CONTINUITY AND DIFFERENTIABILITY

Class 12 - Mathematics

1. If y = $\tan^{-1}$ (sec x + tan x) then $\frac{dy}{dx} = ?$ **[1]**

   a) None of these

   b) $\frac{1}{2}$

   c) 1

   d) $\frac{-1}{2}$

2. If y = $\sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$ then $\frac{dy}{dx} = ?$ **[1]**

   a) $\frac{1}{2\sqrt{1-x^2}}$

   b) None of these

   c) $\frac{1}{2(1+x^2)}$

   d) $\frac{-1}{2\sqrt{1-x^2}}$

3. $f(x) = \begin{cases} \frac{\sqrt{1+px}-\sqrt{1-px}}{x} & -1 \leq x < 0 \\ \frac{2x+1}{x-2} & ,0 \leq x \leq 1 \end{cases}$ is continuous in the interval [-1, 1], then p is equal to **[1]**

   a) 1

   b) $\frac{1}{2}$

   c) -1/2

   d) -1

4. If y = $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

   a) $\frac{-2}{(1-x^2)}$

   b) $\frac{-2}{\sqrt{1-x^2}}$

   c) $\frac{-2}{(1+x^2)}$

   d) None of these

5. If y = $\tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

   a) $\frac{-2x}{(1+x^4)}$

   b) None of these

   c) $\frac{2x}{(1+x^4)}$

   d) $\frac{x}{(1+x^4)}$

6. Derivative of $\sin^3$x w.r.t $\cos^3$x is **[1]**

   a) cot x

   b) – tan x

   c) $\tan^3$x

   d) tan x

7. If $y = e^{1/x}$ then $\frac{dy}{dx} = ?$ **[1]**

   a) $\frac{-e^{1/x}}{x^2}$

   b) $e^{1/x} \log x$

   c) $\frac{1}{x} \cdot e^{(1/x-1)}$

   d) None of these

8. If $y = x\sqrt{1-x^2} + \sin^{-1}x$, then $\frac{dy}{dx}$ is equal to **[1]**

   a) $\frac{1}{\sqrt{1-x^2}}$

   b) $\sqrt{1-x^2}$

   c) $2\sqrt{1-x^2}$

   d) None of these

9. If $y = x^{x^{x+\infty}}$ then $\frac{dy}{dx} = ?$ **[1]**

   a) None of these

   b)

$\frac{y}{x(1-\log x)}$

c) $\frac{y^2}{x(1-y\log x)}$ d) $\frac{y^2}{x(1-\log x)}$

10. If x = a sec θ, y = b tan θ then $\frac{dy}{dx}$ = ? **[1]**

a) $\frac{b}{a}\sec\theta$ b) None of these

c) $\frac{b}{a}cosec\theta$ d) $\frac{b}{a}\cot\theta$

11. The function $f(x) = \begin{cases} x^2a & , \quad 0 \le x < 1 \\ a & , \quad 1 \le x < \sqrt{2} \\ \frac{2b^2-4b}{x^2} & , \quad \sqrt{2} \le x < \infty \end{cases}$ is continuous for $0 \le x < \infty$, then the most suitable values of a and b are **[1]**

a) a = -1, b = 1 b) a = -1, b = 1^+

c) a = - 1, b = -1 d) none of these

12. If $y = \sqrt{\sin x + y}$ then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{\cos x}{2y-1}$ b) $\frac{\sin x}{1-2y}$

c) $\frac{\cos x}{1-2y}$ d) $\frac{\sin x}{2y-1}$

13. If the function f(x) $= \begin{cases} \frac{\sin^2 ax}{x^2}, \text{ when } x \neq 0 \\ k, \text{ when } x = 0 \end{cases}$ is continuous at x = 0 then k = ? **[1]**

a) -4 b) a

c) -2 d) a^2

14. If $f(x) = \begin{cases} \frac{1-\sin^2 x}{3\cos^2 x} & , \quad x < \frac{\pi}{2} \\ a & , \quad x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & , \quad x > \frac{\pi}{2} \end{cases}$ Then, f(x) is continuous at $x = \frac{\pi}{2}$, if **[1]**

