SOLUTION OF Q BANK

Class 12 - Mathematics

1.

(b) $\frac{1}{2}$

Explanation: Given that y = tan^{-1} (sec x + tan x)

Hence, $y=\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

Using $\cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2}$, $\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta+\sin^2\theta$ = 1

Hence, $y=\tan^{-1}\left(\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right)=\tan^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}\right)$

$\Rightarrow y=\tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)$

Dividing by cos $\frac{x}{2}$ in numerator and denominator, we obtain

$y=\tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$

Using $\tan\left(\frac{\pi}{4}+x\right)=\frac{1+\tan x}{1-\tan x}$, we obtain

$y=\tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)=\frac{\pi}{4}+\frac{x}{2}$

Differentiating with respect to x, we

$\frac{dy}{dx}=\frac{1}{2}$

2.

(d) $\frac{-1}{2\sqrt{1-x^2}}$

Explanation: Put x = cos 2θ ,we get

$\Rightarrow y=\sin^{-1}\left(\frac{\sqrt{1+\cos 2\theta}}{2}+\frac{\sqrt{1-\cos 2\theta}}{2}\right)$

$\Rightarrow y=\sin^{-1}\left(\frac{\sqrt{2\cos^2 2\theta}}{2}+\frac{\sqrt{2\cdot\sin^2\theta}}{2}\right)$

$\Rightarrow y=\sin^{-1}\left(\frac{\cos 2\theta}{\sqrt{2}}+\frac{\sin 2\theta}{\sqrt{2}}\right)$

$\Rightarrow y=\sin^{-1}\left(\sin\left(\frac{\pi}{4}+2\theta\right)\right)$

$\Rightarrow y=\frac{\pi}{4}+2\theta$

$\Rightarrow\frac{dy}{d\theta}=2$

Put $\theta=\frac{\cos^{-1}x}{2}$,we get

$\Rightarrow\frac{\mathrm{d}\theta}{\mathrm{dx}}=\frac{-1}{4\sqrt{1-\mathrm{x}^2}}$

$\therefore\frac{dy}{dx}=\frac{-1}{2\sqrt{1-x^2}}$

3.

(c) -1/2

Explanation: $\lim\limits_{x\to 0^-}f(x)=\lim\limits_{x\to 0^+}f(x)$

$\lim\limits_{x\to 0}\frac{\sqrt{1+px}-\sqrt{1-px}}{x}=\lim\limits_{x\to 0}\frac{2x+1}{x-2}$

$\lim\limits_{x\to 0}\frac{\sqrt{1+px}-\sqrt{1-px}}{x}\times\frac{\sqrt{1+px}+\sqrt{1-px}}{\sqrt{1+px}+\sqrt{1-px}}=\frac{-1}{2}$

$\lim\limits_{x\to 0}\frac{1+px-(1-px)}{x}\times\frac{1}{\sqrt{1+px}+\sqrt{1-px}}=\frac{-1}{2}$

$\lim\limits_{x\to 0}\frac{2px}{x}\times\frac{1}{\sqrt{1+px}+\sqrt{1-px}}=\frac{-1}{2}$

$\frac{2p}{2}=\frac{-1}{2}$

$p=\frac{-1}{2}$

4.

(b) $\frac{-2}{\sqrt{1-x^2}}$

Explanation: $\Rightarrow y=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$\Rightarrow \sec y=\frac{1}{2x^2-1}$

$\Rightarrow$ cos y = $2x^2$ - 1

$\Rightarrow$ y = $\cos^{-1}$ ($2x^2$ - 1)

Put x = cos θ ,we get

$\Rightarrow$ y =$\cos^{-1}$ ($2\cos^2\theta$ - 1)

$\Rightarrow$ y =$\cos^{-1}$ (cos 2θ)

$\Rightarrow$ y = 2θ

But θ = $\cos^{-1}$ x, we get

$\Rightarrow \frac{dy}{dx}=\frac{d\left(\cos^{-1}x\right)}{dx}$

$\Rightarrow \frac{dy}{dx}=2\cdot\frac{d\left(\cos^{-1}x\right)}{dx}$

$\Rightarrow \frac{dy}{dx}=2\cdot\left(\frac{-1}{\sqrt{1-x^2}}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{-2}{\sqrt{1-x^2}}$

5.

(c) $\frac{2x}{(1+x^4)}$

Explanation: Given that $y=\tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

Let x^2 = tan θ

$\Rightarrow \theta=\tan^{-1}x^2$

Hence, $y=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4}+x\right)=\frac{1+\tan x}{1-\tan x}$, we obtain

$y=\tan^{-1}\tan\left(\frac{\pi}{4}+\theta\right)=\frac{\pi}{4}+\theta=\frac{\pi}{4}+\tan^{-1}\left(x^2\right)$

Differentiating with respect to x, we obtain

$\frac{dy}{dx}=\frac{1}{1+x^4}\times 2x=\frac{2x}{1+x^4}$

6.

(b) – tan x

Explanation: Let y = $\sin^3$x and z = $\cos^3$x, then , $\frac{dy}{dz}=\frac{\frac{dy}{dx}}{\frac{dz}{dx}}=\frac{3\sin^2x\cos x}{3\cos^2x(-\sin x)}=-\tan x$.

Which is the required solution.

7. **(a)** $\frac{-e^{1/x}}{x^2}$

Explanation: Here $y=e^{\frac{1}{x}}$

Taking log both sides, we get

$\log_e y=\frac{1}{x}$ (Since $\log_a b^c = c\log_a b$)

Differentiating with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx}=-\frac{1}{x^2}$ or $\frac{dy}{dx}=-\frac{1}{x^2}\times y$

Therefore, $\frac{dy}{dx}=-\frac{1}{x^2}\times e^{\frac{1}{x}}$

8.

(c) $2\sqrt{1-x^2}$

Explanation: $y=x\sqrt{1-x^2}+\sin^{-1}(x)$

$\Rightarrow \frac{dy}{dx}=x\left\{\frac{1}{2\sqrt{1-x^2}}(-2x)\right\}+\sqrt{1-x^2}+\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx}=\frac{-x^2}{\sqrt{1-x^2}}+\sqrt{1-x^2}+\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{-x^2+1-x^2+1}{\sqrt{1-x^2}}$
$\Rightarrow \frac{dy}{dx} = \frac{-2x^2+2}{\sqrt{1-x^2}}$
$\Rightarrow \frac{dy}{dx} = 2\sqrt{1-x^2}$

9.

(c) $\frac{y^2}{x(1-y\log x)}$
Explanation: Given:
$y = x^{x^{x+\infty}}$
We can write it as
$\Rightarrow$ y = x^y
Taking log of both sides we obtain
log y = y log x
Differentiating with respect to x,we get
$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log x + y \cdot \frac{1}{x}$
$\Rightarrow \left(\frac{1}{y} - \log x\right)\frac{dy}{dx} = \frac{y}{x}$
$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y}{1-y\log x}\right)$
$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}$.
Which is the required solution.

10.

(c) $\frac{b}{a} cosec\theta$
Explanation: x = a secθ ,we get
$\therefore \frac{dx}{d\theta} = \text{asec}\,\theta \cdot \tan\theta$
$\therefore \frac{d\theta}{dx} = \frac{1}{\text{asec}\,\theta \cdot \tan\theta}$
y = b tan θ ,we get
$\therefore \frac{dy}{d\theta} = b \cdot \sec^2\theta$
$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$
$\Rightarrow \frac{dy}{dx} = b \cdot \sec^2\theta \times \frac{1}{\text{asec}\,\theta \cdot \tan\theta}$
$\Rightarrow \frac{dy}{dx} = \frac{b\sec\theta}{\text{atan}\,\theta}$
$\Rightarrow \frac{\text{dy}}{\text{dx}} = \frac{\text{b} \cdot \frac{1}{\cos\theta}}{\text{a} \cdot \frac{\sin\theta}{\cos\theta}}$
$\Rightarrow \frac{dy}{dx} = \frac{b}{a} cosec\theta$

11.

