Solution

APPLICATIONS OF DERIVATIVES MCQS

Class 12 - Mathematics

1.

(b) -1 < k < 1

Explanation: -1 < k < 1

2.

(d) $e^{1/e}$

Explanation: Let y = f(x) = $\frac{1}{x}^{x}$

Then, log y = log$\frac{1}{x}^{x}$ = xlog$\frac{1}{x}$ = -xlogx

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\left(x.\frac{1}{x} + logx.1\right) = -(1+\log x)$

$\Rightarrow f'(x) = -\frac{1}{x}^{x}(1+\log x)$

$f'(x) = 0 \Rightarrow (1+\log x) = 0$

$\Rightarrow \log x = -1 \Rightarrow x = \frac{1}{e}$

The maximum value of f(x) = $= f(\frac{1}{e}) = e^{1/e}$

3. **(a)** $\frac{1}{2}$

Explanation: Let, the numbers whose sum is 8 are 8, 8 - x.

Given f(x) = $\frac{1}{x} + \frac{1}{8-x}$

$\Rightarrow$ f'(x) = $\frac{-1}{x^2} + \frac{1}{(8-x)^2}$

to find minima or maxima

f'(x) = 0

$\Rightarrow$ $\frac{-1}{x^2} + \frac{1}{(8-x)^2}$ = 0

$\Rightarrow$ x = 4

f''(x) = $\frac{2}{x^3} - \frac{2}{(8-x)^3}$

$\Rightarrow$ f''(4) = $\frac{2}{4^3} - \frac{2}{(8-4)^3}$ = 0

Minimum value of the sum of their reciprocals = $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

4. **(a)** 0

Explanation: we have, f(x)=sinx,then, f'(x)=cosx

For stationary points,we must have f'(x)=0 $\Rightarrow$ cosx=0$\Rightarrow$x=$\frac{3\pi}{2}$[$\because x \in [\pi, 2\pi]$]

Now ,f(π)=sinπ=0, f($\frac{3\pi}{2}$)=sin$\frac{3\pi}{2}$=-1 and f(2π)=sin2π=0

Hence,the maximum value of f(x) is 0.

5.

(c) $a \leq -\frac{1}{2}$

Explanation: $a \leq -\frac{1}{2}$

6.

(b) 0 < x < 1

Explanation: 0 < x < 1

7. **(a)** $e^{\frac{-1}{e}}$

Explanation: Here,it is given the function f(x) = x^x

$\Rightarrow$ Keeping f'(x)= $x^x(1+logx)$ = 0

We get

$\Rightarrow$ x = 0 or $x = \frac{1}{e}$

$\Rightarrow$ $f''(x) = x^x(1 + logx + \frac{1}{x})$

When x is greater than zero. Then $f''(x) \leq 0$

We get a maximum value as the function will be negative

Thus,

f(x) = x^x

f(e) = $\left(\frac{1}{e}\right)^{1/e} = e^{-\frac{1}{e}}$

8.

(c) $\lambda > 1/2$

Explanation: $\lambda > 1/2$

9. **(a)** 2

Explanation: Here ,it is given the function f(x) = $e^x + e^{-x}$

$\Rightarrow f(x) = e^x + \frac{1}{e^x}$

$\Rightarrow f(x) = \frac{e^{2x}+1}{e^x}$

f(x) is always increasing at x = 0 it has the least value

$\Rightarrow f(x) = \frac{1+1}{1}$ = 2

∴ The least value is 2

10.

(b) $\left(0, \frac{3}{2}\right) \cup (3, \infty)$

Explanation: Given,f(x) = $[x(x - 3)]^2$

⇒ f'(x) = 2 [x(x - 3)] = 0|

⇒ x = 3 and $x = \frac{3}{2}$

When x > $\frac{3}{2}$ the function in increasing

X < 3 function is increasing

$\Rightarrow \left(0, \frac{3}{2}\right) \cup (3, \infty)$ Function is increasing

11.

(b) ab $\geq \frac{c^2}{4}$

Explanation: f(x) = ax + $\frac{b}{x}$

⇒ f'(x) = a - $\frac{b}{x^2}$

f'(x) = 0

a - $\frac{b}{x^2}$ = 0

⇒ x = $\pm\sqrt{\frac{b}{a}}$

f''(x) = $\frac{2b}{x^3}$

f''($\sqrt{\frac{b}{a}}$) = $\frac{2b}{(\sqrt{\frac{b}{a}})^3} > 0$

⇒ x = $\sqrt{\frac{b}{a}}$ has a minima.

$f(\sqrt{\frac{b}{a}})$ = $2\sqrt{ab} \geq c$

$\frac{c}{2} \leq \sqrt{ab}$

$\Rightarrow \frac{c^2}{4} \leq ab$

12.