a) $a = \frac{1}{3}$, b = 2 b) none of these

c) $a = \frac{2}{3}, b = \frac{8}{3}$ d) $a = \frac{1}{3}, b = \frac{8}{3}$

15. If y = $\cos^{-1}$ ($4x^3$ - 3x) then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{-3}{\sqrt{1-x^2}}$ b) $\frac{-4}{(3x^2-1)}$

c) $\frac{4}{\sqrt{1-x^2}}$ d) $\frac{3}{\sqrt{1-x^2}}$

16. If $x^y = y^x$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{y(y-x\log y)}{x(x-y\log x)}$ b) $\frac{y(y+x\log y)}{x(x+y\log x)}$

c) $\frac{(y-x\log y)}{(x-y\log x)}$ d) none of these

17. If y = $\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{-1}{(1+x^2)}$ b) None of these

c) $\frac{2}{(1+x^2)}$ d) $\frac{-2}{(1+x^2)}$

18. If the function $f(x) = \frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$ is continuous at each point of its domain, then the value of f(0) is **[1]**

a) $\frac{1}{3}$

b) $\frac{2}{3}$

c) 2

d) $-\frac{1}{3}$

19. The values of the constants a, b and c for which the function $f(x) = \begin{cases} (1+ax)^{1/x} & ,x < 0 \\ b & ,x = 0 \\ \frac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1} & ,x > 0 \end{cases}$ may be **[1]**

continuous at x = 0, are

a) $a = \log_e\left(\frac{2}{3}\right), b = \frac{2}{3}, c = -1$

b) none of these

c) $a = \log_e\left(\frac{2}{3}\right), b = -\frac{2}{3}, c = 1$

d) $a = \log_e\left(\frac{2}{3}\right), b = \frac{2}{3}, c = 1$

20. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is **[1]**

a) $\frac{1-\log x}{1+\log x}$

b) $\frac{1+x}{1+\log x}$

c) $\frac{\log x}{(1+\log x)^2}$

d) not defined

21. If x = a $\cos^2\theta$, y = b $\sin^2\theta$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{a}{b}\cot\theta$

b) None of these

c) $\frac{-a}{b}$

d) $\frac{-b}{a}$

22. The function $f(x) = e^{-|x|}$ is **[1]**

a) continuous everywhere but not differentiable at x = 0

b) continuous and differentiable everywhere

c) none of these

d) not continuous at x = 0

23. If $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{1}{2}$

b) None of these

c) $\frac{-1}{2}$

d) $\frac{1}{(1+x^2)}$

24. The function f(x) = |cos x| is **[1]**

a) everywhere continuous but not differentiable at (2n + 1) $\frac{\pi}{2}$, n $\in$ Z

b) everywhere continuous and differentiable

c) neither continuous nor differentiable at (2n + 1) $\frac{\pi}{2}$, n $\in$ Z

d) none of these

25. If $y^{1/n} + y^{-1/n} = 2x$, then $(x^2 - 1)\, y_2 + xy_1 =$ **[1]**

a) 0

b) none of these

c) n^2y

d) $-n^2y$

26. The function f(x) = $\begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k & \text{, if } x = 0 \end{cases}$ is continuous at x = 0, then the value of k is **[1]**