(c) a = - 1, b = -1
Explanation: f(x) is continuous at x = 1
$\Rightarrow \lim_{x\to 1} f(x) = f(1)$
$\Rightarrow \lim_{h\to 0} f(1+h) = a$
$\Rightarrow \lim_{h\to 0} \frac{(1+h)^2}{a} = a$
$\Rightarrow \frac{1}{a} = a$
$\Rightarrow$ a^2 = 1
$\Rightarrow a = \pm 1$
Consider,
$\lim_{x\to\sqrt{2}} \frac{2b^2-4b}{x^2} = a$
b^2 - 2b = ± 1
for a = 1

using formulas for quadratic equation

$b^2 - 2b - 1 = 0 \Rightarrow b = 1 \pm \sqrt{2}$

for a = -1

$b^2 - 2b = -1$

$b^2 - 2b + 1 = 0$

$(b - 1)^2 = 0$

b = 1

a = -1, b = 1

12. **(a)** $\frac{\cos x}{2y-1}$

Explanation: $\because y = (\sin x + y)^{1/2}$

$\therefore \frac{dy}{dx} = \frac{1}{2}(\sin x + y)^{-1/2} \cdot \frac{d}{dx}(\sin x + y)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{(\sin x+y)^{1/2}} \cdot \left(\cos x + \frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}\left(\cos x + \frac{dy}{dx}\right) \left[\because (\sin x + y)^{1/2} = y\right]$

$\Rightarrow \frac{dy}{dx}\left(1 - \frac{1}{2y}\right) = \frac{\cos x}{2y}$

$\therefore \frac{dy}{dx} = \frac{\cos x}{2y} \cdot \frac{2y}{2y-1} = \frac{\cos x}{2y-1}$.

Which is the required solution.

13.

(d) a^2

Explanation: Given,F(x) is continuous at x = 0.

$\Rightarrow f(x) = \lim_{x\to 0} \frac{\sin^2 ax}{x^2}$

$\Rightarrow f(x) = \lim_{x\to 0} \frac{\sin^2 ax}{x^2} \times \frac{a^2}{a^2}$

$\Rightarrow f(x) = \lim_{x\to 0} \left(\frac{\sin ax}{ax}\right)^2 \times a^2$

$\Rightarrow$ f(x) = a^2

$\therefore$ k = a^2

14.

(d) $a = \frac{1}{3}, b = \frac{8}{3}$

Explanation: Let f(x) be continuous at x = $\frac{\pi}{2}$

Then

LHL = $\lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{1-\sin^2 x}{3\cos^2 x}$

$\Rightarrow$ a = $\lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{\cos^2 x}{3\cos^2 x}$ = $\frac{1}{3}$

$\Rightarrow$ a = $\frac{1}{3}$

Again,

RHL = $\lim_{x\to\left(\frac{\pi}{2}\right)^+} \frac{b(1-\sin x)}{(\pi-2x)^2}$

$\Rightarrow$ a = $\lim_{h\to 0} \frac{b(1-\sin(\frac{\pi}{2}+h))}{[\pi-2(\frac{\pi}{2}+h)]^2}$

= b $\lim_{h\to 0} \frac{(1-\cos\ h)}{[\pi-\pi-2h]^2}$

= b $\lim_{h\to 0} \frac{(1-\cos\ h)}{4h^2}$

= b $\lim_{h\to 0} \frac{2\sin^2\frac{h}{2}}{\frac{h^2}{4}\times 4^2}$

$\Rightarrow$ a = $\frac{2b}{16}$

$\Rightarrow$ $\frac{1}{3}$ = $\frac{b}{8}$

$\Rightarrow$ b = $\frac{8}{3}$

15. **(a)** $\frac{-3}{\sqrt{1-x^2}}$

Explanation: Given that y = $\cos^{-1}(4x^3 - 3x)$

Let x = cos θ

$\Rightarrow \theta = \cos^{-1} x$

Then, $y = \cos^{-1}\left(4\cos^3\theta - 3\cos\theta\right)$

Using $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, we obtain

$y = \cos^{-1}(\cos 3\theta) = 3\theta = 3\cos^{-1} x$

Differentiating with respect to x, we obtain

$\frac{dy}{dx} = \frac{-3}{\sqrt{1-x^2}}$

16. **(a)** $\frac{y(y-x\log y)}{x(x-y\log x)}$

Explanation: given that $x^y = y^y$

Taking log both sides, we obtain

y $\log_e x$ = c $\log_e y$

(Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we obtain

$\frac{y}{x} + \log_e x\frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log_e y$

$\Rightarrow \frac{x-y\log_e x}{y}\frac{dy}{dx} = \frac{y-x\log_e y}{x}$

Hence $\frac{dy}{dx} = \frac{y(y-x\log_e y)}{x(x-y\log_e x)}$

17.

(d) $\frac{-2}{(1+x^2)}$

Explanation: Given that $y = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

$\Rightarrow \sec y = \frac{x^2+1}{x^2-1}$

Since $\tan^2 x = \sec^2 x - 1$, thus

$\tan^2 y = \left(\frac{x^2+1}{x^2-1}\right)^2 - 1 = \frac{4x^2}{\left(x^2-1\right)^2}$

Hence, $\tan y = -\frac{2x}{1-x^2}$ or $y = \tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

Hence, $y = \tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, we obtain

$y = \tan^{-1}(-\tan 2\theta)$

Uisng -tan x = tan(-x), we obtain

y = $\tan^{-1}(\tan(-2\theta)) = -2\theta = -2\tan^{-1} x$

Differentiating with respect to x, we obtain

$\frac{dy}{dx} = \frac{-2}{1+x^2}$

18. **(a)** $\frac{1}{3}$

Explanation: $f(0) = \lim\limits_{x\to 0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$

$f(0) = \lim\limits_{x\to 0}\frac{2-\frac{\sin^{-1}x}{x}}{2+\frac{\tan^{-1}x}{x}}$

$f(0) = \frac{2-1}{2+1} = \frac{1}{3}$

19.

(d) $a = \log_e\left(\frac{2}{3}\right), b = \frac{2}{3}, c = 1$

Explanation: $f(0) = \lim\limits_{x\to 0}(1+ax)^{\frac{1}{x}}$

$b = \lim\limits_{x\to 0}(1+ax)^{\frac{1}{ax}}$

$b = e^a$

$a = \log_e b$

$f(0) = \lim\limits_{x \to 0^+} \frac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1}$

Here, c = 1

x + 1 = y

$x \to 0 \Rightarrow y \to 1$

$f(0) = \lim\limits_{y \to 1} \frac{y^{1/3}-1}{y^{1/2}-1}$

$b = \lim\limits_{y \to 1} \frac{\frac{y^{1/3}-1}{y-1}}{\frac{y^{1/2}-1}{y-1}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$

a = log b = log $\frac{2}{3}$

20.

(c) $\frac{\log x}{(1+\log x)^2}$

Explanation: $x^y = e^{x-y}$

Taking log on both sides,

$\log x^y = \log e^{x-y}$

y log x = x - y

y log x + y = x

$y = \frac{x}{\log x+1}$

Differentiate with respect to x,

$\frac{dy}{dx} = \frac{(\log x+1)-x \times \frac{1}{x}}{(\log x+1)^2}$

$\frac{dy}{dx} = \frac{(\log x+1)-1}{(\log x+1)^2}$

$\frac{dy}{dx} = \frac{\log x}{(\log x+1)^2}$

21.

