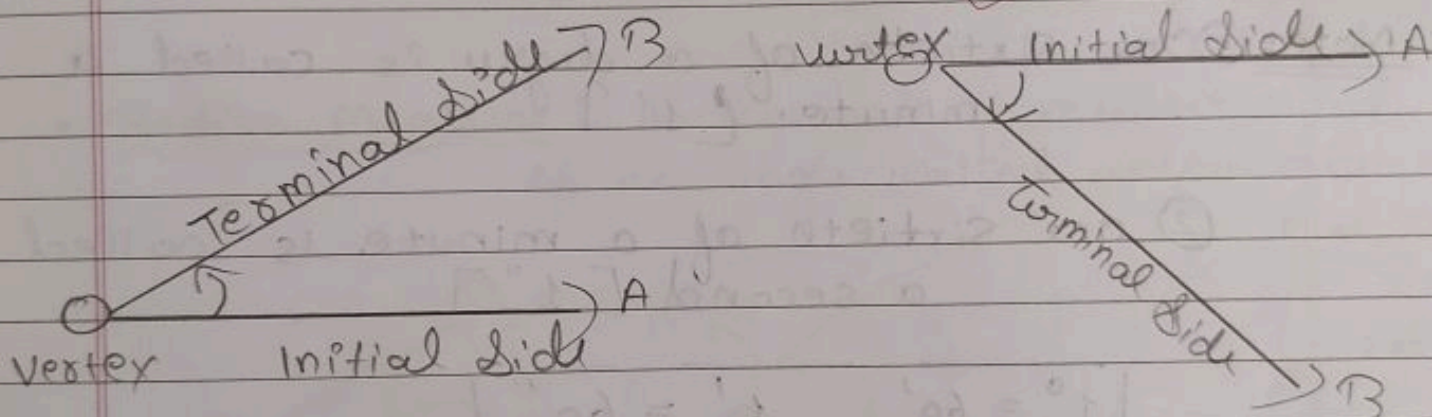


Trigonometric Functions

Notes:

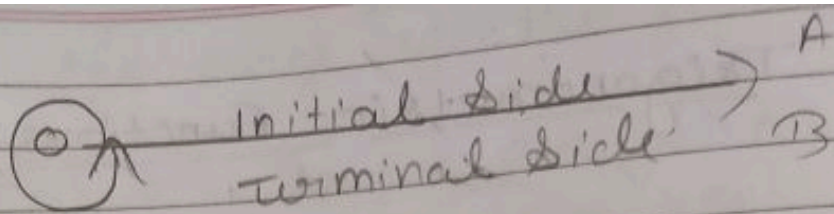
- The word 'trigonometry' is derived from greek words 'trigon' and 'metron' it means "measuring the sides of triangle".
- Angles: Angle is a measure of rotation of a given ray about its initial point.



(i) Positive Angle

(ii) Negative Angle

- The original ray is called initial side and the final position of the ray after rotation is called the terminal side of the angle.
- The point of rotation is called vertex.
- If the direction of rotation is anticlockwise then, the angle is positive and if the direction of rotation is clockwise, then the angle is negative.



• The measure of an angle is the amount of rotation performed to get the terminal side from the initial side.

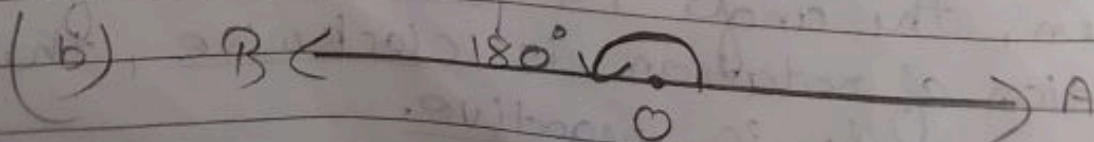
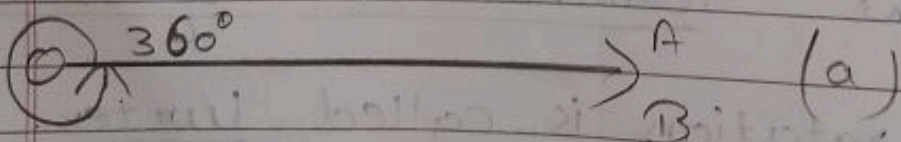
• Degree Measure: If a rotation from the initial side to terminal side is $\frac{1}{360}$ of a revolution, the angle is said to be one degree $[1^\circ]$.