(b) $\frac{1}{2}$

Explanation: Let f(x) = sinx . cosx

$\Rightarrow f(x) = \frac{1}{2}(sin2x)$

Now, f'(x)=$\frac{1}{2}(cos2x).2 = cos2x$

For maximum and minimum values of x, we have f'(x) = 0

f'(x) = 0$\Rightarrow cos2x = 0$

$\Rightarrow x = \frac{\pi}{4}$

Now, f''(x) = -2sin2x

i.e, f''($\frac{\pi}{4}$) = -2sin$\frac{\pi}{2}$ = -2 < 0

Hence, f(x) has a maximum value at x = $\frac{\pi}{4}$ and the max value of f($\frac{\pi}{4}$) = sin$\frac{\pi}{4}$ cos$\frac{\pi}{4}$ = $\frac{1}{2}$.

13.

(c) -39

Explanation: Given function,

$f(x) = 3x^4 - 8x^3 - 48x + 25$

$F'(x) = 12x^3 - 24x^2 - 48 = 0$

$F'(x) = 12(x^3 - 2x^2 - 4) = 0$

Differentiating again, we obtain

$F''(x) = 3x^2 - 4x = 0$

$x(3x - 4) = 0$

$x = 0$ or $x = \frac{4}{3}$

Putting the value in equation, we obtain

$f(x) = -39$

14.

(d) neither maximum nor minimum

Explanation: We have, $f(x) = 2 \sin 3x + 3 \cos 3x$

$\therefore f'(x) = 6 \cos 3x - 9 \sin 3x$

$\therefore f'\left(\frac{5\pi}{6}\right) = 6\cos\left(3 \cdot \frac{5\pi}{6}\right) - 9\sin\left(3 \cdot \frac{5\pi}{6}\right)$

$= 6\cos\frac{5\pi}{2} - 9\sin\frac{5\pi}{2} = 0 - 9 \neq 0$

So, $x = \frac{5\pi}{6}$ cannot be point of maxima or minima

15.

(b) $-\infty, \infty$

Explanation: $(-\infty, \infty)$

$f(x) = \cot^{-1} x + x$

$f'(x) = \frac{-1}{1+x^2} + 1$

$= \frac{-1+1+x^2}{1+x^2}$

$= \frac{x^2}{1+x^2} \geq 0, \forall x \in R$

So, f (x) is increasing on $(-\infty, \infty)$

16.

(b) $0 < X < 2$

Explanation: $0 < X < 2$

$f(x) = x^2e^{-x}$

$f'(x) = 2xe^{-x} - x^2e^{-x}$

$= e^{-x}(2 - x)$

For f(x) to be monotonic increasing, we must have

$f'(x) > 0$

$\Rightarrow e^{-x}x(2 - x) > 0$ $[\because e^{-x} > 0]$

$\Rightarrow x(2 - x) > 0$

$\Rightarrow x(x - 2) < 0$

$\Rightarrow 0 < x < 2$

17.

(c) $\frac{4}{27}$

Explanation: Here,it is given function $ff(x) = (x - 2)(x - 3)^2$

$f(x) = (x - 2)(x^2 - 6x + 9)$

$f(x) = x^3 - 8x^2 + 21x + 18$

$f'(x) = 3x^2 - 16x + 21$

$f''(x) = 6x - 16$

For maximum or minimum value $f'(x) = 0$

$\therefore 3x^2 - 9x - 7x + 21 = 0$

$\Rightarrow 3x(x - 3) - 7(x - 3) = 0$

$\Rightarrow x = 3$ or $x = \frac{7}{3}$

f''(c) at x = 3

$\therefore$ f''(x) = 2

f''(x) > 0 it is decreasing and has minimum value at x = 3

at x = $\frac{7}{3}$

f'' (x) = -2

f''(x) < 0 it is increasing and has maximum value at x = $\frac{7}{3}$

Putting, x = $\frac{7}{3}$ in f(x) we obtain,

$\Rightarrow \left(\frac{7}{3}-2\right)\left(\frac{7}{3}-3\right)^2$

$= \left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)^2$

$= \frac{4}{27}$

18.

(b) only one minima

Explanation: Given, $f(x) = |x| = \begin{matrix} -x, & x < 0 \\ x, x > 0 & \end{matrix}$

$\Rightarrow f'(x) = -1$ when x< 0 and 1 when x > 0

But, we have f'(x) does not exist at x = 0, hence we have x = 0 is a critical point

At x = 0, we get f(0) = 0

For any other value of x, we have f(x) > 0 , hence f(x) has a minimum at x= 0.