(d) $\frac{-b}{a}$

Explanation: $x = a.\cos^2\theta$,we get

$\therefore \frac{dx}{d\theta} = -2\,a\cos\theta \cdot \sin\theta$

$\Rightarrow \frac{d\theta}{dx} = \frac{-1}{2a \cdot \cos\theta \cdot \sin\theta}$

$y = b.\sin^2\theta$,we get

$\therefore \frac{dy}{d\theta} = 2b\sin\theta \cdot \cos\theta$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = 2\mathrm{b}\sin\theta \cdot \cos\theta \times \frac{-1}{2\,a\cos\theta \cdot \sin\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{-b}{a}$

22. **(a)** continuous everywhere but not differentiable at x = 0

Explanation: Given that $f(x) = e^{-|x|}$

$\Rightarrow \mathrm{f(x)} = \begin{Bmatrix} \mathrm{e^x, x < 0} \\ 1, \mathrm{x} = 0 \\ \mathrm{e^{-x}, x > 0} \end{Bmatrix}$

Checking continuity and differentiability at x = 0,

LHL:

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \mathrm{lim} \mathrm{e}^{-h}_{h \to 0} = 1$

RHL:

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} e^{-h} = 1$

And f(0)= 1

$\therefore$ LHL = RHL = f(0)

f(x) is continuous at x = 0.

LHD at x =0,

$\lim\limits_{x \to 0^-} \frac{f(x)-f(0)}{x-0} = \lim\limits_{h \to 0} \frac{f(0-h)-f(0)}{0-h-0}$

$= \lim\limits_{h \to 0} \frac{e^{-h}-(0)}{-h} = \infty$

∴LHD does not exist, so f(x) is not differentiable at x = 0

23. **(a)** $\frac{1}{2}$

Explanation: Given that y = $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

Using 1 - cos x = 2 $\sin^2 \frac{x}{2}$ and 1 + cos x = 2 $\cos^2 \frac{x}{2}$, we obtain

$y = \tan^{-1}\sqrt{\frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}} = \tan^{-1}\tan\left(\frac{x}{2}\right) = \frac{x}{2}$

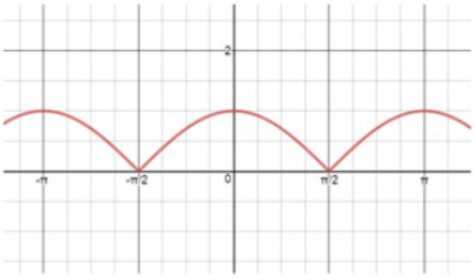
Differentiating with respect to x, we

$\frac{dy}{dx} = \frac{1}{2}$

24. **(a)** everywhere continuous but not differentiable at (2n + 1) $\frac{\pi}{2}$, n ∈ Z

Explanation:

Given that f(x) = |cos x|



From the graph it is evident that it is everywhere continuous but not differentiable at (2n + 1) $\frac{\pi}{2}$, n ∈ Z

25.

(c) n^2y

Explanation: $y^{1/n} + y^{-1/n} = 2x$

Differentiating both sides we get

$\frac{y_1}{n}\left(y^{\frac{1}{n}-1} - y^{\frac{-1}{n}-1}\right) = 2$

$\Rightarrow y_1\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) = 2ny$

Again differentiating both sides we get

$y_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + \frac{y_1}{n}\left(y^{\frac{1}{n}^{-1}} + y^{\frac{-1}{n}-1}\right) = 2ny_1$

$\Rightarrow ny_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + \frac{y_1^2}{y}\left(y^{\frac{1}{n}} + y^{\frac{-1}{n}}\right) = 2n^2y_1$

$\Rightarrow nyy_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + 2xy_1^2 = 2n^2yy_1$

$\Rightarrow nyy_2\frac{2ny}{y_1} + 2xy_1^2 = 2n^2yy_1$

$\Rightarrow \frac{n^2y^2y_2}{y_1^2} + xy_1 = n^2y$

$\Rightarrow y_2\frac{\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right)^2}{4} + xy_1 = n^2y$

$\Rightarrow y_2\frac{\left(y^{\frac{1}{n}} + y^{\frac{-1}{n}}\right)^2 - 4}{4} + xy_1 = n^2y$

$\Rightarrow y_2\frac{4x^2-4}{4} + xy_1 = n^2y$

$\Rightarrow (x^2 - 1)\, y_2 + xy_1 = n^2y$

26.

(d) 2

Explanation: Since the given function is continuous,

$\therefore$ k = $\lim_{x \to 0} \frac{Sinx}{x} + Cosx$

$\Rightarrow$ k = 1 + 1 = 2

27.

(c) m^2y

Explanation: $y = ae^{mx} + be^{-mx} \Rightarrow y_1 \ = ame^{mx} + (-m)be^{-mx} \Rightarrow y_2 = am^2e^{mx} + (m^2)be^{-mx}$

$\Rightarrow y_2 = m^2(ae^{mx} + be^{-mx}) \Rightarrow y_2 = m^2y$

28.

(d) $\frac{1}{|x|}$

Explanation: $\frac{d}{d|x|}(\log|x|) = \frac{1}{|x|}$

29.

(d) continuous at x = 0

Explanation: Given f(x) = $\sin^{-1}$(cos x),

Checking differentiability and continuity,

LHL at x = 0,

$\lim_{\mathbf{x} \to 0^-} \mathbf{f(x)} = \lim_{\mathbf{h} \to 0} \mathbf{f}(0 - \mathbf{h}) = \lim_{\mathbf{h} \to 0} \sin^{-1}(\cos(0 - \mathbf{h})) = \lim_{\mathbf{h} \to 0} \sin^{-1}(\cos(-\mathbf{h})) = \sin^{-1} 1 = \frac{\pi}{2}$

RHL at x =0,

$\lim_{\mathbf{x} \to 0^+} \mathbf{f(x)} = \lim_{\mathbf{h} \to 0} \mathbf{f}(0 + \mathbf{h}) = \lim_{\mathbf{h} \to 0} \sin^{-1}(\cos(0 + \mathbf{h})) = \lim_{\mathbf{h} \to 0} \sin^{-1}(\cos(\mathbf{h})) = \sin^{-1} 1 = \frac{\pi}{2}$

And $f(0) = \frac{\pi}{2}$

Hence, f(x) is continuous at x =0.

LHD at x = 0,

$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{h \to 0} \frac{f(0-h) - f(0)}{0 - h - 0}$

$= \lim_{h \to 0} \frac{\sin^{-1}(\cos(0-h)) - \left(\frac{\pi}{2}\right)}{-h} = 1$

RHD at x =0,

$\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{h \to 0} \frac{f(0+h) - f(0)}{0 + h - 0}$

$= \lim_{h \to 0} \frac{\sin^{-1}(\cos(0+h)) - \left(\frac{\pi}{2}\right)}{h} = -1$

$\because$ LHD $\neq$ RHD

$\therefore$ f(x) is not differentiable at x =0.

30.

(c) $\frac{1}{2\sqrt{x}(1+x)}$

Explanation: Given that $y = \tan^{-1} \frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}$

Let $\sqrt{a} = \tan A$ and $\sqrt{\mathrm{x}} = \tan \mathrm{B}$, then $A = \tan^{-1} \sqrt{a}$ and $B = \tan^{-1} \sqrt{x}$

Hence, $y = \tan^{-1} \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Using tan (A + B) $= \frac{\tan A + \tan B}{1 - \tan A \tan B}$, we obtain

y = $\tan^{-1}$ tan (A + B) = A + B

$= \tan^{-1} \sqrt{a} + \tan^{-1} \sqrt{x}$

Differentiating with respect to x, we obtain

$\frac{dy}{dx} = 0 + \frac{1}{1+(\sqrt{x})^2} \times \frac{1}{2\sqrt{\mathrm{x}}} = \frac{1}{2\sqrt{\mathrm{x}}(1+\mathrm{x})}$

31.

