Solution

LINEAR INEQUALITIES

Class 11 - Mathematics

1.

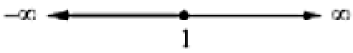
(d) $x \in (10, \infty)$
Explanation: - 3x + 17 < - 13
⇒ - 3x + 17 - 17 < - 13 - 17
⇒ - 3x < - 30
⇒ $\frac{-3x}{-3} > \frac{-30}{-3}$
⇒ x > 10
⇒ $x \in (10, \infty)$

2.

(d) $x \leq -\frac{5}{2}$
Explanation: 2x + 5 $\leq$ 0
⇒ 2x < -5
⇒ x $\leq \frac{-5}{2}$
⇒ $x \epsilon \left(-\infty, \frac{-5}{2}\right]$
Now x - 3 $\leq$ 0
⇒ x $\leq$ 3
⇒ $x \in (-\infty, 3]$
Hence the solution set is $\left(-\infty, \frac{-5}{2}\right] \cap (-\infty, 3]$ = $\left(-\infty, \frac{-5}{2}\right]$
⇒$x \leq -\frac{5}{2}$

3. **(a)** no solution
Explanation: We have given: $4x + 3 \geq 2x + 17$
$\Rightarrow 4x - 2x \geq 17 - 3 \Rightarrow 2x \geq 1$
$\Rightarrow x \geq \frac{14}{2}$ [Dividing by 2 on both sides]
$\Rightarrow x \geq 7$ ..... (i)

Also we have 3x - 5 < -2
$\Rightarrow \quad 3x < -2 + 5 \Rightarrow 3x < 3$
$\Rightarrow \quad x < 1$



On combining (i) and (ii), we see that solution is not possible because nothing is common between these two solutions.(i.e., x < 1, x $\geq$ 7)

4.

(c) [$\frac{1}{2}$, $\frac{5}{6}$]
Explanation: |3x - 2| $\leq \frac{1}{2}$
$\Rightarrow \frac{-1}{2} \leq 3x - 2 \leq \frac{1}{2}$
$\Rightarrow \frac{-1}{2} + 2 \leq 3x - 2 + 2 \leq \frac{1}{2} + 2$
$\Rightarrow \frac{3}{2} \leq 3x \leq \frac{5}{2}$ $[\because |x| \leq a \Leftrightarrow -a \leq x \leq a]$
$\Rightarrow \frac{3}{2} \cdot \frac{1}{3} \leq 3x \cdot \frac{1}{3} \leq \frac{5}{2} \cdot \frac{1}{3}$
$\Rightarrow \frac{1}{2} \leq x \leq \frac{5}{6}$
$\Rightarrow x \in \left[\frac{1}{2}, \frac{5}{6}\right]$

5.

(b) $\{1\}$
Explanation: 3x < 5
$\Rightarrow x < \frac{5}{3}$

$\Rightarrow x < 1\frac{2}{3}$

Hence solution set = $\{x : x < 1\frac{2}{3}, x \in N\}$ = {1}

6.

(b) $\left(\frac{-2}{3}, \infty\right)$

Explanation: (x + 3) + 4 > -2x + 5

$\Rightarrow$ x + 7 > -2x + 5

$\Rightarrow$ x + 7 + 2x > -2x + 5 + 2x

$\Rightarrow$ 3x + 7 > 5

$\Rightarrow$ 3x + 7 - 7 > 5 - 7

$\Rightarrow$ 3x > -2

$\Rightarrow$ x > $\frac{-2}{3}$

$\Rightarrow x \in \left(\frac{-2}{3}, \infty\right)$

7.

(d) 8 $\leq$ x $\leq$ 22

Explanation: Let the length of the shortest piece be x cm. Then we have the length of the second and third pieces are x + 3 and 2x centimeters respectively.

According to the question,

x + (x + 3) + 2x $\leq$ 91

$\Rightarrow$ 4x + 3 $\leq$ 91

$\Rightarrow$ 4x $\leq$ 88

$\Rightarrow$ x $\leq$ 22

Also 2x $\geq$ (x + 3) + 5

$\Rightarrow$ 2x $\geq$ x + 8

$\Rightarrow$ x $\geq$ 8

$\Rightarrow$ 8 $\leq$ x $\leq$ 22

Hence the shortest piece may be atleast 8 cm long but it cannot be more than 22cm in length.

8.

(d) x $\in$ (–∞, – 13] $\cup$ [7, ∞)

Explanation: since |x + 3| $\geq$ 10 , $\Rightarrow$ x + 3 $\leq$ – 10 or x + 3 $\geq$ 10

$\Rightarrow$ x $\leq$ – 13 or x $\geq$ 7

$\Rightarrow$ x $\in$ (– ∞, – 13] $\cup$ [7, ∞)

solution set = (– ∞, – 13] $\cup$ [7, ∞)

9.