19. **(a)** a = 2, b = $-\frac{1}{2}$

Explanation: Let f(x) = alogx + bx^2 + x

$f'(x) = a.\frac{1}{x} + 2bx + 1$

For maximum and minimum value of f(x) we have f'(x) = 0

Therefore, at x = -1 and x = 2 we have 2bx^2 + x + a = 0

i.e, a + 2b = 1....(i) and a + 8b = 2....(ii)

(ii) - (i) gives b = $-\frac{1}{2}$

Now , from (i) we get a = 2

$\Rightarrow$ a = 2, b = $-\frac{1}{2}$

20. **(a)** x $\in$ R

Explanation: x $\in$ R

21.

(d) R

Explanation: R

22.

(d) 0

Explanation: Let f(x) = x^3 - 18x^2+ 96x

$\Rightarrow$ f'(x) = 3x^2 - 36x + 96 = 3[x^2 - 12x +32] = 3(x - 4)(x - 8)

For maximum and minimum values of x, we have f'(x) = 0

$\Rightarrow$ 3(x - 4)(x - 8) $\Rightarrow$ x = 4, 8

Both of these values lies in the given interval [0,9]

Now, f''(x) = 6x - 36

When x = 8 we get f''(x) = 12 > 0

Since at x = 8, f'(x)= 0 and f''(x)>0, we get f(x) is minimum at x = 8 in (0,9)

Now, we have to find minimum values at the end points of the given interval

We have, f(0) = 0 and f(9) = 135

Hence, the minimum value of f(x) = 0 at x= 0 in [0,9]

23. **(a)** a > 0

Explanation: f(x) = ax

f'(x) = a

f(x) is increasing on $\frac{1}{2}$ if a > 0

24.

(d) $\frac{\pi}{2}$

Explanation: f(x) = x + cosx

f'(x) = 1 - sinx

For maximum and minimum values of f(x) , we have f'(x) = 0

Now, f'(x) = 0 ⇒ $1 - sinx = 0 \Rightarrow x = \frac{\pi}{2}$

Hence, maximum value of f(x) is $f(\frac{\pi}{2}) = \frac{\pi}{2}$

25. **(a)** cos x

Explanation: $f_1(x) = \sin^2 x$, increases from '0' to '1' in $\left(0, \frac{\pi}{2}\right)$

$f_2(x)$ = tan x is increasing function in each quadrant

$f_3(x)$ = cos x, decreases from '1' to '0' in $\left(0, \frac{\pi}{2}\right)$

$f_4(x)$ = cos 3x, decreases if $3x \in \left(0, \frac{\pi}{2}\right)$ or $x \in \left(0, \frac{\pi}{6}\right)$

26.

(d) $\frac{1}{e}$

Explanation: Consider f(x) = $\frac{logx}{x}$

Then, f'(x) == $\frac{x.\frac{1}{x} - logx.1}{x^2} \quad = \frac{1 - logx}{x^2}$

For maximum or minimum values of x we have f'(x) = 0

f'(x) = 0 ⇒ (1 - logx) = 0

⇒ logx = 1 ⇒ x = e.

Now, f'(x) $= \frac{x^2.\frac{-1}{x} - (1 - logx)2x}{x^4} = \left[\frac{-3 + 2logx}{x^3}\right]$

f''(x) at $at \quad x = e = \frac{-3}{e^3} < 0$

Therefore f(x) is maximum at x = e and the max. value= $\frac{loge}{e} = \frac{1}{e}$

27.

(c) Decreasing on R

Explanation: Given, f'(x) = $-x^3 + 3x^2 - 3x + 4$

f'(x) = $-3x^2 + 6x - 3$

f'(x) = $-3(x^2 - 2x + 1)$

f'(x) = $-3(x - 1)^2$

As f'(x) has -ve sign before 3

⇒ f'(x) is decreasing over R.

28.

(d) 126

Explanation: Marginal revenue (MR) is the rate of change of total revenue with respect to the number of units sold.

So, MR = $\frac{dR}{dx}$ = 6x + 36 = 6x + 36

∴ when x = 15, then

MR = 6(15) + 36 = 126

Therefore, the marginal revenue when x = 15 is 126.

29.

(d) 75

Explanation: f(x) = $x^2 + \frac{250}{x}$

⇒f'(x) = 2x - $\frac{250}{x^2}$

For the local minima or maxima we must have

f'(x) = 0

$2x - \frac{250}{x^2} = 0$

$\Rightarrow$ x = 5

f''(x) = 2 + $\frac{500}{x^3}$

f''(5) = 2 + $\frac{500}{125}$ > 0

function has minima at x = 5

f(5) = 75

30.