(d) |sec θ|

Explanation: $x = \mathrm{a}\cos^3 \theta \Rightarrow \cos^2 \theta = \left(\frac{x}{a}\right)^{\frac{2}{3}}$

$y = \mathrm{a}\sin^3 \theta \Rightarrow \sin^2 \theta = \left(\frac{y}{a}\right)^{\frac{2}{3}}$

We know that

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$

$x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{x^3}$

Differentiating with respect to x,

$\frac{2}{3}x^{\frac{-1}{3}}+\frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx}=0$

$\frac{dy}{dx}=\left(\frac{y}{x}\right)^{\frac{1}{3}}$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\sqrt{1+\left[\left(\frac{y}{x}\right)^{\frac{1}{3}}\right]^2}$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\sqrt{1+\left[\left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{1}{3}}\right]^2}$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2}=|\sec\theta|$.

Which is the required solution.

32.

(b) $n=\frac{m\pi}{2}$

Explanation: We have, $f(x)=\begin{cases} mx+1, & \text{if } x\leq\frac{\pi}{2} \\ \sin x+n, & \text{if } x>\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$

$\therefore LHL=\lim\limits_{x\to\frac{\pi}{2}}(mx+1)=\lim\limits_{h\to0}\left[m\left(\frac{\pi}{2}-h\right)+1\right]=\frac{m\pi}{2}+1$

and $RHL=\lim\limits_{x\to\frac{\pi'}{2}}(\sin x+n)=\lim\limits_{h\to\infty}\left[\sin\left(\frac{\pi}{2}+h\right)+n\right]$

$=\lim\limits_{n\to0}\cos h+n=1+n$

Since the function is continuous, we have

$LHL=RHL$

$\Rightarrow m\cdot\frac{\pi}{2}+1=n+1$

$\therefore\ n=m\cdot\frac{\pi}{2}$

33.

(b) 1.5

Explanation: [x] is always continuous at non-integer value of x. Hence, f(x) = [x] will be continuous at x = 1.5.

34.

(c) f(x) and g(x) both are continuous at x = 0

Explanation: Given f(x) = |x| and g(x) = $|x^3|$,

$f(x)=\begin{cases}-x, & x\leq0 \\ x, & x>0\end{cases}$

Checking differentiability and continuity,

LHL at x =0,

$\lim\limits_{x\to0^-}f(x)=\lim\limits_{h\to0}f(0-h)=\lim\limits_{h\to0}-(0-h)=0$

RHL at x =0,

$\lim\limits_{x\to0^+}f(x)=\lim\limits_{h\to0}f(0+h)=\lim\limits_{h\to0}(0+h)=0$

And f(0)=0

Hence, f(x) is continuous at x =0.

LHD at x =0,

$\lim\limits_{x\to0^-}\frac{f(x)-f(0)}{x-0}=\lim\limits_{h\to0}\frac{f(0-h)-f(0)}{0-h-0}$

$=\lim\limits_{h\to0}\frac{(0-h)-(0)}{-h}=-1$

RHD at x =0,

$\lim\limits_{x\to0^+}\frac{f(x)-f(0)}{x-0}=\lim\limits_{h\to0}\frac{f(0+h)-f(0)}{0+h-0}$

$=\lim_{h\to0}\frac{(0+h)-(0)}{h}=1$

$\because$ LHD $\neq$ RHD

$\therefore$ f(x) is not differentiable at x =0.

$$g(x)=\begin{cases}-x^3, & x\le 0\\ x^3, & x>0\end{cases}$$
Checking differentiability and continuity,
LHL at x =0,
$$\lim_{x\to 0^-} g(x)=\lim_{h\to 0} g(0-h)=\lim_{h\to 0} -(0-h)^3=0$$
RHL at x =0,
$$\lim_{x\to 0^+} g(x)=\lim_{h\to 0} g(0+h)=\lim_{h\to 0}(0+h)^3=0$$
And g(0)=0
Hence, g(x) is continuous at x =0.
LHD at x =0,
$$\lim_{x\to 0^-}\frac{g(x)-g(0)}{x-0}=\lim_{h\to 0}\frac{g(0-h)-g(0)}{0-h-0}$$
$$=\lim_{h\to 0}\frac{(0-h)^3-(0)}{-h}=0$$
RHD at x =0,
$$\lim_{x\to 0^+}\frac{g(x)-g(0)}{x-0}=\lim_{h\to 0}\frac{g(0+h)-g(0)}{0+h-0}$$
$$=\lim_{h\to 0}\frac{(0+h)^3-(0)}{h}=0$$
∵ LHD = RHD
∴ g(x) is differentiable at x =0.

35. **(a)** $-(n - 1)^2 y$

Explanation: $y = x^{n-1} \log x$

$\frac{dy}{dx} = (n - 1)^{xn-2} \log x + \frac{1}{x} x^{n-1}$

$= (n - 1)x^{n-2}\log x + x^{n-2}$

$= x^{n-2}[(n - 1)\log x + 1]$

xy1 = xn-1[(n - 1)log x + 1]

$= (n - 1)y + x^{n-1}$

$(3 - 2n)xy_1 = (3 - 2n) [(n - 1)y + x^{n-1}]$

$= (3n - 3 - 2n^2 + 2n)y + 3x^{n-1} - 2nx^{n-1}]$ ...(1)

$\frac{d^2y}{dx^2} = (n - 1)(n - 2)x^{n-3} \log x + \frac{1}{x} (n - 1)x^{n-2} + (n - 2)x^{n-3}$

$= (n - 1)(n - 2)x^{n-3} \log x + (n - 1)x^{n-3} + (n - 2)x^{n-3}$

$= x^{n-3}[(n - 1)(n - 2)\log x + (n - 1) + (n - 2)]$

$x^2y_2 = x^{n-1}[(n - 1)(n - 2)\log x + (2n - 3)]$

$= (n^2 - 3n + 2)y + 2nx^{n-1} - 3x^{n-1}$ ...(2)

$x^2y_2 + (3 - 2n) xy_1$

$= (n^2 - 3n + 2)y + 2nx^{n-1} - 3x^{n-1} + (3n - 3 - 2n^2 + 2n)y + 3x^{n-1} - 2nx^{n-1}$

$= (-n^2 + 2n - 1)y$

$= -(n - 1)^2y$

36. **(a)** $(1 + \sin 2x) y_1$

Explanation: $y = e^{\tan x}$

$y_1 = \sec^2 x\, e^{\tan x}$

$\Rightarrow \cos^2 xy_1 = e^{\tan x}$

Again differentiating w.r.t. x we get

$\cos^2 (x) .y_2 - 2 \cos x \sin xy_1 = \sec^2 xe^{\tan x}$

$\Rightarrow \cos^2 (x).y_2 = y_1 \sin 2x + y_1.$

37.

(c) -1/2

Explanation: $y=\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$y=\tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right)$

$y=\tan^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right)$

$y=\tan^{-1}\left(\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)$

$y=\tan^{-1}\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)$

$y=\frac{\pi}{4}-\frac{x}{2}$

$\frac{dy}{dx}=-\frac{1}{2}$

38.

