DELHI PUBLIC SCHOOL, BANGALORE- EAST

UNIT TEST-I (2023 – 2024)

SUBJECT: MATHEMATICS (CODE-041)

CLASS: XII

DATE : 17/07/2023

MAX- MARKS: 40

TIME: $1\frac{1}{2}$ HOURS

NAME :

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 10 MCQ's and 1 Assertion-Reason-based questions of 1 mark each.
3. Section B has 3 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 3 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 2 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 1 source based/case based/passage based/integrated unit of assessment of 4 marks with sub-parts.

SECTION A
(Multiple Choice Questions)
Each question carries 1 mark

1) A function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & if\ x \neq 0 \\ 2k & if\ x = 0 \end{cases}$ is continuous at $x = 0$ for

a) $k = 1$ b) $k = 2$ c) $k = \frac{1}{2}$ d) $k = \frac{3}{2}$

2) If $f(x) = log_{x^2}(logx), then\ f'(e)$ is

a) 0 b) 1 c) $\frac{1}{e}$ d) $\frac{1}{2e}$

3) The solution set of the constraints $x + 2y \geq 11, 3x + 4y \leq 30, 2x + 5y \leq 30, x \geq 0, y \geq 0,$ include the point

a) (2,3) b) (3,2) c) (3,4) d) (4,3)

4) If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector ,then the angle between $\vec{a}\ and\ \vec{b}$ is

a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) $\frac{\pi}{4}$ d) $-\frac{\pi}{3}$

5) The value of $'a'$ for which the two vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} + a\hat{\jmath} + \hat{k}$ are perpendicular is

a) 2 b) -4 c) 6 d) 8

6) The straight line $\frac{x-2}{3} = \frac{y-2}{1} = \frac{z+1}{0}$ is

a) Parallel to x-axis b) Parallel to y-axis

c) Parallel to z-axis d) Perpendicular to z-axis

7) If $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ then , $y_1 = ?$

a) $\frac{y}{x}$ b) $\sqrt{\frac{y}{x}}$ c) $-\sqrt[3]{\frac{y}{x}}$ d) $\sqrt[3]{\frac{y}{x}}$

8) $y = \sin^{-1}(6x\sqrt{1-9x^2})$, $-\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$,then $\frac{dy}{dx} = ?$

a) $\frac{6}{\sqrt{1-9x^2}}$ b) $\frac{2}{\sqrt{1-9x^2}}$ c) $\frac{6x}{\sqrt{1-9x^2}}$ d) $\frac{-6}{\sqrt{1-9x^2}}$

9) If α, β, γ are the angles that a line makes with x,y and z axis respectively ,then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is

a) 1 b) -1 c) 2 d) -2

10) Determine the value of λ so that the function $f(x) = \begin{cases} \frac{\lambda x}{|x|}, & if\ x < 0 \\ 3, & if\ x \geq 0 \end{cases}$ is continuous at $x = 0$.

a) 1 b) 3 c) -3 d) -1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true

11) **Assertion (A) :** $f(x) = |x| \sin x$ is differentiable at $x = 0$.

Reason (R) : If $f(x)$ is not differentiable and $g(x)$ is differentiable at $x = a$,then $f(x).g(x)$ will be differentiable at $x = a$.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

12) The scalar product of $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\alpha\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of $'\alpha'$.

13) If $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, prove that $\frac{dy}{dx} = 1 - y^2$.

OR

If $y = A \sin x + B \cos x$, then prove that $\frac{d^2y}{dx^2} + y = 0$.

14) If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$,then prove that vector $2\vec{a} + \vec{b}$ is perpendicular to vector $\vec{b}$.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

15) Determine graphically the minimum value of the objective function ,

$Z = -50x + 20y$ subject to the constraints

$2x - y \geq 5$, $3x + y \geq 3$, $2x - 3y \leq 12$, $x \geq 0, y \geq 0$

16) Find the coordinates of the foot of the perpendicular and the length of the perpendicular drawn from the point P (5,4,2) to the line $\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.

17) If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors . Suppose $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$.

Prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

OR

If $\vec{a} + \vec{b} + \vec{c} = 0$, than prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

18) If $(x-a)^2 + (y-b)^2 = c^2, for\ some\ c > 0, prove\ that\ \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b .

19) If $\sqrt{1-x^4} + \sqrt{1-y^4} = a(x^2 - y^2)$, show that $\frac{dy}{dx} = \frac{x}{y}\sqrt{\frac{1-y^4}{1-x^4}}$.

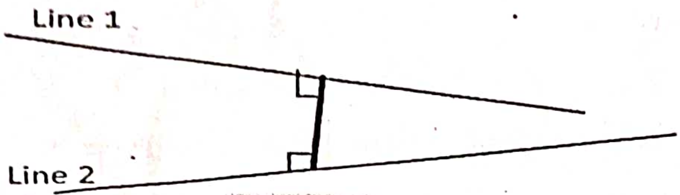
OR

Differentiate $(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$ with respect to x.

SECTION E

(This section comprises of 1 case-study/ passage-based questions of 4 marks with two sub- parts. It has two sub-parts (i), (ii) of marks 2, 2 respectively.)

The shortest distance between two lines is the length of the perpendicular drawn from a point on one line onto the other line.



Skew Lines

Parallel Lines

here are two types of lines in the space.
1. Skew Lines 2. Parallel Lines

1. Skew Lines: These are the lines in the space that are neither intersecting nor parallel.
2. Parallel Lines: These are the lines in the space that are parallel. They do not intersect anywhere in the space.

20) Based on the above information, answer the following questions.

(i) Find the shortest distance between the lines whose vector equations are:

$$\vec{r} = (1-t)\,\hat{i} + (t-2)\hat{j} + (3-2t)\,\hat{k} \text{ and } \vec{r} = (s+1)\,\hat{i} + (2s-1)\hat{j} - (2s-1)\hat{k}$$

(ii) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ *and* $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z+1}{0}$ are intersecting. Also find the point of their intersection.

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