(b) {x : x > 1, x $\in$ R}

Explanation: 6x - 1 > 5

$\Rightarrow$ 6x - 1 + 1 > 5 + 1

$\Rightarrow$ 6x > 6

$\Rightarrow$ x > 1

Hence the solution set is {x : x > 1, x $\in$ R}

10.

(c) ac $\geq$ bc

Explanation: The sign of the inequality is to be reversed ($\leq$ to $\geq$ or $\geq$ to $\leq$) if both sides of an inequality are multiplied by the same negative real number.

11.

(d) $(-\infty, -7) \cup (7, \infty)$

Explanation: |x| > 7

$\Rightarrow$ -7 > x > 7

$\Rightarrow$ x < -7 or x > 7 [$\because$ |x| > a $\Leftrightarrow$ x < -a or x > a]

$\Rightarrow x \in (-\infty, -7)$ or $x \in (7, \infty)$

$\Rightarrow x \epsilon (-\infty, -7) \cup (7, \infty)$

12.

(b) ac < bc

Explanation: The sign of the inequality is to be reversed (< to > or > to <) if both sides of an inequality are multiplied by the same negative real number.

13. **(a)** { }

Explanation: $\mathrm{x} \geq 2 \Rightarrow x \epsilon [2, \infty)$

$\mathrm{x} \leq -3 \Rightarrow x \epsilon (-\infty, -3]$

Hence solution set of $\mathrm{x} \geq 2$ and $\mathrm{x} \leq -3$ is $[2, \infty) \cap (-\infty, -3] = \Phi$

14. **(a)** $-\mathrm{x} > -5$

Explanation: Given $\mathrm{x} < 5$

Multiplying both sides of the above inequality by -1,we get

$-\mathrm{x} > -5$ [The sign of the inequality is to be reversed if both sides of an inequality are multiplied by the same negative real number]

15.

(b) [-7, 3]

Explanation: |x + 2| $\leq$ 5

$\Rightarrow$ -5 $\leq$ x + 2 $\leq$ 5

$\Rightarrow$ -7 $\leq$ x $\leq$ 3

$\Rightarrow$ x $\in$ [-7, 3]

16.

(d) x $\in (2, \infty)$

Explanation: Since $\frac{|x-2|}{x-2} \geq 0$, for |x - 2| $\geq$ 0, and x - 2 $\neq$ 0 solution set (2,∞)

17. **(a)** -x > -7

Explanation: x < 7

We know that when we change the sign of inequalities then greater tan changes to less than and vice versa also true.

$\Rightarrow$ -x > -7

18. **(a)** - 3 < x < 3

Explanation: We have $|x| < a \Leftrightarrow -a < x < a$

19. **(a)** - 5 < x < 5

Explanation: |x| < 5

$\Rightarrow$ -5 < x < 5

20. **(a)** 2 < x < 6

Explanation: x - 2 > 0

$\Rightarrow$ x > 2

$\Rightarrow x \in (2, \infty)$

Now 3x < 18

$\Rightarrow$ x < 6

$\Rightarrow x \in (-\infty, 6)$

So solution set is $(2, \infty) \cap (-\infty, 6)$ = (2, 6)

$\Rightarrow$ 2 < x < 6

21. **(a)** $-\frac{1}{6} \leq x < \frac{1}{2}$

Explanation: -2 $\leq$ 6x - 1 < 2

$\Rightarrow$ -2 + 1 $\leq$ 6x - 1 + 1 < 2 + 1

$\Rightarrow$ -1 $\leq$ 6x < 3

$\Rightarrow \frac{-1}{6} \leq \frac{6x}{6} < \frac{3}{6}$

$\Rightarrow \frac{-1}{6} \leq x < \frac{1}{2}$

22.

(b) $x \in (-\infty, -a) \cup (a, \infty)$

Explanation: |x| > a

$\Rightarrow$ x < -a or x > a
$\Rightarrow$x $\in$ $(-\infty$, -a) $\cup$ (a, ∞)

23.