(d) 1

Explanation: Given f(x) = x^2 - 8x + 17

$\Rightarrow$ f'(x) = 2x - 8

For minimum valye of f(x) we have f'(x) = 0

$\Rightarrow$ 2x - 8 = 0 $\Rightarrow$ x = $\frac{8}{2}$ = 4

Now, f''(x) = 2 > 0, hence the minimum of f(x) exist at x = 4 and minimum value = f(4) = 1

31.

(b) -1

Explanation: f(x) = $2x^3 - 3x^2$ - 12x + 5

$\Rightarrow$ f'(x) = $6x^2$ - 6x - 12

For local maxima or minima we have

f'(x) = 0

$6x^2$ - 6x - 12 = 0

$\Rightarrow$ x^2 - x - 2 = 0

$\Rightarrow$ x = 2 or x = -1

f''(x) = 12x - 6

f''(2) = 18 > 0

function has local minima at x = 2.

f''(-1) = -18 < 0

function has local maxima at x = -1.

32. **(a)** local minima at x = 1

Explanation: Given, f(x) = x^3 - 3x

f'(x) = $3x^2$ - 3

For point of inflexion we have f'(x) = 0

$f'(x) = 0 \Rightarrow 3x^2 - 3 = 0 = 3(x-1)(x+1) \Rightarrow x = \pm 1$

Hence, f(x) has a point of inflexion at x = 0.

When , x is slightly less than 1, f'(x) = (+)(-)(+) i.e, negative

When x is slightly greater than 1, f'(x)= (+)(+)(+) i.e, positive

Hence, f'(x) changes its sign from negative to positive as x increases through 1 and hence x = 1 is a point of local minimum.

33. **(a)** 2

Explanation: f(x) = $2x^3 - 15x^2$ + 36x + 4

$\Rightarrow$ f'(x) = $6x^2$ - 30x + 36

for local minima or maxima

f'(x) = 0

$6x^2$ - 30x + 36 = 0

x^2 - 5x + 6 = 0

$\Rightarrow$ x = 2 or x = 3

f''(x) = 12x - 30

f''(2) = -6 < 0

x = 2 has maxima.

f''(3) = 6 > 0

x = 3 has minima.

34.

(c) increasing

Explanation: increasing

35. **(a)** k > 3

Explanation: $f(x) = kx^3 - 9x^2 + 9x + 3$

$f'(x) = 3kx^2 - 18x + 9$

$= 3(kx^2 - 6x + 3)$

Given: f(x) is monotonically increasing in every interval.

$\Rightarrow f'(x) > 0$

$\Rightarrow 3(kx^2 - 6x + 3) > 0$

$\Rightarrow (kx^2 - 6x = 3) > 0$

$\Rightarrow K > 0$ and $(-6)^2 - 4(k)(3) < 0$ [$\because ax^2 + bx + c > 0$ and D is c < 0]

$\Rightarrow k > 0$ and $(-6)^2 - 4(k)(3) < 0$

$\Rightarrow k > 0$ and $36 - 12k < 0$

$\Rightarrow k > 0$ and $12k > 36$

$\Rightarrow k > 0$ and $k > 3$

$\Rightarrow k > 3$

36. **(a)** point of inflexion at x = 0

Explanation: Given $f(x) = x^3$

$f'(x) = 3x^2$

For point of inflexion, we have f'(x) = 0

$f'(x) = 0 \Rightarrow 3x^2 = 0 \Rightarrow x = 0$

Hence, f(x) has a point of inflexion at x = 0.

But x = 0 is not a local extremum as we cannot find an interval I around x = 0 such that

$f(0) \geq f(x) \quad or \quad f(0) \leq f(x) \qquad for \quad all \quad x \epsilon I$

37. **(a)** $a = \frac{1}{2}$

Explanation: $a = \frac{1}{2}$

38. **(a)** -128

Explanation: $f(x) = 2x^3 - 21x^2 + 36x - 20$

$\Rightarrow f'(x) = 6x^2 - 42x + 36$

For local maxima or minima

$6x^2 - 42x + 36 = 0$

$x^2 - 7x + 6 = 0$

$\Rightarrow x = 1$ or $x = 6$

$f''(x) = 12x - 42$

$\Rightarrow f''(1) = -30 < 0$

also, $f''(6) = 30 > 0$

function has minima at x = 6

$\Rightarrow f(6) = -128$

39. **(a)** $(1, \infty)$

Explanation: Given, function

$\Rightarrow f(x) = (x + 1)^3. (x - 3)^3$

$\Rightarrow f'(x) = 3(x + 1)^2 (x - 3)^3 + 3(x - 3)^3 (x + 1)^3$

Put f'(x) = 0

$\Rightarrow 3(x + 1)^2 (x - 3)^3 = -3(x - 3)^2 (x + 1)^3$

$\Rightarrow x - 3 = -(x + 1)$

$\Rightarrow 2x = 2$

$\Rightarrow x = 1$

When x > 1 the function is increasing

x < 1 function is decreasing

Therefore f(x) is increasing in $(1, \infty)$.