a) 1

b) 3

c) 1.5

d) 2

27. If $y = ae^{mx} + be^{-mx}$, then y_2 is equal to **[1]**

a) my_1

b) $-m^2y$

c) m^2y

d) None of these

28. Derivative of log|x| w.r.t. |x| is **[1]**

a) None of these

b) $\frac{1}{x}$

c) $\pm\frac{1}{x}$

d) $\frac{1}{|x|}$

29. The function f(x) = $\sin^{-1}$(cos x) is **[1]**

a) None of these

b) differentiable at x = 0

c) discontinuous at x = 0

d) continuous at x = 0

30. It y = $\tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{2}{\sqrt{x}(1+x)}$

b) $\frac{1}{(1+x)}$

c) $\frac{1}{2\sqrt{x}(1+x)}$

d) $\frac{1}{\sqrt{x}(1+x)}$

31. If x = a $\cos^3\theta$, y = a $\sin^3\theta$, then $\sqrt{1+\left(\frac{dy}{dx}\right)^2} =$ **[1]**

a) sec θ

b) $\tan^2\theta$

c) $\sec^2\theta$

d) |sec θ|

32. If $f(x) = \begin{cases} mx+1, & \text{if } x \leq \frac{\pi}{2} \\ \sin x + n, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ then **[1]**

a) $m = n = \frac{\pi}{2}$

b) $n = \frac{m\pi}{2}$

c) m = 1, n = 0

d) $m = \frac{n\pi}{2} + 1$

33. The function f(x) = [x], where [x] denotes the greatest integer function, is continuous at **[1]**

a) -2

b) 1.5

c) 1

d) 4

34. Let f(x) = |x| and g(x) = $|x^3|$, then **[1]**

a) f(x) and g(x) both are differentiable at x = 0

b) f(x) is differentiable but g(x) is not differentiable at x = 0

c) f(x) and g(x) both are continuous at x = 0

d) f(x) and g(x) both are not differentiable at x = 0

35. If y = x^{n-1} log x then $x^2\ y_2$ + (3 - 2n) xy_1 is equal to **[1]**

a) $-(n-1)^2\ y$

b) $(n-1)^2\ y$

c) $-n^2y$

d) n^2y

36. If y = $e^{\tan x}$, then $(\cos^2 x)\ y_2$ = **[1]**

a) (1 + sin 2x) y_1

b) none of these

c) (1 - sin 2x) y_1

d) -(1 + sin 2x) y_1

37. $\frac{d}{dx}\left\{\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right\}$ equal **[1]**

a) -1 b) 1

c) -1/2 d) 1/2

38. If f (x) = x $\tan^{-1}$ x then f ' (1) is equal to **[1]**

a) None of these b) $\frac{1}{2}-\frac{\pi}{4}$

c) $\frac{\pi}{4}-\frac{1}{2}$ d) $\frac{\pi}{4}+\frac{1}{2}$

39. If $y=\tan^{-1}\left\{\frac{\log_e(e/x^2)}{\log_e(ex^2)}\right\}+\tan^{-1}\left(\frac{3+2\log_e x}{1-6\log_e x}\right)$, then $\frac{d^2y}{dx^2}$ = **[1]**

a) 1 b) 0

c) -1 d) 2

40. If y = $\cot^{-1}\left(\frac{1-x}{1+x}\right)$ then $\frac{dy}{dx}=$? **[1]**

a) $\frac{1}{(1+x^2)}$ b) $\frac{-1}{(1+x^2)}$

c) None of these d) $\frac{1}{(1+x^2)^{3/2}}$

41. If $f(x)=\begin{cases}\frac{|x+2|}{\tan^{-1}(x+2)} & ,x\neq -2\\ 2 & ,x=-2\end{cases}$ then f(x) is **[1]**

a) continuous but derivable at x = –2 b) differentiable at x = –2

c) continuous at x = –2 d) not continuous at x = –2

42. If $y=\log\left(\frac{1-x^2}{1+x^2}\right)$ then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{4x^3}{1-x^4}$ b) $\frac{-4x^3}{1-x^4}$

c) $\frac{1}{4-x^4}$ d) $\frac{-4x}{1-x^4}$

43. If x sin (a + y) = sin y, then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{\sin a}{\sin(a+y)}$ b) $\frac{\sin^2(a+y)}{\sin a}$

c) $\frac{\sin a}{\sin^2(a+y)}$ d) $\frac{\sin(a+y)}{\sin a}$

44. The function $f(x)=\frac{\sin(\pi[x-\pi])}{4+[x]^2}$,where [.] denotes the greatest integer function, is **[1]**

a) differentiable for all x but not continuous at some x. b) none of these

c) continuous for all x but not differentiable at some x d) continuous as well as differentiable for all x $\in$ R