(c) $(\frac{3}{2}, \frac{9}{2})$

Explanation: $\left|\frac{2(3-x)}{5}\right| < \frac{3}{5}$

$\Rightarrow -\frac{3}{5} < \frac{2(3-x)}{5} < \frac{3}{5}$

$\Rightarrow -\frac{3}{5} \cdot \frac{5}{2} < \frac{2(3-x)}{5} \cdot \frac{5}{2} < \frac{3}{5} \cdot \frac{5}{2}$

$\Rightarrow -\frac{3}{2} < 3 - x < \frac{3}{2}$

$\Rightarrow -\frac{3}{2} - 3 < 3 - x - 3 < \frac{3}{2} - 3$ $[\because |x| < a \Leftrightarrow -a < x < a]$

$\Rightarrow -\frac{9}{2} < -x < \frac{-3}{2}$

$\Rightarrow \frac{9}{2} > x > \frac{3}{2}$

$\Rightarrow x \in \left(\frac{3}{2}, \frac{9}{2}\right)$

24.

(b) $(-\infty, -2) \cup (-1, 1) \cup (2, \infty)$

Explanation: Given $\frac{|x|-1}{|x|-2} \geq 0, x \neq \pm 2$

$\frac{|x|-1}{|x|-2} \geq 0$

$\Rightarrow$ |x| - 1 $\geq$ 0 and |x| - 2 $\geq$ 0 or |x| - 1 $\leq$ 0 and |x| - 2 $\leq$ 0 [$\because \frac{a}{b} \geq 0 \Rightarrow$ (a $\geq$ 0 and b $\geq$ 0) or (a $\leq$ 0 and b $\leq$ 0)]

$\Rightarrow$ |x| $\geq$ 1 and |x| $\geq$ 2 or |x| $\leq$ 1 and |x| $\leq$ 2

$\Rightarrow$ |x| $\geq$ 2 or |x| $\leq$ 1 [$\because$ |x| $\geq$ a $\Rightarrow$ x $\geq$ a or x $\leq$ -a and |x| $\leq$ a $\Rightarrow -a \leq x \leq a$]

$\Rightarrow$ x $\geq$ 2 or x $\leq$ -2 or $-1 \leq x \leq 1$

$\Rightarrow x \in (2, \infty)$ or $x \in (-\infty, -2)$ or $x \epsilon (-1, 1)$

$\Rightarrow x \in (2, \infty) \cup (-\infty, -2) \cup (-1, 1)$

25.

(c) $x \in (-\infty, -4) \cup (6, \infty)$

Explanation: $|\text{ x} - 1| > 5$

$\Rightarrow x - 1 < -5 \quad or \quad x - 1 > 5$ $[\because |x| > a \Leftrightarrow x < -a \quad or \quad x > a]$

$\Rightarrow x - 1 + 1 < -5 + 1 \quad or \quad x - 1 + 1 > 5 + 1$

$\Rightarrow x < -4 \quad or \quad x > 6$

$\Rightarrow x \in (-\infty, -4) \cup (6, \infty)$

26.

(b) $-4 \leq \text{ x} \leq 1$

Explanation: (x + 5) - 7(x - 2) $\geq$ 4x + 9

$\Rightarrow$ x + 5 - 7x + 14 $\geq$ 4x + 9

$\Rightarrow$ -6x + 19 $\geq$ 4x + 9

$\Rightarrow$ -6x - 4x $\geq$ 9 - 19

$\Rightarrow$ -10x $\geq$ -10

$\Rightarrow$ x $\leq$ 1

$\Rightarrow x \in (-\infty, 1]$

2(x - 3) - 7(x + 5) $\leq$ 3x - 9

$\Rightarrow$ 2x - 6 - 7x - 35 $\leq$ 3x < 9

$\Rightarrow$ -5x - 41 $\leq$ 3x - 9

$\Rightarrow$ -5x - 3x $\leq$ 41 - 9

$\Rightarrow$ -8x $\leq$ 32

$\Rightarrow$ -x $\leq \frac{32}{8}$ = 4

$\Rightarrow$ x $\geq$ -4

$\Rightarrow x \epsilon [-4, \infty)$

Hence the solution set is $[-4, \infty) \cap (-\infty, 1]$ = [-4, 1]

Which means $-4 \leq \text{ x} \leq 1$

27.

(c) x < -6

Explanation: 3x + 5 < x - 7

⇒ 3x + 5 - x < x - 7 - x

⇒ 2x + 5 < -7

⇒ 2x + 5 - 5 < -7 - 5

⇒ 2x < -12

⇒ $\frac{2x}{2}$ < - $\frac{12}{2}$

⇒ x < -6

28.