40. **(a)** $x = \frac{-\pi}{2}$

Explanation: We can go through options for this question

Option a is wrong because 0 is not included in $(-\pi, 0)$

At $x = \frac{-\pi}{4}$ value of f(x) is $-\sqrt{2}$ = -1.41

At $x = \frac{-\pi}{3}$ value of f(x) is -2.

At $x = \frac{-\pi}{2}$ value of f(x) = -1.

$\therefore$ f(x) has max value at $x = \frac{-\pi}{2}$

Which is -1.This is the required solution.

41.

(d) monotonically decreasing

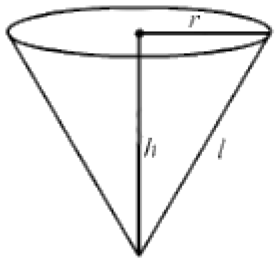
Explanation: monotonically decreasing

42.

(b) $54\pi \mathrm{cm}^2/\mathrm{min}$

Explanation:

Let r be the radius, h be the height and S be the lateral surface area of the cone at any time t.



Given: $\frac{dr}{dt}$ =3cm/min and $\frac{dh}{dt} = -4cm/\min$

Here,

$l^2 = h^2 + r^2$

$\Rightarrow l = \sqrt{(24)^2 + (7)^2}$

$\Rightarrow l = \sqrt{625}$

$\Rightarrow$ l = 25

$S = \pi r l$

$\Rightarrow S^2 = (\pi r l)^2$

$\Rightarrow S^2 = \pi^2 r^2 \left(h^2 + r^2\right)$

$\Rightarrow S^2 = \pi^2 r^4 + \pi^2 h^2 r^2$

$\Rightarrow 2S\frac{dS}{dt} = 4\pi^2 r^3 \frac{dr}{dt} + 2\pi^2 r^2 h \frac{dh}{dt} + 2\pi^2 h^2 r \frac{dr}{dt}$

$\Rightarrow 2\pi r l \frac{dS}{dt} = 2\pi^2 r h \left[\frac{2r^2}{h}\frac{dr}{dt} + r\frac{dh}{dt} + h\frac{dr}{dt}\right]$

$\Rightarrow 25\frac{dS}{dt} = 24\pi\left[\frac{2(7)^2}{24} \times 3 - 7 \times 4 + 24 \times 3\right]$ [Given: r = 7, h = 24]

$\Rightarrow 25\frac{dS}{dt} = 24\pi\left[\frac{49}{4} - 28 + 72\right]$

$\Rightarrow 25\frac{dS}{dt} = 24\pi\left[\frac{49+288-112}{4}\right]$

$\Rightarrow \frac{dS}{dt} = 24\pi\left[\frac{225}{100}\right]$

$\Rightarrow \frac{dS}{dt} = 24\pi(2.25)$

$\Rightarrow \frac{dS}{dt} = 54\pi \mathrm{cm}^2/\mathrm{sec}$

43.

(c) $(-\infty, 1) \cup (2, 3)$

Explanation: Given that;

f(x) = 2 log(x - 2) - x^2 + 4x +1

$f'(x) = \frac{2}{(x-2)}$ - 2x + 4

= $\frac{2}{(x-2)}$ - 2(x - 2)

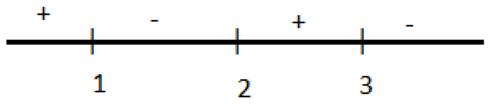
= $\frac{2(1-(x-2)^2)}{(x-2)}$

= $\frac{2(1-x+2)(1+x-2)}{(x-2)}$

= $\frac{2(3-x)(x-1)}{(x-2)}$

Critical points are;

1, 2 and 3



F(x) is increasing in $(-\infty, 1) \cup (2, 3)$

44.

(d) relative minimum > relative maximum

Explanation: f(x) = x + $\frac{1}{x}$

Then, f'(x) = 1 - $\frac{1}{x^2}$

For, relative maximum and minimum values of x, we have f'(x) =0

$\Rightarrow$ 1 - $\frac{1}{x^2}$ = 0

$\Rightarrow$ x^2 = 1

$\Rightarrow$ x = ± 1

Now, f''(x) = $\frac{2}{x^3}$

When , x = 1, we get f''(x) = 2>0 and when x= -1, we get f''(x) = -2 <0

f(x) = x + $\frac{1}{x}$ has a local maximum at x = -1 and a local minimum at x= 1.

Now, the maximum value = f(-1) =-2 and minimum value = f(1) =2

45.