45. If $f(x)=x^2\sin\frac{1}{x}$ where $x\neq 0$ then the value of the function f at x = 0, so that the function is continuous at x = 0, is **[1]**

a) -1 b) 1

c) 0 d) None of these

46. If the function f(x) = $\begin{cases}\frac{1-\cos 4x}{8x^2} & ,x\neq 0\\ k & ,x=0\end{cases}$ is continuous x = 0 then k = ? **[1]**

a) $\frac{-1}{2}$

b) $\frac{1}{2}$

c) 2

d) 1

47. If $f(x) = \sqrt{1-\sqrt{1-x^2}}$, then f(x) is **[1]**

a) none of these

b) continuous on [–1, 1] and differentiable on (–1, 1)

c) continuous on [–1, 1] and differentiable on (–1, 0) ∪ (0, 1)

d) continuous and differentiable on [–1, 1]

48. f(x) = $|\log_e |x||$, then **[1]**

a) f(x) is continuous and differentiable for all x in its domain

b) f(x) is continuous for all x in its domain but not differentiable at x = ± 1

c) none of these

d) f(x) is neither continuous nor differentiable at x = ± 1

49. Let f(x) = $\begin{cases} e^{1/x}, x < 0 \\ x, x \geqslant 0 \end{cases}$ $, then \underset{x \to 0}{Lt} \ f(x)$ **[1]**

a) does not exist

b) is equal to 0

c) is equal to non – zero real number

d) None of these

50. $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$ where **[1]**

a) $-1 \leqslant x \leqslant 1$

b) -1 < x < 1

c) $-1 \leqslant x < 1$

d) $-1 < x \leqslant 1$

51. If y = $\log\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$ then $\frac{dy}{dx}$ =? **[1]**

a) $\frac{-2}{\sqrt{1+x^2}}$

b) $\frac{2\sqrt{1+x^2}}{x^2}$

c) none of these

d) $\frac{2}{\sqrt{1+x^2}}$

52. Find the value of b for which the function $f(x) = \begin{cases} 5x-4 & , 0 < x \leq 1 \\ 4x^2+3bx & , 1 < x < 2 \end{cases}$ is continuous at every point of its domain, is **[1]**

a) $\frac{13}{3}$

b) -1

c) 1

d) 0

53. If $y = \frac{ax+b}{x^2+c}$, then $(2xy_1 + y)\ y_3$ = **[1]**

a) $3\ (xy_1 + y_2)\ y_2$

b) $3\ (xy_2 + y_1)\ y_2$

c) none of these

d) $3\ (xy_2 + y_1)\ y_1$

54. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \ldots + \frac{x^2}{(1+x^2)^n} + \ldots$ then at x = 0, f(x)ṁ **[1]**

a) has no limit

b) in continuous but not differentiable

c) is discontinuous

d) is differentiable

55. If f(x) = |3 – x| + (3 + x), where (x) denotes the least integer **[1]**

a) neither differentiable nor continuous at x = 3

b) continuous but not differentiable at x = 3

c) differentiable but not continuous at x = 3

d) continuous and differentiable at x = 3

56. If y = $(\tan x)^{\cot x}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $-(\tan x)^{\cot x} . \operatorname{cosec}^2 x$