(d) {0, 1, 2, 3}

Explanation: Given 2(x - 1) < 3x - 1

⇒ 2x - 2 < 3x - 1

⇒ 2x - 2 + 2 < 3x - 1 + 2

⇒ 2x < 3x + 1

⇒ 2x - 3x < 3x + 1 - 3x

⇒ -x < + 1

⇒ x > -1 but x ∈ Z

Hence A = {0, 1, 2, 3, 4, ...}

Now 4x - 3 ≤ 8 + x

⇒ 4x - 3 + 3 ≤ 8 + x + 3

⇒ 4x ≤ 11 + x

⇒ 4x - x ≤ 11 + x - x

⇒ 3x ≤ 11

⇒ $\frac{3x}{3} \leq \frac{11}{3}$

⇒ x ≤ $\frac{11}{3}$

⇒ x ≤ $3\frac{2}{3}$, but x ∈ Z

Therefore B = {....., -2, -1, 0, 1, 2, 3}

Hence A ∩ B = {0, 1, 2, 3}

29.

(d) x ∈ (– ∞, – 4] ∪ [3, ∞)

Explanation: Common solution of the inequalities is from – ∞ to – 4 and 3 to ∞

{(– ∞, – 4] ∪ [3, ∞)} ∩ {(– ∞, – 3] ∪ [1, ∞)} = (– ∞, – 4] ∪ [3, ∞)

30.

(c) x ∈ [- 11 , 7]

Explanation: $|x+2| \leq 9$

⇒ $-9 \leq x+2 \leq 9 \quad [\because |x| \leq a \Leftrightarrow -a \leq x \leq a]$

⇒ $-9-2 \leq x+2-2 \leq 9-2$

⇒ $-11 \leq x \leq 7$

$x \epsilon [-11, 7]$

31. **(a)** x > 5

Explanation: x - 5 > 0

⇒ x > 5

⇒ $x \in (5, \infty)$

Now $\frac{2x-4}{x+2}$ < 2

$\frac{2x-4}{x+2}$ - 2 < 0

⇒ $\frac{2x-4-2(x+2)}{x+2}$ < 0

⇒ $\frac{2x-4-2x-4}{x+2}$ < 0

⇒ $\frac{-8}{x+2}$ < 0

⇒ x + 2 > 0 [∵ $\frac{a}{b}$ < 0, a < 0 ⇒ b > 0]

⇒ x > -2
⇒ $x \in (-2, \infty)$
Hence the solution set is $(5, \infty) \cap (-2, \infty) = (5, \infty)$
Which means x > 5.

32.

(c) null set
Explanation: $\frac{2x-1}{3} - \frac{3x}{5} + 1 < 0$
⇒ $15 \cdot \frac{2x-1}{3} - 15 \cdot \frac{3x}{5} + 15 < 0$ [Multiply the inequality throughout by the L.C.M]
⇒ 5(2x - 1) -3(3x) + 15 < 0
⇒ 10x - 5 - 9x + 15 < 0
⇒ x + 10 < 0
⇒ x < -10, but given x ∈ W
Hence the solution set will be null set.

33.

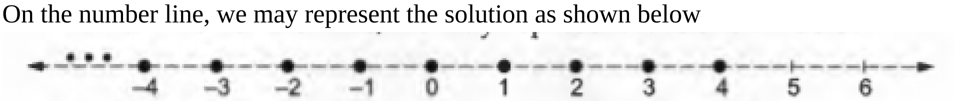
(b) -23 < x ≤ 2
Explanation: $-15 < \frac{3(x-2)}{5} \leq 0$
⇒ $-15 \cdot \frac{5}{3} < \frac{3(x-2)}{5} \cdot \frac{5}{3} \leq 0 \cdot \frac{5}{3}$
⇒ -25 < (x - 2) ≤ 0 + 2
⇒ -25 + 2 < x - 2 + 2 ≤ 2
⇒ -23 < x ≤ 2

34.

(c) $x \in (-\infty, -b) \cup (b, \infty)$
Explanation: We have $|x| > a \Leftrightarrow x < -a$ or x > a
So |x| $> b \Rightarrow x < -b$ or x > b
⇒ $x \in (-\infty, -b) \cup (b, \infty)$

35.

(d) -2 < x < 1
Explanation: -2 < 1 - 3x < 7
⇒ -2 - 1 < 1 - 3x - 1 < 7 - 1
⇒ -3 < -3x < 6
⇒ $\frac{-3}{-3} > \frac{-3x}{-3} > \frac{6}{-3}$
⇒ 1 > x > -2
⇒ -2 < x < 1

36. 5x < 24 , $x \in Z$
⇒ $\{x \in Z : x < 4.8\}$
⇒ x = $\{4, 3, 2, 1, 0, -1, -2, -3, \ldots\}$
⇒ x = $(\ldots, -3, -2, -1, 0, 1, 2, 3, 4\}$
On the number line, we may represent the solution as shown below


The darkened circles show the integers contained in the set and three dark dots above the left part of the line show that the negative integers are continued indefinitely.