(c) $\frac{1}{6}$

Explanation: f(x) = $\frac{x}{4+x+x^2}$

$\Rightarrow$ f'(x) = $\frac{4-x^2}{(4+x+x^2)^2}$

For a local maxima or minima,

f'(x) = 0

$\frac{4-x^2}{(4+x+x^2)^2}$ = 0

$\Rightarrow$ x = $\pm 2 \in$ [-1, 1]

f(1) = $\frac{1}{6}$ > 0

f(-1) = $\frac{-1}{4}$ < 0

$\Rightarrow$ $\frac{1}{6}$ is the maximum value.

46.

(d) $\left(\frac{1}{e}\right)$

Explanation: $\Rightarrow f(x) = \frac{\log x}{x}$

$\therefore \mathrm{f}'(\mathrm{x}) = \frac{\log \mathrm{x} - \mathrm{x}\frac{1}{\mathrm{x}}}{\mathrm{x}^2}$

$\Rightarrow$ f'(x) = log x - 1

$\Rightarrow$ substitute f'(x) = 0

We get x = e

F''(x) = $\frac{1}{x}$

Substitute x = e in f''(x)

$\frac{1}{e}$ is point of maxima

$\therefore$ The max value is $\frac{1}{e}$

47.

(c) decreasing in $\left(\frac{\pi}{2}, \pi\right)$

Explanation: We have, f(x) = 4 $\sin^3$x - 6 $\sin^2$x + 12 sin x + 100

$\therefore$ f'(x) = 12 $\sin^2$ x $\cdot$ cos x - 12 sin x $\cdot$ cos x + 12 cos x

= 12 cos x [$\sin^2$x - sin x + 1]

= 12 cos x [$\sin^2$ x + (1 - sinx)]

Now 1 - sin x $\geq$ 0 and $\sin^2$ x $\geq$ 0

$\therefore$ $\sin^2$ x + 1 - sin x > 0

Hence f'(x) > 0, when cos x > 0 i.e., $x \in\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

So, f(x) is increasing when $x \in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

and f'(x) < 0, when cos x < 0 i.e., $x \in\left(\frac{\pi}{2}, \frac{3 \pi}{2}\right)$

Hence, f(x) is decreasing when $x \in\left(\frac{\pi}{2}, \frac{3 \pi}{2}\right)$

Hence, f(x) is decreasing in $\left(\frac{\pi}{2}, \pi\right)$

48.

(b) $\frac{4}{3}$

Explanation: f(x) = $\frac{1}{4 x^2+2 x+1}$

$\Rightarrow$ f'(x) = 8x + 2

For local minima or maxima we have

f'(x) = 8x + 2 = 0

$\Rightarrow$ x = $\frac{-1}{4}$

f''(x) = 8 > 0

$\Rightarrow$ function has maxima at x = $\frac{-1}{4}$

$f\left(\frac{-1}{4}\right)=\frac{4}{3}$

49. **(a)** decreases on [0, a]

Explanation: decreases on [0, a]

50.

(d) $x=\frac{1}{e}$

Explanation: Consider f(x) = y =x^x

Then, log y = log x^x = x.logx

$\Rightarrow f^{\prime}(x)=x^x(1+\log x)$

$\Rightarrow(1+\log x)=0 \ldots \ldots(\because x^x \neq 0)$

$\Rightarrow \log x=-1 \Rightarrow x=e^{-1}$

51.

(b) $\lambda>2$

Explanation: $\lambda>2$

52. **(a)** always increases

Explanation: We have, f(x) = tan x - x

$\therefore$ f'(x) = $\sec^2$ x - 1

$\Rightarrow f^{\prime}(x) \geq 0, \forall x \in R$

So, f(x) always increases

53. **(a)** $\frac{a+b+c}{3}$

Explanation: f(x) = $(x - a)^2 + (x - b)^2 + (x - c)^2$

$\Rightarrow$f'(x) = 2(x - a) + 2(x - b) + 2(x - c)

to find minima or maxima

f'(x) = 0

2(x - a) + 2(x - b) + 2(x - c) = 0

$\Rightarrow$ x = $\frac{a+b+c}{3}$
f''(x) = 6 > 0
function has minima at x = $\frac{a+b+c}{3}$.