(d) $\frac{\pi}{4}+\frac{1}{2}$

Explanation: $f'(x)=\frac{d}{dx}(x\tan^{-1}x)=\frac{x}{1+x^2}+\tan^{-1}x\ \Rightarrow f'(1)=\frac{1}{1+1}+\tan^{-1}1=\frac{1}{2}+\frac{\pi}{4}$

39.

(b) 0

Explanation: $y=\tan^{-1}\left\{\frac{\log_e(e/x^2)}{\log_e(ex^2)}\right\}+\tan^{-1}\left(\frac{3+2\log_e x}{1-6\log_e x}\right)$

$=\tan^{-1}\left\{\frac{1-2\log_e \times}{1+2\log_e \times}\right\}+\tan^{-1}\left\{\frac{3+2\log_e \times}{1-6\log_e \times}\right\}$

$=\tan^{-1}$ - $\tan^{-1}$ (2 $\log_e$ x) + $\tan^{-1}$ (3) + $\tan^{-1}$ (2 $\log_e$ x)

$=\tan^{-1}$ 1 + $\tan^{-1}$ (3)

$\frac{d^2y}{dx^2}=0$

40. **(a)** $\frac{1}{(1+x^2)}$

Explanation: Given that $y=\cot^{-1}\left(\frac{1-x}{1+x}\right)$

Let x = $\tan\theta\Rightarrow\theta=\tan^{-1}x$ and using $\cot^{-1}x=\frac{\pi}{2}-\tan^{-1}x$

Hence, $y=\frac{\pi}{2}-\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4}-x\right)=\frac{1-\tan x}{1+\tan x}$, we obtain

$y=\frac{\pi}{2}-\tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)=\frac{\pi}{2}-\left(\frac{\pi}{4}-\theta\right)=\frac{\pi}{4}+\theta=\frac{\pi}{4}+\tan^{-1}x$

Differentiating with respect to x, we obtain

$\frac{dy}{dx}=\frac{1}{1+x^2}$

41.

(d) not continuous at x = –2

Explanation: Given that $f(x)=\begin{cases}\frac{|x+2|}{\tan^{-1}(x+2)}, x\neq-2\\ 2, x=-2\end{cases}$

$\Rightarrow f(x)=\begin{cases}\frac{-(x+2)}{\tan^{-1}(x+2)}, x<-2\\ \frac{(x+2)}{\tan^{-1}(x+2)}, x>-2\\ 2 \quad , x=-2\end{cases}$

Checking the continuity at x = -2,

LHL:

$\lim\limits_{x\to-2^-}f(x)=\lim\limits_{h\to0}f(-2-h)=\lim\limits_{h\to0}\frac{-(-2-h+2)}{\tan^{-1}(-2-h+2)}=\frac{h}{\tan^{-1}(-h)}=-1$

RHL:

$\lim\limits_{x\to-2^+}f(x)=\lim\limits_{h\to0}f(-2+h)=\lim\limits_{h\to0}\frac{(-2+h+2)}{\tan^{-1}(-2+h+2)}=\frac{h}{\tan^{-1}(h)}=1$

And f(-2) = 2

$\because$ LHL $\neq$ RHL $\neq$ f(-2)

Hence, f(x) is not continuous at x =-2.

42.

(d) $\frac{-4x}{1-x^4}$

Explanation: We have, $y = \log\left(\frac{1-x^2}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\frac{1-x^2}{1+x^2}} \times \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{1+x^2}{1-x^2} \times \frac{\left[(1+x^2)(-2x)-(1-x^2)(+2x)\right]}{(1+x^2)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(1+x^2)}{(1-x^2)} \times \frac{\left[-2x-2x^3-2x+2x^3\right]}{(1+x^2)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\frac{1-x^2}{1+x^2}} \cdot \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$

$= \frac{-2x\left[1+x^2+1-x^2\right]}{(1-x^2)\cdot(1+x^2)} = \frac{-4x}{1-x^4}$

$\Rightarrow \frac{dy}{dx} = \frac{1\times-4x}{(1-x^2)(1+x^2)}$

$\therefore \frac{dy}{dx} = \frac{-4x}{1-x^4}$

43.

(b) $\frac{\sin^2(a+y)}{\sin a}$

Explanation: $x\sin(a+y) = \sin y \ \Rightarrow x = \frac{\sin y}{\sin(a+y)}$

$\Rightarrow \frac{dx}{dy} = \frac{\sin(a+y)\cos y-\sin y\cos(a+y)}{\sin^2(a+y)}$

$= \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\text{sina}}{\sin^2(a+y)}$

$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

44.

(d) continuous as well as differentiable for all x $\in$ R

Explanation: Given that $f(x) = \frac{\sin(\pi[x-\pi])}{4+[x]^2}$

$\therefore$We know that π (x - π) = nπ and sin nπ = 0

So, $4+[x]^2 \neq 0$

$\Rightarrow$ f(x) = 0 for all x

Thus f(x) is a constant function and continuous as well as differentiable for all x $\in$ R

45.

(c) 0

Explanation: We have, $f(x) = x^2\sin\left(\frac{1}{x}\right)$, where x $\neq$ 0

Since the function is continuous at x = 0, we have

f(0) = $\lim\limits_{x\to 0} x^2\sin\left(\frac{1}{x}\right)$ ...(i)

Now, $-1 \leq sin\frac{1}{x} \leq 1$

$\Rightarrow -x^2 \leq x^2 sin\frac{1}{x} \leq x^2$

$\Rightarrow \lim\limits_{x\to 0}(-x^2) \leq \lim\limits_{x\to 0}(x^2 sin\frac{1}{x}) \leq \lim\limits_{x\to 0}(x^2)$

$\Rightarrow 0 \leq \lim\limits_{x\to 0}(x^2 sin\frac{1}{x}) \leq 0$

Therefore by squeeze principle, we have

f(0) = $\lim\limits_{x\to 0}(x^2 sin\frac{1}{x}) = 0$

Hence, value of the function f at x=0 so that it is continuous at x=0 is 0.

46.

(d) 1

Explanation: Here, given

$\Rightarrow f(x)=\frac{1-\cos 4x}{8x^2}$ is continuous at x = 0

$\Rightarrow f(x)=\lim_{x\to 0}\frac{2\sin^2 2x}{8x^2}$

$\Rightarrow f(x)=\lim_{x\to 0}\frac{2\sin^2 2x}{2\times 4x^2}$

$\Rightarrow f(x)=\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)^2$

$\Rightarrow$ f(x) = 1

$\therefore$ k = 1

47.

(c) continuous on [–1, 1] and differentiable on (–1, 0) ∪ (0, 1)

Explanation: Given that $f(x)=\sqrt{1-\sqrt{1-x^2}}$

So, the function will be defined for those values of x for which

$1-x^2 \geq 0$

$\Rightarrow x^2 \leq 1$

$\Rightarrow |x| \leq 1$

$\Rightarrow -1 \leq x \leq 1$

$\therefore$ Function is continuous in [-1, 1].

Now, we will check the differentiability at x = 0

LHD at x =0,

$\lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0}=\lim_{h\to 0}\frac{f(0-h)-f(0)}{0-h-0}$

$=\lim_{h\to 0}\frac{\sqrt{1-\sqrt{1-(0-h)^2}}-(0)}{-h}=-\infty$

$\because$ LHD does not exist, so f(x) is not differentiable at x = 0

$\therefore$ f(x) is not differentiable at x =0.

48.