37. $(x^2 - 2x + 1)(x - 4) \geq 0$
⇒ $(x - 1)^2(x - 4) \geq 0$
⇒ $(x - 1)^2 \geq 0$ and $(x - 4) \geq 0$
⇒ Square term is always positive and x ≥ 4
So, solution set is x ∈ [4, ∞)

38. Given:
$\frac{5x}{4} - \frac{4x-1}{3} > 1$, where x ∈ R.
⇒ $\frac{3(5x)-4(4x-1)}{12} > 1$

$\Rightarrow \frac{15x-16x+4}{12} > 1$

$\Rightarrow \frac{-x+4}{12} > 1$

Now, multiplying by 12 on both the sides in the above equation,

$\Rightarrow \left(\frac{-x+4}{12}\right)\cdot(12) > 12$

⇒ -x + 4 > 12

Now, subtracting 4 from both the sides in the above equation

⇒ -x + 4 – 4 > 12 – 4

⇒ -x > 8

Multiplying by -1 on both the sides of the above equation

⇒ x < - 8

x ∈ $(-\infty$,-8)

39. We have

|4 - x| < 2

⇒ - 2 < 4 - x < 2

⇒ - 2 < 4 - x < 2

⇒ - 2 - 4 < - x and - x < 2 - 4

⇒ - 6 < -x and ~x < -2

⇒ 6 > x and x > 2

⇒ 2 < x < 6.

$= \{x \in R: 2 < x < 6\} = (2, 6)$

40. $\frac{(x-1)}{3} + 4 < \frac{x-5}{5} - 2$

$\frac{(x-1)+12}{3} < \frac{x-5-10}{5}$

$\frac{x+11}{3} < \frac{x-15}{5}$

5x + 55 < 3x -45

2x < -100

x < -50

Hence, the solution of the given inequation is $(-\infty, -50)$.

41. Given:

3x – 4 > x + 6, where x ∈ R.

⇒ 3x – 4 > x + 6

⇒ 3x – 4 + 4 > x + 6 + 4

⇒ 3x > x + 10

⇒ 3x – x > x + 10 – x

⇒ 2x > 10

$\Rightarrow \frac{2x}{2} > \frac{10}{2}$

⇒ x > 5

Therefore, x ∈ (5, ∞)

42. We have, $0 < \frac{-x}{3} < 1$

Multiplying inequality by 3, we get

0 < - x < 3

or 0 > x > -3

or -3 < x < 0 ⇒ x ∈ (-3, 0)

43. Given, 4x - 2 < 8, x ∈ N

4x < 10

x < $\frac{10}{4}$ [divide both sides by 4]

x < $\frac{5}{2}$

As $2 < \frac{5}{2} < 3$, when x is a natural number, the maximum possible value of x is 2 and we know the natural numbers start from 1.

[divide both sides by 4]

Thus, the solution of the given inequation is {1, 2}.

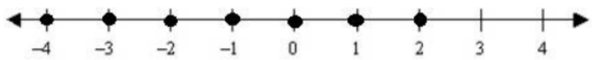
44. 5x < 24 $\Rightarrow x < \frac{24}{5}$
$\Rightarrow$ x < 4.8
$\Rightarrow \{x \in N; x < 4.8\}$
$\Rightarrow$ x = {1, 2, 3, 4 }
On the number line, we may represent it as shown below

The darkened circles indicate the natural numbers contained in the set

45. $\frac{2x+3}{5} - 2 < \frac{3(x-2)}{5}$
2x + 3 - 10 < 3x - 6
-x - 7 < -6
-x < 7 - 6
-x < 1
x > -1
Hence, the solution set of the given inequation is (-1, ∞)

46. Given, 5x + 2 < 17
Subtracting 2 from both the sides in above equation
$\Rightarrow$ 5x + 2 – 2 < 17 – 2
$\Rightarrow$ 5x < 15
Dividing both the sides by 5 in above equation
$\Rightarrow \frac{5x}{5} < \frac{15}{5}$
$\Rightarrow$ x < 3
Therefore, x $\in$ (-∞, 3)

47. Given, 5x + 2 < 17
Subtracting 2 from both the sides in the above equation,
$\Rightarrow$ 5x + 2 – 2 < 17 – 2
$\Rightarrow$ 5x < 15
Dividing both the sides by 5 in the above equation,
$\Rightarrow \frac{5x}{5} < \frac{15}{5}$
$\Rightarrow$ x < 3
Since x is an integer.
Therefore, possible values of x can be
x = {…, -2, -1, 0, 1, 2}



48. $\Rightarrow$ 3x - x > 1 + 7
$\Rightarrow$ 2x > 8
$\Rightarrow$ x > $\frac{8}{2}$
$\Rightarrow$ x > 4
$\therefore$ (4, ∞) is the solution set.