b) $(\tan x)^{\cot x} . \operatorname{cosec}^2 x (1 - \log \tan x)$

c) $\cot x . (\tan x)^{\cot x - 1} . \sec^2 x$

d) None of these

57. If $y = \sin^{-1}(3x - 4x^3)$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{3}{\sqrt{1-x^2}}$

b) $\frac{-4}{\sqrt{1-x^2}}$

c) None of these

d) $\frac{3}{\sqrt{1+x^2}}$

58. If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ $\frac{dy}{dx} = ?$ **[1]**

a) a cot θ

b) cot θ

c) tan θ

d) a tan θ

59. If $y = \tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{2}{(1+x^2)}$

b) $\frac{1}{(1+x^2)}$

c) $\frac{1}{2(1+x^2)}$

d) None of these

60. Let f(x) = (x + |x|) |x|. Then, for all x **[1]**

a) f is differentiable for some x

b) f'' is continuous

c) f' is continuous

d) f is continuous

61. If $y = x^{\sqrt{x}}$ then $\frac{dy}{dx} = ?$ **[1]**

a) None of these

b) $\sqrt{x} \cdot x^{(\sqrt{x}-1)}$

c) $x^{\sqrt{x}}\left\{\frac{2+\log x}{2\sqrt{x}}\right\}$

d) $\frac{x^{\sqrt{x}}\log x}{2\sqrt{x}}$

62. $\frac{d}{dx}(\tan^{-1}(\cot x))$ is equal to **[1]**

a) None of these

b) – 1

c) $\sin^2 x$

d) $-\operatorname{cosec}^2 x$

63. The value of k for which $f(x) = \begin{cases} \frac{3x + 4\tan x}{2}, \text{ when } x \neq 0 \\ k, \text{ when } x = 0 \end{cases}$ is continuous at x = 0, is **[1]**

a) 3

b) 7

c) None of these

d) 4

64. If x = a cos nt - b sin nt, then $\frac{d^2x}{dt^2}$ is **[1]**

a) $-n^2x$

b) n^2x

c) -nx

d) nx

65. If $x = at^2$,y = 2 a t, then $\frac{d^2y}{dx^2}$ = **[1]**

a) None of these

b) 0

c) $\frac{1}{t^2}$

d) $-\frac{1}{2a\ t^3}$

66. If y = x^x then $\frac{dy}{dx} = ?$ **[1]**

a) x(1 + log x)

b) None of these

c) x^x (1 + log x)

d) x^x log x

67. $\lim_{x\to\infty} \left(1 + \frac{3}{x}\right)^x$ is equal to **[1]**

a) 3 e

b) None of these

c) e^3

d) $e^{1/3}$

68. If y = $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

a) None of these

b) $\frac{-2}{(1+x^2)}$

c) $\frac{2}{(1+x^2)}$

d) $\frac{2x}{(1+x^2)}$

69. If the function f(x) = $\begin{cases} \frac{k\cos x}{(\pi-2x)}, & \text{when } x \neq \frac{\pi}{2} \\ 3, & \text{when } x = \frac{\pi}{2} \end{cases}$ be continuous at x = $\frac{\pi}{2}$, then the value of k is **[1]**

a) 6

b) 3

c) -3

d) -5

70. If e^{x+y} = xy then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{(x-xy)}{(xy-y)}$

b) none of these

c) $\frac{y(1-x)}{x(y-1)}$

d) $\frac{x(1-y)}{y(x-1)}$

71. If y = 2^x then $\frac{dy}{dx} = ?$ **[1]**

a) 2^x (log 2)

b) None of these

c) $\frac{2^x}{(\log 2)}$

d) $x(2^{x-1})$

72. If $f(x) = \frac{1-\sin x}{(\pi-2x)^2}$, when $x \neq \frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right) = \lambda$, then f(x) will be continuous function at $x = \frac{\pi}{2}$, where λ = **[1]**

a) 1/4

b) none of these

c) 1/2

d) 1/8

73. If $y = \log\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is given by **[1]**

a) 0

b) ∞

c) $\frac{1}{2}$

d) 1

74. If $f(x) = \begin{cases} \frac{\sin(\cos x)-\cos x}{(\pi-2x)^2} & , x \neq \frac{\pi}{2} \\ k & , x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then k is equal to **[1]**

a) 1

b) -1

c) 0

d) $\frac{1}{2}$

75. If y = ax^2 + bx + c, then $y^3\frac{d^2y}{dx^2}$ is **[1]**

a) a constant

b) a function of x only

c) a function of y only

d) a function of x and y