(b) f(x) is continuous for all x in its domain but not differentiable at x = ± 1

Explanation: Here,the given function is f(x) = |log|x|| where

$$|x|=\begin{cases}-x, -\infty<x<-1\\ -x, -1<x<0\\ x, 0<x<1\\ x, 1<x<\infty\end{cases}$$

$$\log|x|=\begin{cases}\log(-x), -\infty<x<-1\\ \log(-x), -1<x<0\\ \log x, 0<x<1\\ \log x, 1<x<\infty\end{cases}$$

$$|\log|x||=\begin{cases}\log(-x), -\infty<x<-1\\ -\log(-x), -1<x<0\\ -\log x, 0<x<1\\ \log x, 1<x<\infty\end{cases}$$

We can see that function is continuous for all x. Now, checking the points of non differentiability.

Now,L.H.D at x =1,we get

$\lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1}=\lim_{h\to 0}\frac{f(1-h)-f(1)}{1-h-1}$

$=\lim_{h\to 0}\frac{\log(1-h)-\log 1}{-h}=-1$

RHD at x =1,

$\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}=\lim_{h\to 0}\frac{f(1+h)-f(1)}{1+h-1}$

$=\lim_{h\to 0}\frac{\log(1+h)-\log 1}{h}=1$

$\because \text{L.H.D} \neq \text{R.H.D}$

Thus, function is not differentiable at x =1.

L.H.D at x =-1,

$\lim\limits_{x \to -1^-} \frac{f(x)-f(-1)}{x-(-1)} = \lim\limits_{h \to 0} \frac{f(-1-h)-f(-1)}{-1-h-(-1)}$

$= \lim\limits_{h \to 0} \frac{\log(-1-h)-\log(-1)}{-h} = -1$

R.H.D at x =-1,

$\lim\limits_{x \to -1^+} \frac{f(x)-f(-1)}{x-1} = \lim\limits_{h \to 0} \frac{f(-1+h)-f(-1)}{(-1)+h-(-1)}$

$= \lim\limits_{h \to 0} \frac{\log(-1+h)-\log(-1)}{h} = 1$

$\because \mathrm{L.H.D} \neq \mathrm{R.H.D}$

So, function is not differentiable at x =-1.

At x =0 function is not defined.

∴ Function is not differential at x = 0 and ±1.

49.

(b) is equal to 0

Explanation: $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} e^{\frac{1}{x}} = 0, \quad \lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} x = 0 \therefore \lim\limits_{x \to 0} f(x) = 0$

50.

(b) -1 < x < 1

Explanation: $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$, is valid only if , $1 - \ x^2 > \ 0,$ i.e. if $x^2 < \ 1 \; i.e. \; if \, |x| < \ 1$

51.

(d) $\frac{2}{\sqrt{1+x^2}}$

Explanation: Given that y $= \log_e\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$

Differentiating with respect to x, we obtain

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}+x} \times \frac{(\sqrt{1+x^2}-x)\times\left(\frac{1}{2\sqrt{1+x^2}}\times 2x+1\right)-(\sqrt{1+x^2}+x)\times\left(\frac{1}{2\sqrt{1+x^2}}\times 2x-1\right)}{(\sqrt{1+x^2}-x)^2}$

Hence, $\frac{dy}{dx} = \frac{2}{\sqrt{1+x^2}}$

52.

(b) -1

Explanation: $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x)$

$\lim\limits_{x \to 1} 5x - 4 = \lim\limits_{x \to 1} 4x^2 + 36x$

5 - 4 = 4 + 3b

1 = 4 + 3b

b = -1

53.

(b) 3 $(xy_2 + y_1) y_2$

Explanation: $y = \frac{ax+b}{x^2+c}$

⇒ $y(x^2 + c) = ax + b$

Differentiating both sides w.r.t. x we get

$y_1 (x^2 + c) + 2xy = a$

Again differentiating w.r.t to x we get

$y_2 (x^2 +c) + 2xy_1 + 2y + 2xy_1 = 0$

⇒ $y_2 (x^2 + c) = -(4xy_1 + 2y)$ ...(i)

Again differentiating w.r.t to x we get

$y_3 (x^2 + c) + 2xy_2 + 4xy_2 + 4y_1 + 2y_1 = 0$

⇒ $y_3 (x^2 + c) = -(6xy_2 + 6y_1)$ ...(ii)

$\frac{y_2}{y_3} = \frac{2xy_1+y}{3xy_2+3y_1}$

$\Rightarrow y_3 (2xy_1 + y) = 3y_2 (xy_2 + y_1)$

54.

(c) is discontinuous

Explanation: Given $f(x) = x^2 + \dfrac{x^2}{1+x^2} + \dfrac{x^2}{(1+x^2)^2} + \ldots\ldots + \dfrac{x^2}{(1+x^2)^n} + \cdots$

For x =0, $x^2 = 0$

$\Rightarrow$ f(x) = 0

For x $\neq$ 0,

$x^2 + 1 > x^2$

$\Rightarrow 0 < \frac{x^2}{1+x^2} < 1$

$\therefore \lim\limits_{x\to0} f(x) = \lim\limits_{x\to0}\ x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \cdots\ldots + \frac{x^2}{(1+x^2)^n} + \cdots$

$\therefore \lim\limits_{x\to0} f(x) = \lim\limits_{x\to0} x^2 \left(1 + \frac{1}{1+x^2} + \frac{1}{(1+x^2)^2} + \cdots\ldots + \frac{1}{(1+x^2)^n} + \cdots\right)$

$\therefore \lim\limits_{x\to0} f(x) = \lim\limits_{x\to0} \left\{x^2 \left(\frac{1}{1-\frac{1}{1+x^2}}\right)\right\}$

$\because$ Sum of infinite series where $r = \frac{1}{1+x^2}$

$\Rightarrow \lim\limits_{x\to0} f(x) = \lim\limits_{x\to0} x^2 \left(\frac{1+x^2}{x^2}\right)$

$\Rightarrow \lim\limits_{x\to0} f(x) = \lim\limits_{x\to0} x^2 + 1$

$\Rightarrow \lim\limits_{x\to0} f(x) = 1 \neq f(0)$

So, f(x) is discontinuous at x = 0

55. **(a)** neither differentiable nor continuous at x = 3

Explanation: Given that f(x) = |3 – x| + (3 + x), where (x) denotes the least integer greater than or equal to x.

$f(x) = \left\{\begin{array}{l} 3 - x + 3 + 3, 2 < x < 3 \\ x - 3 + 3 + 4, 3 < x < 4 \end{array}\right\}$

$\Rightarrow \mathrm{f(x)} = \left\{\begin{array}{l} 9 - \mathrm{x}, 2 < \mathrm{x} < 3 \\ \mathrm{x} + 4, 3 < \mathrm{x} < 4 \end{array}\right\}$

Checking continuity at x =3,

Here, LHL at x = 3

$\lim\limits_{x\to3^-} 9 - x = 6$

RHL at x = 3

$\lim\limits_{x\to3^+} x + 4 = 7$

$\because$ LHL $\neq$ RHL

$\therefore$ f(x) is neither continuous nor differentiable at x = 3.

56.

(b) $(\tan x)^{\cot x}$. cosec^2 x (1 - log tan x)

Explanation: Given that y = $(\tan x)^{\cot x}$

Taking log both sides, we obtain

$\log_e y = \cot x \times \log_e \tan x$ (Since $\log_a b^c = c\log_a b$)

Differentiating with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx} = \cot x \times \frac{1}{\tan x} \times \sec^2 x - \log_e$ tanx $\times$ cosec^2x = cosec^2x (1 - $\log_e$ tanx)

Therefore, $\frac{dy}{dx} =$ cosec^2x (1 - $\log_e$ tan $\times$ y) = cosec^2x (1 - $\log_e$ tanx) $(\tan x)^{\cot x}$

57. **(a)** $\frac{3}{\sqrt{1-x^2}}$

Explanation: Given that y = $\sin^{-1}(3x - 4x^3)$

Let x = sin θ

$\Rightarrow \theta = \sin^{-1} x$

Then, $y=\sin^{-1}\left(3\sin\theta-4\sin^3\theta\right)$
Using $\sin 3\theta=3\sin\theta-4\sin^3\theta$, we get
$y=\sin^{-1}(\sin 3\theta)=3\theta=3\sin^{-1}x$
Differentiating with respect to x, we obtain
$\frac{dy}{dx}=\frac{3}{\sqrt{1-x^2}}$

58.