49. Given 12x < 50
$\Rightarrow \frac{12x}{12} < \frac{50}{12}$ [divide both sides by 12]
$\therefore x < \frac{25}{6}$
x $\in$ R
When x is a real number, the solumintion of the given inequation is $\left(-\infty, \frac{25}{6}\right)$.

50. We have $6 \leq -3(2x-4) < 12$
$\Rightarrow 2 \geq (2x-4) > -4 \Rightarrow 2 \geq 2x > 0$
$\Rightarrow 1 \geq x > 0 \Rightarrow 0 < x \leq 1$

51. Given 12x < 50
$\Rightarrow \frac{12x}{12} < \frac{50}{12}$ [divide both sides by 12]
$\therefore x < \frac{25}{6}$

$x \in N$

As $4 < \frac{25}{6} < 5$, when x is a natural number, the maximum possible value of x is 4 and we know the natural numbers start from 1.

Thus, the solution of the given inequation is {1, 2, 3, 4}.

52. We have $7 \leqslant \frac{(3x+11)}{2} \leqslant 11$

$\Rightarrow 14 \leqslant 3x + 11 \leqslant 22 \Rightarrow 3 \leqslant 3x \leqslant 11$

$\Rightarrow 1 \leqslant x \leqslant \frac{11}{3}$

53. We have 24x < 100

$\Rightarrow \quad \frac{24x}{24} < \frac{100}{24}$ [dividing both sides by 24]

$\Rightarrow \quad x < \frac{25}{6}$

When x is a natural number, then solutions of the inequality are given by $x < \frac{25}{6}$ i.e., all natural numbers x which are less than $\frac{25}{6}$.

Hence, the solution set is {1, 2, 3, 4}

54. Given that $\frac{4}{x+1} \leq 3 \leq \frac{6}{x+1}, (x > 0)$

$\Rightarrow 4 \leq 3(x+1) \leq 6$

$\Rightarrow 4 \leq 3x+3 \leq 6$

$\Rightarrow 4-3 \leq 3x \leq 6 - 3$

$\Rightarrow 1 \leq 3x \leq 3$

$\Rightarrow \frac{1}{3} \leq x \leq 1$

Therefore, solution set = $[\frac{1}{3}, 1]$

55. $\Rightarrow \frac{5x-6x}{x+6} < 1$

$\Rightarrow \frac{5x-6}{x+6} < 1$

$\Rightarrow \frac{5x-6}{x+6} - 1 < 0$

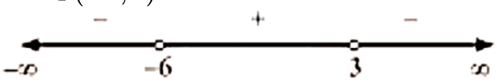
$\Rightarrow \frac{5x-6-x-6}{x+6} < 0$

$\Rightarrow \frac{4x-12}{x+6} < 0$

$\Rightarrow$ {4x - 12 > 0 and x + 6 < 0} or {4x - 12 < 0 and x + > 0}

$\Rightarrow$ {x > 3 and x < -6} or {x < 3 and x > -6}

$\Rightarrow x \in (-6, 3)$



$\therefore x \in (-6, 3)$

56. Given:

$\frac{|x-3|}{x-3} < 0, x \in R$

|x - 3| < 0

The above condition can’t be true because the absolute value cannot be less than 0

but, it is possible only when denominator x-3 is negative so,

$x - 3 < 0 \Rightarrow x < 3$

Therefore,

Solution set is; $x \in (-\infty, 3)$

57. $\frac{(2x+3)}{4} - 3 < \frac{x-4}{3} - 2$

$\frac{2x-9}{4} < \frac{x-10}{3}$

6x - 27 < 4x - 40

2x < - 13

$x < -\frac{13}{2}$

Hence, the solution of the given inequation is $\left(-\infty, \frac{-13}{2}\right)$.

58. We have $-15 < \frac{3(x-2)}{5} \leqslant 0$

$\Rightarrow -75 < 3(x-2) \leqslant 0 \Rightarrow -25 < x - 2 \leqslant 0$

$\Rightarrow -23 < x \leqslant 2$

59. Given, $6x \leq 25, x \in Z$

Dividing both the sides by 6 in the above equation,

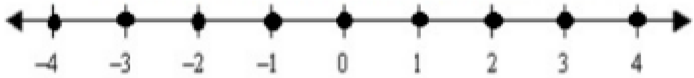
$\frac{6x}{6} \leq \frac{25}{6}$

⇒ x ≤ $\frac{25}{6}$

⇒ x ≤ 4.166

Since x is an integer so the possible values of x can be:

x = {…, -3, -2, -1, 0, 1, 2, 3, 4} for x ∈ Z



60. Given 12x < 50

⇒ $\frac{12x}{12} < \frac{50}{12}$ [divide both sides by 12]

$\therefore x < \frac{25}{6}$

x ∈ Z

As $4 < \frac{25}{6} < 5$, when x is an integer, the maximum possible value of x is 4.

