mathrm{A} \cap (\mathrm{B} \cup \mathrm{C})$

$\Rightarrow (A \cap B) \cup (A \cap C) \subset A \cap (B \cup C)$ ...... (ii)

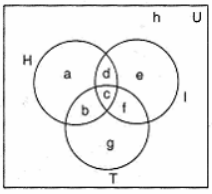
From eqn. (i) and (ii) we get

$\mathrm{A} \cap (\mathrm{B} \cup \mathrm{C}) = (\mathrm{A} \cap \mathrm{B}) \cup (\mathrm{A} \cap \mathrm{C})$

69. Here

n(U) = a + b + c + d + e + f + g + h = 60 ....(i)

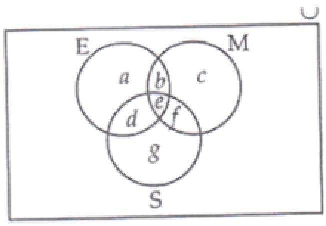
n (H) = a + b + c +d = 25 ....(ii)
n(T) = b + c + f + g = 26 .....(iii)
n(I) = c + d + e + f = 26 ....(iv)
$n(H \cap I)$ = c + d = 9 .....(v)
$n(H \cap T)$ = b + c = 11 .....(vi)
$n(T \cap I)$ = c + f = 8 ....(vii)
$n(H \cap T \cap I)$ = c = 3 ....(viii)



Putting value of c in (vii),
3 + f = 8 ⇒ f = 5
Putting value of c in (vi),
3 + b = 11 ⇒ b = 8
Putting values of c in (v),
3 + d = 9 ⇒ d = 6
Putting value of c, d, f in (iv),
3 + 6 + e + 5 = 26 ⇒ e = 26 - 14 = 12
Putting value of b, c, f in (iii),
8 + 3 + 5 + g = 26 ⇒ g = 26 - 16 = 10
Putting value of b, c, d in (ii)
a + 8 + 3 + 6 = 25 ⇒ a = 25 - 17 = 8
Number of people who read at least one of the three newspapers
= a + b + c + d + e + f + g
= 8 + 8 + 3 + 6 + 12 + 5 + 10 = 52

70. Let $X \in A \Rightarrow \{x\} \in P(A)$
$\Rightarrow \{x\} \in P(B)\, [\because P(A) = P(B)]$
$\Rightarrow X \notin B$
$\therefore A \subset B \ldots$ (i)
Let $X \in B \Rightarrow \{x\} \in P(B)$
$\Rightarrow \{X\} \in P(A)\, [\because\ P(A) = P(B)]$
$\Rightarrow X \in A \ldots$ (ii)
$\therefore B \subset A$
From (i) and (ii) we have A = B

71. Let the set of students who passed in Mathematics be M, the set of students who passed in English be E and the set of students who passed in Science be S.
Then n(U) =100, n(M) = 12, n(E) = 15, n(S) =8, n(E ∩ M) = 6, n(M ∩ S) = 7, n(E ∩ S) = 4 and n(E ∩ M ∩ S) = 4
Let us draw a Venn diagram



According to the Venn diagram,
n(E ∩ S) = 4 ⇒ e = 4
n(E ∩ M) = 6 ⇒ b + e = 6 ⇒ b + 4 = 6 ⇒ b = 2
n(M ∩ S) = 7 ⇒ e + f = 7 ⇒ 4 + f = 7 ⇒ f = 3
n(E ∩ S) = 4 ⇒ d + e = 4 ⇒ d + 4 = 4 ⇒ d = 0
n(E) = 15 ⇒ a + b + d + e = 15 ⇒ a + 2 + 0 + 4 = 15 ⇒ a = 9
n(M) = 12 ⇒ b + c + e + f = 12 ⇒ 2 + c + 4 + 3 = 12 ⇒ c = 3