(c) $\tan\theta$
Explanation: $x=a(\cos\theta+\theta\sin\theta)$,we get
$\therefore\frac{dx}{d\theta}=a(-\sin\theta+\sin\theta+\theta\cos\theta)$
$\Rightarrow\frac{d\theta}{dx}=\frac{1}{a\theta\cos\theta}$
$y=a(\sin\theta-\theta\cos\theta)$,we get
$\therefore\frac{\mathrm{dy}}{\mathrm{d}\theta}=a(\cos\theta-(\cos\theta+\theta(-\sin\theta))$
$\Rightarrow\frac{dy}{d\theta}=\mathrm{a}\cos\theta-\mathrm{a}\cos\theta+\theta\mathrm{a}\sin\theta$
$\Rightarrow\frac{\mathrm{dy}}{\mathrm{d}\theta}=\mathrm{a}\theta\sin\theta$
$\Rightarrow\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{\mathrm{dy}}{\mathrm{d}\theta}\times\frac{\mathrm{d}\theta}{\mathrm{dx}}$
$\Rightarrow\frac{dy}{dx}=a\theta\sin\theta\times\frac{1}{a\theta\cos\theta}$
$\Rightarrow\frac{dy}{dx}=\tan\theta$

59.

(c) $\frac{1}{2\left(1+x^2\right)}$
Explanation: Put x = tan θ ,we get
$\Rightarrow y=\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$
$\Rightarrow y=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$
$\Rightarrow y=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$
$\Rightarrow \mathrm{y}=\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}\right)$
$\Rightarrow y=\tan^{-1}\left(\tan\frac{\theta}{2}\right)$
$\Rightarrow y=\frac{\theta}{2}$
$\theta=\tan^{-1}x$,we get
$\Rightarrow y=\frac{\tan^{-1}x}{2}$
$\Rightarrow\frac{dy}{dx}=\frac{1}{2\left(1+x^2\right)}$

60.

(d) f is continuous
Explanation: Given that f(x) = (x + |x|) |x|
$\Rightarrow \mathrm{f(x)}=\left\{\begin{array}{c}(\mathrm{x}-\mathrm{x})(-\mathrm{x}),\mathrm{x}<0\\0,\mathrm{x}=0\\(\mathrm{x}+\mathrm{x})(\mathrm{x}),\mathrm{x}>0\end{array}\right\}$
$\Rightarrow \mathrm{f(x)}=\left\{\begin{array}{c}0,\mathrm{x}<0\\0,\mathrm{x}=0\\2\mathrm{x}^2,\mathrm{x}>0\end{array}\right\}$
So, we can say that f(x) is continuous for all x.
Now, checking the differentiability at x =0
Given that f(x) = (x + |x|) |x|
$\Rightarrow \mathrm{f(x)}=\left\{\begin{array}{c}(\mathrm{x}-\mathrm{x})(-\mathrm{x}),\mathrm{x}<0\\0,\mathrm{x}=0\\(\mathrm{x}+\mathrm{x})(\mathrm{x}),\mathrm{x}>0\end{array}\right\}$
$\Rightarrow \mathrm{f(x)}=\left\{\begin{array}{c}0,\mathrm{x}<0\\0,\mathrm{x}=0\\2\mathrm{x}^2,\mathrm{x}>0\end{array}\right\}$

So, we can say that f(x) is continuous for all x.
Now, checking the differentiability at x =0
LHD at x =0,
$\lim\limits_{x \to 0^-} \frac{f(x)-f(0)}{x-0} = \lim\limits_{h \to 0} \frac{f(0-h)-f(0)}{0-h-0}$
$= \lim\limits_{h \to 0} \frac{0-(0)}{-h} = 0$
RHD at x =0,
$\lim\limits_{x \to 0^+} \frac{f(x)-f(0)}{x-0} = \lim\limits_{h \to 0} \frac{f(0+h)-f(0)}{0+h-0}$
$= \lim\limits_{h \to 0} \frac{2h^2-(0)}{h} = \lim\limits_{h \to 0} \frac{2h}{1} = 0$
$\therefore$ LHD = RHD
So, f(x) is differentiable for all x.
LHD at x =0,
$\lim\limits_{x \to 0^-} \frac{f(x)-f(0)}{x-0} = \lim\limits_{h \to 0} \frac{f(0-h)-f(0)}{0-h-0}$
$= \lim\limits_{h \to 0} \frac{0-(0)}{-h} = 0$
RHD at x =0,
$\lim\limits_{x \to 0^+} \frac{f(x)-f(0)}{x-0} = \lim\limits_{h \to 0} \frac{f(0+h)-f(0)}{0+h-0}$
$= \lim\limits_{h \to 0} \frac{2h^2-(0)}{h} = \lim\limits_{h \to 0} \frac{2h}{1} = 0$
$\therefore$ LHD = RHD
So, f(x) is differentiable for all x.

61.
(c) $x^{\sqrt{x}}\left\{\frac{2+\log x}{2\sqrt{x}}\right\}$
Explanation: Let y = f(x) $= x^{\sqrt{x}}$
Taking log both sides, we obtain
$\log_e$ y $= \sqrt{x} \times \log_e x - (1)$
(Since $\log_a b^c = c \log_a b$)
Differentiating (i) with respect to x, we obtain
$\frac{1}{y}\frac{dy}{dx} = \sqrt{x} \times \frac{1}{x} + \log_e x \times \frac{1}{2\sqrt{x}}$
$\Rightarrow \frac{dy}{dx} = y \times \left(\frac{2+\log_e x}{2\sqrt{x}}\right)$
$\Rightarrow \frac{dy}{dx} = f'(x)$
$= x^{\sqrt{x}}\left(\frac{2+\log_e x}{2\sqrt{x}}\right)$

62.
(b) – 1
Explanation: $\frac{d}{dx}(\tan^{-1}(\cot x))$ $=\frac{d}{dx}(\tan^{-1}(\tan(\frac{\pi}{2} - x))) = \frac{d}{dx}(\frac{\pi}{2} - x) = -1$

63.
(b) 7
Explanation: $\Rightarrow f(x) = \frac{3x+4\tan x}{x}$ is continuous at x = 0
$\Rightarrow f(x) = \lim\limits_{x \to 0} \frac{3x+4\tan x}{x}$
$\Rightarrow f(x) = \lim\limits_{x \to 0} \frac{3x}{x} + \frac{4\tan x}{x}$
$\Rightarrow f(x) = 3 + 4\lim\limits_{x \to 0} \frac{\tan x}{x}$
$\Rightarrow$ f(x) = 3 + 4
$\therefore$ k = 7

64. **(a)** $-n^2x$
Explanation: Here,
x = a cos nt - b sin nt
Differentiating w.r.t. t, we get

$\frac{dx}{dt}$ = -an sin nt - bn cos nt

Differentiating again w.r.t. t, we get

$\frac{d^2x}{dt^2}$ = -an^2 cos nt + bn^2 sin nt

= -n^2 (a cos nt - b sin nt)

= -n^{2}x

65.

(d) $-\frac{1}{2a\ t^3}$

Explanation: $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2a}{2at}=\frac{1}{t}$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{1}{t}\right)=-\frac{1}{t^2}\frac{dt}{dx}=-\frac{1}{t^2}.\frac{1}{2at}=-\frac{1}{2at^3}$

66.