Thus, the solution of the given inequation is {…, -2, -1, 0, 1, 2, 3, 4}.

61. We know,

$|x| > a \Leftrightarrow$ x < -a or x > a

⇒ $|x+1| > 4 \Leftrightarrow x+1 < -4$ or $x+1 > 4$

⇔ x < - 4 - 1 or x > 4 - 1

⇔ x < -5 or x > 3

⇔ $x \in (-\infty, -5)$ or $x \in (3, \infty)$

∴ Solution set $= (-\infty, -5) \cup (3, \infty)$

62. Now, 12x < 50

⇒ x < $\frac{50}{12} < \frac{25}{6}$

i. Since x ∈ R, x = $\left(-\infty, \frac{25}{6}\right)$

ii. Since x ∈ Z, x = {....,-3,-2,-1,0,1,2,3,4}

63. ⇒ $\frac{3x}{5} - \frac{2}{5} \leq \frac{4x}{2} - \frac{3}{2}$

⇒ $\frac{3x}{5} - \frac{4x}{2} \leq \frac{-3}{2} + \frac{2}{5}$

⇒ $\frac{6x-20x}{10} \leq \frac{-15+4}{10}$

⇒ -14x ≤ -11

⇒ 14x ≥ 11

⇒ x ≥ $\frac{11}{14}$

Therefore, $\left[\frac{11}{14}, \infty\right)$ is the solution set.

64. We have,

4x + 3 < 6x + 7

⇒ 4x + 3 - 3 < 6x + 7 - 3

⇒ 4x < 6x + 4

⇒ 4x – 6x < 6x + 4 – 6x

⇒ – 2x < 4 or x > – 2

i.e., all the real numbers which are greater than –2, are the solutions of the given inequality.

Hence, the solution set is (–2, ∞).

65. Given, 3x + 8 > 2, x ∈ R

Subtracting 8 from both the sides in above equation

⇒ 3x + 8 – 8 > 2 – 8

⇒ 3x > - 6

Dividing both the sides by 3 in above equation

⇒ $\frac{3x}{3} > \frac{-6}{3}$

Thus, x > -2

x ∈ (-2, ∞)

66. We have, x - 2 > 0

⇒ x > 2

⇒ x ∈ (2, ∞)

Also. 3x < 18

$\Rightarrow$ x < 6

$\Rightarrow$ x $\in$ (-∞, 6)

Solution of the given set of the inequations is intersection of (2, ∞) $\cap$ (- ∞, 6) = (2, 6)

Thus, (2, 6) is the solution of the given set of inequalities.

67. We have

2 x - 3 < x + 2 < 3 x + 5

$\Rightarrow$ 2x - 3 < x + 2 and x + 2 < 3x + 5

Now, 2x - 3 < x + 2 $\Rightarrow$ x - 3 < 2

$\Rightarrow$ x < 5

$\Rightarrow x \in (-\infty, 5)$

And $x + 2 \leq 3x + 5 \Rightarrow 3x + 5 \geq x + 2$

$\Rightarrow 2x + 5 \geq 2$

$\Rightarrow 2x \geq -3$

$\Rightarrow x \geq \frac{-3}{2}$

$\Rightarrow x \in \left[\frac{-3}{2}, \infty\right)$

Solution set is, $(-\infty, 5) \cap \left(\frac{-3}{2}, \infty\right) = \left[\frac{-3}{2}, 5\right)$

We may represent it on the number line, as shown below.

68. We have, 4x - 1 $\leq$ 0

$\Rightarrow$ 4x $\leq$ 1

$\Rightarrow$x $\leq \frac{1}{4}$ $\Rightarrow$x $\in$(-∞, $\frac{1}{4}$] ...... (i)

Also, 3 - 4x < 0

$\Rightarrow$ 0 > 3 - 4x

$\Rightarrow$ 4x > 3

$\Rightarrow$x > $\frac{3}{4}$

$\Rightarrow$ x $\in$ ($\frac{3}{4}$, ∞) ......(ii)

Hence, the solution set of inequalities is the intersection of (i) and (ii). But, (- ∞, $\frac{1}{4}$) $\cap$ ($\frac{3}{4}$, ∞) = ϕ

Thus, the given set of inequations has no solution.