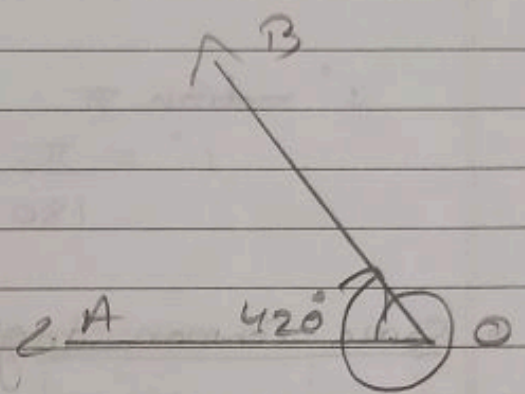
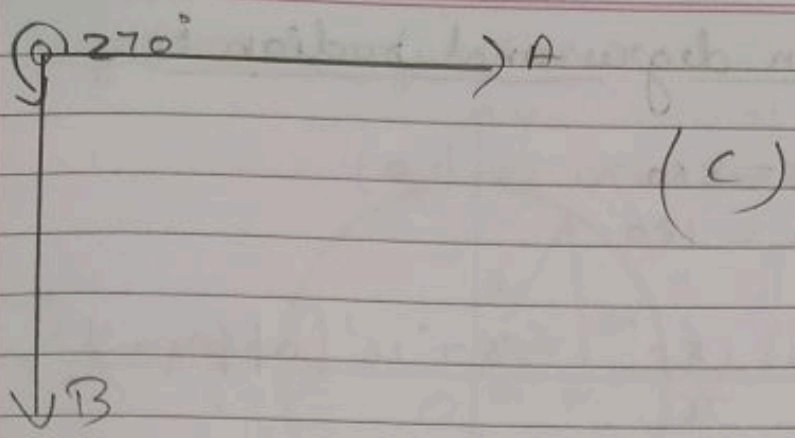
Note: ① One sixtieth of a degree is called a minute. $[1']$