54.
(c) increasing on $\left(0, \frac{\pi}{2}\right)$
Explanation: increasing on $\left(0, \frac{\pi}{2}\right)$

55.
(d) monotonic function
Explanation: monotonic function

56.
(d) $-\frac{1}{e}$
Explanation: f(x) = x $\log_e$x
$\Rightarrow$ f'(x) = 1 + $\log_e$x
to find maxima or minima
f'(x) = 0
$\Rightarrow$ 1 + $\log_e$x = 0
$\Rightarrow$ x = $\frac{1}{e}$
f''(x) = $\frac{1}{x}$
f''$\left(\frac{1}{e}\right)$ = e > 0
x = $\frac{1}{e}$ is a local minima.
$\Rightarrow$ Minimum value of the function is
f$\left(\frac{1}{e}\right)$ = $\frac{1}{e}\log_e\left(\frac{1}{e}\right) = \frac{-1}{e}$

57.
(d) (-∞, 4)
Explanation: $f(x) = 2x^2 - kx + 5$
$f'(x) = 4x - k$
for f(x) to be increasing, we must have
f(x) > 0
4x - k > 0
K < 4 x
since $x \in [1, 2], 4x \in [4, 8]$
so, the minimum value of 4 x is 4.
since K < 4x, K < 4.
k $\in (-\infty, 4)$

58.
(b) none of these
Explanation: f(x) = x + $\frac{1}{x}$
$\Rightarrow$ f'(x) = 1 -$\frac{1}{x^2}$
For minimum or maximum value of the function
f'(x) = 0
$\Rightarrow$ 1 - $\frac{1}{x^2}$ = 0
$\Rightarrow$ x = ± 1
f''(x) = $\frac{2}{x^3}$
$\Rightarrow$ f''(x) = 1 > 0 $\Rightarrow$ function has minima at x = 1.
f''(-1) = -1 > 0 $\Rightarrow$ function has minima at x = -1.

59. **(a)** Strictly increasing on R
Explanation: Given,ff(x) = $x^3 + 6x^2 + 15x - 12$
f'(x) = $3x^2 + 12x + 15$
f'(x) = $3x^2 + 12x + 12 + 3$

f'(x) = $3(x^2 + 4x + 4) + 3$

f'(x) = $3(x + 2)^2 + 3$

As square is a positive number

∴ f'(x) will be always positive for every real number

Hence f'(x) > 0 for all x ∈ R

∴ f'(x) is strictly increasing.

60. **(a)** -1 < x < -3

Explanation: Here,it is given that function,

f(x) = $x^3 + 6x^2 + 9x + 3$

f'(x) = $3x^2 + 12x + 9 = 0$

f'(x) = $3(x^2 + 4x + 3) = 0$

f'(x) = 3(x + 1) (x + 3) = 0

x = -1 or x = -3

for x > -1 f(x) is increasing

for x < -3 f(x) is increasing

But for - 1 < x <-3 it is decreasing

61.

(d) (0 , $\frac{1}{e}$)

Explanation: (0 , $\frac{1}{e}$)

Let y = x^x

$\Rightarrow \log(y)$ = xlogx

$\Rightarrow \frac{1}{y} \times \frac{dy}{dx}$ = 1 + logx

$\Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$

Since the function is decreasing,

$\Rightarrow x^x x(1 + |\log x) < 0$

$\Rightarrow 1 + \log x < 0$

$\Rightarrow \log x < -1$

$\Rightarrow x < \frac{1}{e}$

Therefore, function is decreasing on (0,$\frac{1}{e}$)

62. **(a)** f(x) is invertible

Explanation: f(x) is invertible

63.

(b) e

Explanation: $f(x) = \frac{x}{\log x}$

$\Rightarrow f'(x) = \frac{logx.1 - x.\frac{1}{x}}{(\log x)^2}$

For maximum or minimum values of x we have f'(x) = 0

$f'(x) = 0 \Rightarrow \frac{\log x - 1}{(\log x)^2} = 0 \Rightarrow (\log x - 1) = 0$

$\Rightarrow \log x = 1 \Rightarrow x = e$

Now, $f''(x) = (\log x - 1)\frac{-2}{(\log x)^3} \quad + (\log x)^{-2}.\frac{1}{x}$

$f''(e) = \frac{1}{e} > 0$

Hence, f(x) has a minimum value f(e) = e.

64.

(d) (-1, 1)

Explanation: We have,$\Rightarrow f(x) = \frac{x}{x^2+1}$

$\Rightarrow \mathrm{f}'(\mathrm{x}) = \frac{\mathrm{x}^2 - 2\mathrm{x}^2 + 1}{\mathrm{x}^2 + 1}$

$\Rightarrow \mathrm{f}'(\mathrm{x}) = -\frac{\mathrm{x}^2 - 1}{\mathrm{x}^2 + 1}$

$\Rightarrow$ for critical points f'(x) = 0
when f'(x) = 0
We get x = 1 or x = -1
When we plot them on number line as f'(x) is multiplied by -ve sign we get
For x > 1 function is decreasing
For x < -1 function is decreasing
But between -1 to 1 function is increasing
$\therefore$ Function is increasing in (-1, 1)

65. **(a)** 2
Explanation: Given xy = 1. To find minimum value of x + y
$\Rightarrow$ y = $\frac{1}{x}$
f(x) = x + $\frac{1}{x}$
$\Rightarrow$ f'(x) = 1 - $\frac{1}{x^2}$
To find local maxima or minima we have
f'(x) = 0
1 - $\frac{1}{x^2}$ = 0
$\Rightarrow$ x = $\pm 1 \Rightarrow$ y =± 1
But given that x > 0 $\Rightarrow$ x = 1, y = 1
f''(x) = $\frac{2}{x^3}$
f''(1)= 2 > 0
function has minima at x = 1
f(1) = 2.