(c) x^x (1 + log x)

Explanation: Given ,y = f(x) = x^x

Taking log both sides, we obtain

$\log_e$ y = x × log x(-1) (Since $\log_a b^c = c \log_a b$)

Differentiating (i) with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx}=x\times\frac{1}{x}+\log_e x\times 1$

$\Rightarrow \frac{dy}{dx}=y\times(1+\log_e x)$

$\Rightarrow \frac{dy}{dx}=f'(x)$ = x^x (1 + $\log_e$ x)

67.

(c) e^3

Explanation: $\lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^x=\lim\limits_{t\to0^+}(1+3t)^{\frac{1}{t}}=\lim\limits_{3t\to0^+}\left[(1+3t)^{\frac{1}{3t}}\right]^3=e^3$

68.

(b) $\frac{-2}{(1+x^2)}$

Explanation: Given that $y=\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

$\Rightarrow \cos y=\frac{x^2-1}{x^2+1}$ or $\sec y=\frac{x^2+1}{x^2-1}$

Since $\tan^2$ x = $\sec^2$ x - 1, therefore

$\tan^2 y=\left(\frac{x^2+1}{x^2-1}\right)^2-1$

$=\frac{4x^2}{(x^2-1)^2}$

Hence, $\tan y=-\frac{2x}{1-x^2}$ or $y=\tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let x = tan θ

$\Rightarrow \theta=\tan^{-1}x$

Hence, $y=\tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$, we get

Using -tan x = tan (-x), we obtain

$=-2\theta$

= -2$\tan^{-1}$ x

Differentiating with respect to x, we obtain

$\frac{dy}{dx}=\frac{-2}{1+x^2}$

69.

(b) 3

Explanation: Here,it is given that the function f(x) is continuous at x = $\frac{\pi}{2}$.

$\therefore \mathrm{L.H.L}=\lim_{x\to\frac{\pi}{2}}$ f(x)

$= \lim_{x\to\frac{\pi}{2}} \frac{k\cos x}{\pi-2x}$
Substituting, $x = \frac{\pi}{2} - h$;
As $x \to \frac{\pi^-}{2}$ then $h \to 0$
$\therefore \lim_{x\to\frac{\pi}{2}} \frac{k\cos\left(\frac{\pi}{2}-h\right)}{\pi-2\left(\frac{\pi}{2}-h\right)} = k \cdot \lim_{h\to 0} \frac{\sin h}{h}$
$\therefore$ L.H.L = k
As it is continuous which implies right hand limit equals left hand limit equals the value at that point.
$\therefore$ k = 3

70.

(c) $\frac{y(1-x)}{x(y-1)}$

Explanation: Given that xy = $e^{x + y}$

Taking log both sides, we get

$\log_e xy$ = x + y (Since $\log_a b^c = c\log_a b$)

Since $\log_a bc = \log_a b + \log_a c$, we get

$\log_e x + \log_e y$ = x + y

Differentiating with respect to x, we obtain
$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$
Or
$\frac{dy}{dx}\left(\frac{y-1}{y}\right) = \frac{1-x}{x}$
Therefore, $\frac{dy}{dx} = \frac{y(1-x)}{x(y-1)}$

71. **(a)** 2^x (log 2)

Explanation: Given that y = 2^x

Taking log both sides, we get

$\log_e y = x \log_e 2$ (Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get
$\frac{1}{y}\frac{dy}{dx} = \log_e 2$ or $\frac{dy}{dx} = \log_e 2 \times y$
Hence $\frac{dy}{dx} = 2^x \log_e 2$

72.

(d) 1/8

Explanation: If f(x) is continuous at $x = \frac{\pi}{2}$ then

$\lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$

$\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{(\pi-2x)^2} = f\left(\frac{\pi}{2}\right)$ ...(i)

Now lets suppose
$\left(\frac{\pi}{2} - x\right)$ = t, then limit becomes
$\lim_{t\to 0}\left[\frac{1-\sin\left(\frac{\pi}{2}-t\right)}{(2t)^2}\right] = f\left(\frac{\pi}{2}\right)$ [from equation (i)]
$\Rightarrow \lim_{t\to 0}\left[\frac{1-\cos t}{4t^2}\right] = f\left(\frac{\pi}{2}\right)$
$\Rightarrow \frac{1}{4}\lim_{t\to 0}\left[\frac{2\sin^2\left(\frac{t}{2}\right)}{t^2}\right] = f\left(\frac{\pi}{2}\right)$
$\Rightarrow \frac{1}{4}\lim_{t\to 0}\left[\frac{\frac{2}{4}\sin^2\left(\frac{t}{2}\right)}{\frac{t^2}{4}}\right] = f\left(\frac{\pi}{2}\right)$
$\Rightarrow \frac{1}{8}\lim_{t\to 0}\left[\frac{\sin^2\left(\frac{t}{2}\right)}{\frac{t^2}{4}}\right] = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \frac{1}{8} \lim\limits_{t \to 0} \left[\frac{\sin\left(\frac{t}{2}\right)}{\frac{t}{2}}\right]^2 = f\left(\frac{\pi}{2}\right)$

$\Rightarrow f\left(\frac{\pi}{2}\right) = \lambda = \frac{1}{8}$

73.

(d) 1

Explanation: $y = \log\sqrt{\tan x}$

$\frac{dy}{dx} = \frac{1}{\sqrt{\tan x}} \times \frac{1}{2\sqrt{\tan x}}\sec^2 x$

$\frac{dy}{dx} = \frac{\sec^2 x}{2\tan x}$

$\left|\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = \frac{\sec^2 \frac{\pi}{4}}{\sqrt{\tan\frac{\pi}{4}}} = \frac{2}{2\times 1} = 1$

74.

(c) 0

Explanation: Since, f is continuous at x = $\frac{\pi}{2}$

$\therefore$ f($\frac{\pi}{2}$) = $\lim\limits_{x \to \frac{\pi}{2}} \frac{\sin(\cos x - \cos x)}{(\pi - 2x)^2}$

i.e. k = $\lim\limits_{x \to \frac{\pi}{2}} \frac{\sin(\cos x - \cos x)}{(\pi - 2x)^2}$

Let x = $\frac{\pi}{2}$ - h,

$\Rightarrow$ k = $\lim\limits_{h \to 0} \frac{\sin(\cos(\frac{\pi}{2} - h) - \cos(\frac{\pi}{2} - h)}{\left(\pi - 2(\frac{\pi}{2} - h)\right)^2}$

= $\lim\limits_{h \to 0} \frac{\sin(\sin\ h) - \sin\ h}{4h^2}$

Using sin x = x - $\frac{x^3}{3!}$ + $\frac{x^5}{5!}$ + ...

$\Rightarrow$ k = $\lim\limits_{h \to 0} \frac{(\sin\ h - \frac{\sin^3 h}{3!} + \frac{\sin^5 h}{5!} \ldots.) - \sin\ h}{4h^2}$

= $\lim\limits_{h \to 0} \left(\frac{-\sin^3 h}{3! \times 4h^2} + \frac{\sin^5 h}{5! \times 4h^2} \ldots.\right)$

= 0

$\therefore$ $\lim\limits_{x \to \frac{\pi}{2}}$ f(x) = 0 = k

$\Rightarrow$ k = 0

75.

(c) a function of y only

Explanation: y = ax^2 + bx + c

$\frac{dy}{dx} = 2ax + b$

$\frac{d^2y}{dx^2} = 2a$

$y^3 \frac{d^2y}{dx^2} = 2ay^3$ = A function of y only