Delhi Public School, Faridabad
Mid Semester Examination 2023-2024
CLASS: XII
Mathematics (Code-041)

7/7/23

Time Allowed :3 Hours

Maximum Marks: 80

General Instructions:
1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A
(Multiple Choice Questions)
Each question carries 1 mark

1. Let $A = \{2,3,4,5, \ldots ,17,18\}$. Let $\simeq$ be the equivalence relation on $A \times A$, cartesian product of A with itself, defined by $(a,b) \simeq (c,d)$ iff $ad = bc$. Then, the number of ordered pairs of the equivalence class of $(3,2)$ is
   (a) 4   (b) 5   (c) 6   (d) 7

2. Consider the set $A = \{1,2\}$. The relation on A which is symmetric but neither reflexive nor transitive is
   (a) $\{(1,1),(2,2)\}$ (b) $\{\}$ (c) $\{(1,2)\}$ (d) $\{(1,2),(2,1)\}$

3. Which of the following function from R to R is one-one function?
   (a) $f(x) = e^x$ (b) $g(x) = e^{x^2}$ (c) $h(x) = \sin x$ (d) $\phi(x) = \cos x$

4. Let $f(x) = e^{\cos^{-1}\left(\sin\left(x+\frac{\pi}{3}\right)\right)}$. Then $f\left(\frac{8\pi}{9}\right) =$
   (A) $e^{5\pi/18}$ (b) $e^{13\pi/18}$ (c) $e^{-2\pi/18}$ (d) none of these

   $\frac{\pi}{2} - \frac{2\pi}{9}$

   $\frac{9\pi - 4\pi}{18}$

   $= \frac{5\pi}{18}$

5. I: If $\cos^{-1} x < \sin^{-1} x$, then $\frac{1}{\sqrt{2}} < x \le 1$

   II: $\tan^{-1} 1 > \tan 1$

   III: The domain of $\cos^{-1}(x^2 - 4) + \cos x$ is $\left[\sqrt{3}, \sqrt{5}\right]$

   (a) Only I is true (b) Only II is true (c) Only III is true (d) none of these

6. Value of $\cos(\cos^{-1} \pi) =$
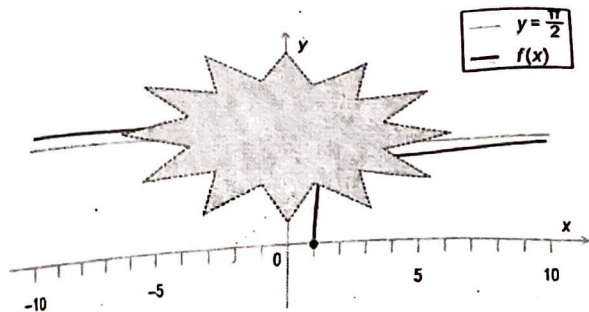   (a) -1 (b) π (c) 0   (d) none of these

3,2 (u, y)

$3y = 2u$ $\frac{u}{y} = \frac{3}{2}$

7. Shown below is the graph of a function $f(x)$ whose domain is $R - (-1,1)$. Some portion of the graph is hidden behind the star. Which of the following is $f(x)$?

   (a) $\tan^{-1} x$
   (b) $\cot^{-1} x$
   (c) $\sec^{-1} x$
   (d) $\csc^{-1} x$

8. If A be any scalar matrix of order 3, such that $a_{12} + a_{22} + a_{32} = 3$, where $A = [a_{ij}]_{3\times3}$, then value of $|A|$ is

   (a) 1   (b) 3   (c) 6   (d) 27

9. Which of the following matrix is both symmetric and skew-symmetric?

   (a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) none of these

10. Given that A is a square matrix of order 3 and $|A| = -2$, then $|adj\ (2A)|$ is equal to

    (a) -2^6 (b) 4   (c) -2^8 (d) 2^8

11. The value of $|A|$, if $A = \begin{bmatrix} 0 & 2x-1 & \sqrt{x} \\ 1-2x & 0 & 2\sqrt{x} \\ -\sqrt{x} & -2\sqrt{x} & 0 \end{bmatrix}$, where $x \in R^+$, is