66.
(d) odd and increasing
Explanation: odd and increasing

67.
(c) $\frac{1}{4}$
Explanation: Given f(x) = $x^{25}(1 - x)^{75}$
f'(x) = $x^{25}. 75(1 - x)^{74}(-1)+(1 - x)^{75}. 25x^{24}$
= $25x^{24}(1 - x)^{74}\{-3x + (1-x)\}$
= $25x^{24}(1 - x)^{74}(1 - 4x)$
For maximum value of f(x) we have f'(x) = 0
$\Rightarrow 25x^{24}(1 - x)^{74}(1 - 4x) = 0$
$\Rightarrow$ x = 0, x = 1, x= $\frac{1}{4}$
All the values of x $\in$ [0,1]
Note that f(0) = f(1) = 0 and f($\frac{1}{4}$) = $\frac{3^{75}}{4^{100}}$
So, f(x) is maximum at x = $\frac{1}{4}$

68. **(a)** -2
Explanation: Given,f(x) = x^2 + kx + 1
For increasing
f'(x) = 2x + k
$k \geq -2x$
thus,
$k \geq -2x$
Least value of -2

69.
(d) (1, 2)
Explanation: $y^2 = 4x \Rightarrow x = \frac{y^2}{4}$

$\Rightarrow$ d = $\sqrt{(x-2)^2+(y-1)^2}$

$\Rightarrow$ $d^2 = (x - 2)^2 + (y - 1)^2$

$\Rightarrow$ $d^2 = \left(\frac{y^2}{4}-2\right)^2 + (y - 1)^2$

Let u = $\left(\frac{y^2}{4}-2\right)^2 + (y - 1)^2$

$\Rightarrow$ $\frac{du}{dy} = 2\left(\frac{y^2}{4}-2\right)\frac{y}{2} + 2(y - 1)$

To find minima

$\frac{du}{dy} = 0$

$2\left(\frac{y^2}{4}-2\right)\frac{y}{2} + 2(y - 1) = 0$

$\Rightarrow$ y = 2 $\Rightarrow$ x = 1 $\left(x=\frac{y^2}{4}\right)$

$\frac{d^2u}{dy^2} = \frac{3y^2}{4}$

$\Rightarrow$ $\left(\frac{d^2u}{dy^2}\right)_{(1,2)} = 3 > 0$

Hence, nearest point is (1, 2).

70.

(c) $\frac{3}{4}$

Explanation: Given, $f(x) = x^2 + x + 1$

$\Rightarrow$ f'(x) = 2x + 1

For minimum value of f(x) we have f'(x) = 0

$\Rightarrow$ 2x + 1 = 0 $\Rightarrow$ x = $\frac{-1}{2}$

Now, f''(x) = 2 >0, hence the minimum of f(x) exist at x = $\frac{-1}{2}$ and minimum value = f($\frac{-1}{2}$) = $\frac{3}{4}$

71.

(d) $k \in (-\infty, 4)$

Explanation: $\because$ $f(x) = X^2 - kx + 5$ is increasing in x $\in$ [2, 4]

f'(x) = 2x - k

f'(x) > 0, for increasing function

2x - k > 0

k < 2x (k should be less than minimum value of 2x)

k < 4

$k \in (-\infty, 4)$

72. **(a)** strictly increasing

Explanation: strictly increasing

73.

(b) local minima at x = 2 and a local maxima at x = – 2

Explanation: Given , f(x) = x + $\frac{4}{x}$

$\Rightarrow f'(x) = 1 - \frac{4}{x^2}$

$\Rightarrow f'(x) = 0$

$\Rightarrow x = \pm 2$

$\Rightarrow f''(x) = \frac{8}{x^3}$

$\Rightarrow f''(2) = \frac{8}{8} = 1 > 0$

$\Rightarrow f''(-2) = \frac{8}{-8} = -1 < 0$

So, f(x) has a local minima at x = 2 and a local minima at x= -2.

74. **(a)** is an increasing function

Explanation: We have, f(x) = 2x + cos x

$\therefore f'(x) = 2 - \sin x > 0, \forall x$

Hence, f(x) is an increasing function

75.

(b) $1 < x < 2$

Explanation: $1 < x < 2$