n(S) = 8 ⇒ d + e + f + g = 8 ⇒ 0 + 4 + 3 + g = 8 ⇒ g = 1

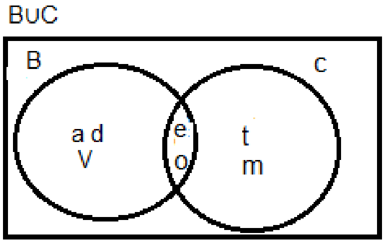
Hence we get,

i. Number of students who passed in English and Mathematics but not in Science, b = 2.
ii. Number of students who passed in Mathematics and Science but not in English, f = 3.
iii. Number of students who passed in Mathematics only, c = 3.
iv. Number of students who passed in more than one subject = b + e + d + f = 2 + 4 + 0 + 3 = 9.

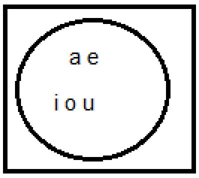
72. Here,it is given: A = {a, e, i, o, u}, B = {a, d, e, o, v} and C = {e, o, t, m}.

B ∩ C = {e, o} and A ∪ (B ∩ C) = {a, e, i, o, u}

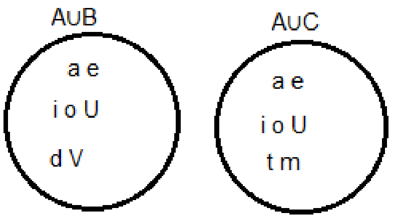
LHS



$A \cup (B \cap C)$



R.H.S: A ∪ B = {a, d, e, I, o, u, v} and A ∪ C = {a, e, I, o, u, t, m}

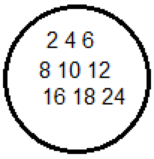


(A ∪ B) ∩ (A ∪ C) = {a, e, I, o, u}

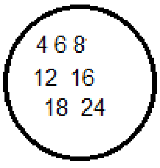
L.H.S = R.H.S. [Verified]

73. L.H.S: A ∪ B = {2, 4, 6, 8, 10, 12, 16},

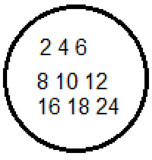
(A ∪ B) ∪ C = {2, 4, 6, 8, 10, 12, 16, 18, 24}



R.H.S: B ∪ C = {4, 6, 8, 10, 12, 16, 18, 24}



A ∪ (B ∪ C) = {2, 4, 6, 8, 10, 12, 16, 18, 24}



Hence proved.

Thus L.H.S = R.H.S. [Verified]

74. Here U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

A = {2, 4, 6, 8} and B = {2, 3, 5, 7}

$A \cap B$ = {2, 4, 6, 8} ∩ {2, 3, 5, 7}

= {2}

$(A \cap B)' = U - (A \cap B)$ = {1, 2, 3, 4, 5, 6, 7, 8, 9} - {2}
= {1, 3, 4, 5, 6, 7, 8, 9} . . . (i)
A' = U - A = {1, 2, 3, 4, 5, 6, 7, 8, 9} - {2, 4, 6, 8}
= {1, 3, 5, 7, 8}
B' = U - B = {1, 2, 3, 4, 5, 6, 7, 8, 9} - {2, 3, 5, 7}
= {1, 4, 6, 8, 9}
$A' \cup B'$ = {1, 3, 5, 7, 9} ∪ {1, 4, 6, 8, 9}
= {1, 3, 4, 5, 6, 7, 8, 9} . . . (ii)
From (i) and (ii) we have
$(A \cap B)' = A' \cup B'$

75. Here $A \cup B = C$
$\therefore (A \cup B) - B = C - B$
$\Rightarrow (A \cup B) \cap B' = C - B \, (\because A - B = A \cap B')$
$\Rightarrow (A \cap B') \cup (B \cap B') = C - B$
$\Rightarrow (A \cap B') \cup \phi = C - B$
$\Rightarrow (A \cap B') = C - B$
$\Rightarrow A - B = C - B$
$\Rightarrow A = C - B \; (\because A \cap B = \phi)$