    (a) $(2x+1)^2$ (b) 0   (c) $(2x+1)^3$ (d) none of these

12. The number of elements below the diagonal of a square matrix of order n is

    (a) $\dfrac{n(n+1)}{2}$ (b) $\dfrac{n(n-1)}{2}$ (c) $\dfrac{(n-1)(n-2)}{2}$ (d) $\dfrac{(n+1)(n+2)}{2}$

13. Suppose $f(x)$ is defined on $[a,b]$. Then the continuity of $f(x)$ at $x = a$ means

    (a) $\lim\limits_{x\to a^+} f(x) = f(a)$ (b) $\lim\limits_{x\to a^-} f(x) = f(a)$

    (c) $\lim\limits_{x\to a^+} f(x) = \lim\limits_{x\to a^-} f(x) = f(a)$ (d) none of these

14. If $f(x) = \begin{cases} \dfrac{kx}{|x|}, & x < 0 \\ 3, & x \geq 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

    (a) -3 (b) 0   (c) 3   (d) any real number

15. The set of all points where the function $f(x) = x + |x|$ is differentiable, is

    (a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty, \infty)$

16. If $\sec^{-1}\left(\dfrac{1+x}{1-y}\right) = a$, then $\dfrac{dy}{dx}$ is equal to

    (a) $\dfrac{x-1}{y-1}$ (b) $\dfrac{x-1}{y+1}$ (c) $\dfrac{y-1}{x+1}$ (d) $\dfrac{y+1}{x-1}$

17. If $y = a\cos(\log_e x) + b\sin(\log_e x)$, then $x^2 y_2 + xy_1 =$

    (a) 0   (b) y (c) $-y$ (d) none of these

18. In a sphere the rate of change of volume is
    (a) π times the rate of change of radius
    (b) surface area times the rate of change of diameter

(c) surface area times the rate of change of radius
(d) none of these

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion(A):** The relation $f:\{1,2,3,4\} \to \{x,y,z,p\}$ defined by $f = \{(1,x),(2,y),(3,z)\}$ is a bijective function.   F

    Reason(R): The function $f:\{1,2,3\} \Rightarrow \{x,y,z,p\}$ such that $f = \{(1,x),(2,y),(3,z)\}$ is   T
    one-one.

20. **Assertion(A):** The rate of change of volume of sphere with respect to its radius is   T
    equal to S, surface area of the sphere.

    Reason(R): The rate of change of volume of sphere with respect to its radius is $\frac{r}{2}$.   ⨯   F

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Let $A = \{a,b,c\}$ and the relation R be defined on A as $R = \{(a,a),(b,c),(a,b)\}$. Then, write the minimum number of ordered pairs to be added in R to make R reflexive and transitive.

OR

Write the smallest and largest equivalence relations on the set $A = \{-1,0,1\}$

22. $\cot^{-1} x = \cos^{-1}(-1) - \operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$.

    Based on the above equation, find $\tan^{-1}\frac{1}{x}$ using principal values of the inverse trigonometric functions. Show your calculation.

23. If area of a triangle is 35 square units with vertices $(2,-6), (5,4)$ and $(k,4)$, find the value of k.

OR

Express the matrix $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

24. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$

25. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2 cm/minute. When $x = 10$cm and $y = 6$cm, find the rate of change of (a) perimeter and (b) the area of the rectangle.