69. Let x be the score obtained by Mohan in the last examination.

Then, $\frac{94+73+72+84+x}{5} \geq 80$

$\Rightarrow$ $\frac{323+x}{5} \geq 80$

$\Rightarrow$ $323 + x \geq 400$

$\Rightarrow$ $323 + x - 323 \geq 400 - 323$ [subtracting 323 from both sides]

$\Rightarrow$ $x \geq 77$

Therefore, Mohan should obtain more than or equal to 77 marks in the last examination. The upper limit being 90.Hence, the required range is $77 \leq x < 90$.

70. We have,

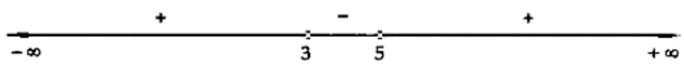
$\frac{x-3}{x-5} > 0$ ...(I)

$\Leftrightarrow$ { x - 3 > 0 & x - 5 > 0 } or { x - 3 < 0 & x - 5 < 0}

$\Leftrightarrow$ { x > 3 & x > 5} or {x < 3 & x < 5 }

$\Leftrightarrow$ x $\in$ (5,∞) or (-∞,3)

$\Leftrightarrow$ x $\in$ (5,∞) $\cup$ (-∞,3)



$\frac{x-3}{x-5} > 0 \Rightarrow x \in (-\infty, 3) \cup (5, \infty)$

Hence, the solution set of the given inequation is $(-\infty$, 3) $\cup$ (5, ∞) as shown in Figure

71. We have,

$3x + 12 \leq 0$

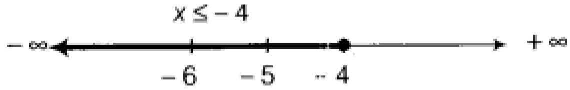
$\Rightarrow$ $3x + 12 - 12 \leq 0 - 12$ [subtacting 12 from both sides]

$\Rightarrow$ $\frac{3x}{3} \leq \frac{-12}{3}$ [dividing both sides by 3]

$\therefore \quad x \leq -4$

Thus, any value of x less than or equal to - 4 satisfies the inequality. So, the solution set is $x \in (-\infty, -4]$.

This solution set can be represented on number line as given below



Hence, the dark portion on number line represents the solution of given inequality.

72. Given that, |2x - 3| > 1

Square both sides

$\Rightarrow (2x - 3)^2 < 1^2$

$\Rightarrow 4x^2 - 12x + 9 < 1$

$\Rightarrow 4x^2 - 12x + 8 < 0$

$\Rightarrow x^2 - 3x + 2 < 0$

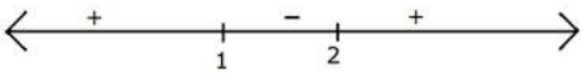
$\Rightarrow x^2 - 2x - x + 2 < 0$

$\Rightarrow x(x - 2) - 1(x - 2) < 0$

$\Rightarrow (x - 1)(x - 2) < 0$

Observe that when x is greater than 2(x – 1)(x – 2) is positive

And for each root the sign changes hence



We want less than 0 that is a negative part

Hence x should be between 1 and 2 for (x – 1)(x – 2) to be negative

Hence $x \in (1, 2)$

Hence the solution set of |2x – 3| < 1 is (1, 2)

73. Let x be the smaller of the two consecutive odd natural numbers. Then the other odd integer is x+2.

It is given that both the natural number are greater than 10 and their sum is less than 40.

$\therefore x > 10$ and, $x + x + 2 < 40$

$\Rightarrow x > 10$ and $2x < 38$

$\Rightarrow x > 10$ and $x < 19$

$\Rightarrow 10 < x < 19$

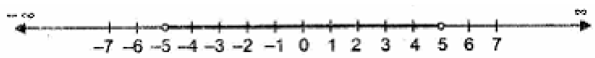
$\Rightarrow x = 11, 13, 15, 17$ [$\because$ x is an odd number]

Hence, the required pairs of odd natural number are (11, 13), (13, 15), (15, 17) and (17, 19).

74. We have 5x + 1 > - 24 and 5x - 1 < 24

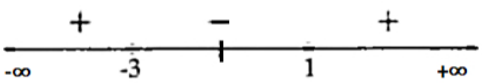
$\Rightarrow 5x > -25$ and 5x < 25

$\Rightarrow x > -5$ and x < 5



75. We have $\frac{x+3}{x-1} > 0$

Equation x - 1 and x + 3 to zero, we obtain x = 1, -3 as critical points. Plot these points on real line as shown in figure. The real line is divided into three regions. In the right most region the expression on LHS of (i) is positive and in the remaining two regions it is alternatively negative



and positive. Since the expression in (i) is positive, so the solution set is given by

$\frac{x+3}{x-1} > 0 \Rightarrow (-\infty, -3) \cup (1, \infty)$