-2 cm/min $\to$ 2 cm²/min

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Let $f: X \rightarrow Y$ be a function. Define a relation R in X given by $R = \{(a,b): f(a) = f(b)\}$. Is this relation (i) reflexive? (ii) symmetric? (iii) transitive? Justify your answer.

yes, yes, yes

27. Evaluate $\cos^{-1} \cos \frac{13\pi}{6} + \tan^{-1} \tan \frac{7\pi}{6}$.

OR

Prove that $3 \cos^{-1} x = \cos^{-1}(4x^3 - 3x), x \in \left[\frac{1}{2}, 1\right]$

28. Examine the consistency of the system of equations
$3x - y - 2z = 2, 2y - z = -1, 3x - 5y = 3$

OR

Prove that a square matrix A is invertible if and only if A is non-singular matrix.

29. If $A = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$, find x such that $A^2 = xA - 2I$. Hence find A^{-1}.

OR

Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}, C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that $CD - AB = 0$.

30. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$.

31. If $y = e^{a \cos^{-1} x}, -1 \le x \le 1$, show that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$. Is f one-one and onto? Justify your answer.

33. If $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$, find A^{-1}. Use A^{-1} to solve the system of equations
$2x - 3y + 5z = 11, 3x + 2y - 4z = 1, x + y - 2z = 3$

34. Differentiate $(x \cos x)^x + (x \sin x)^{1/x}$ with respect to x. —

35. Water is running into a conical vessel, 15 cm deep and 5 cm in radius, at the rate of 0.1 cm^3/sec. When the water is 6 cm deep, find at what rate is
(i) the water level rising?
(ii) the water-surface area increasing?
(iii) the wetter surface of the vessel increasing?

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i),(ii),(iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

$\begin{array}{r} 45 \\ 5 \\ \hline 225 \end{array}$

82×3

$\begin{array}{r} 82 \\ 3 \\ \hline 246 \\ - \quad 10 \\ \hline 236 \end{array}$

36. Raman wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area does not alter, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.
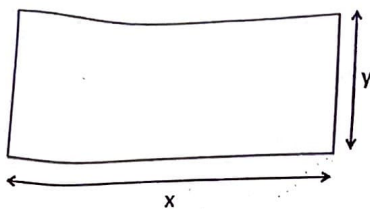
Based on the information given, answer the following questions:

(i) Write the matrix equation representing the system of equations.

(ii) Find the length of the rectangular plot, by solving the matrix equation.

(iii) (a) Find the area of the rectangular plot.

OR

(iii) (b) Find the breadth of the rectangular plot.

37. Let $f(x)$ be a real valued function. Then its

- Left Hand Derivative (L.H.D.): $Lf'(a) = \lim_{h \to 0} \frac{f(a-h)-f(a)}{-h}$

- Right Hand Derivative (R.H.D.): $Rf'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function is said to be differentiable at $x = a$ if its L.H.D. and R.H.D. at $x = a$ exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ answer the following questions:

(i) What is R.H.D. of $f(x)$ at $x = 1$?

(ii) What is L.H.D. of $f(x)$ at $x = 1$?

(iii) (a) Check if the function $f(x)$ is differentiable at $x = 1$.

OR

(iii) (b) Find $f'(2)$ and $f'(1)$.

38. Read the following passage and answer the questions given below:

The relation between the height of the plant ('y' in cm) with respect to its exposure to the sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to the sunlight, for $x \leq 3$.
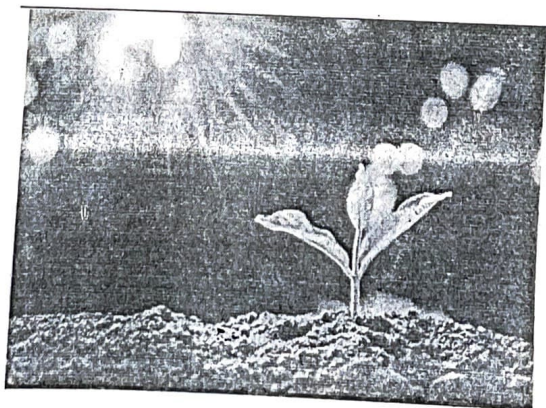
(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.   4-x

(ii) What will be the height of the plant after 2 days?   8cm