② one sixtieth of a minute is called a second $[1'']$

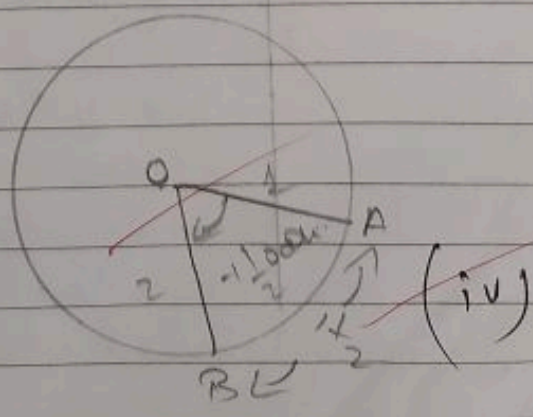
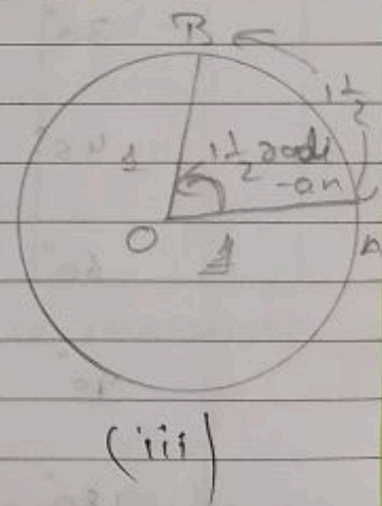
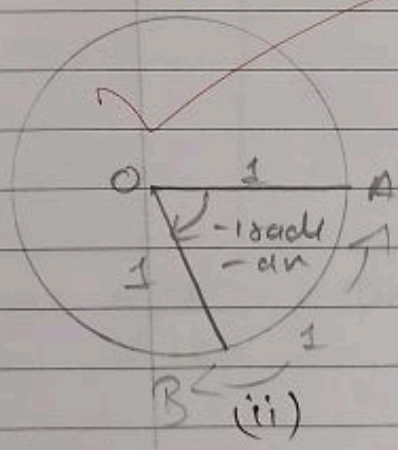
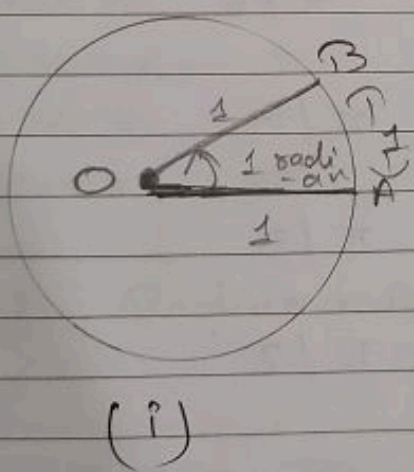
$$1^\circ = 60', \quad 1' = 60''$$

• Some of the Angle Measures:





• Radian measure : Another unit of measurement of an angle called radian measure.



• Relation between degree and radian:

$$2\pi \text{ radian} = 360^\circ$$

$$\pi \text{ radian} = 180^\circ$$

$$1 \text{ radian} = \frac{180^\circ}{\pi} = 57^\circ 16' \text{ (approx.)}$$

1 radian π

$$1^\circ = \frac{\pi}{180} \text{ radian} = 0.01746 \text{ (approx.)}$$

• Some Common angles radian:

Degree

30°

45°

60°

90°

180°

270°

360°

Radian

$\pi/6$

$\pi/4$

$\pi/3$

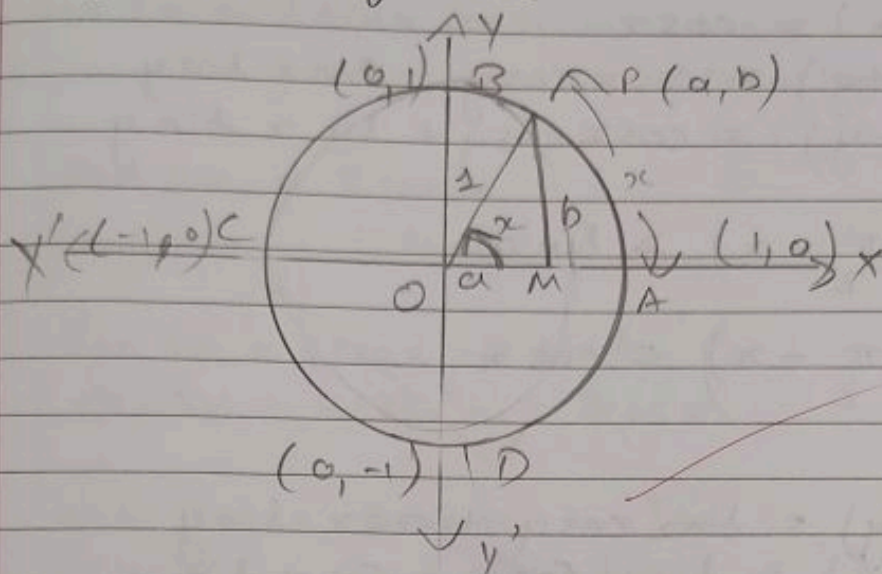
$\pi/2$

π

$3\pi/2$

2π

• Trigonometric function:



• Trigonometric function in quadrants:

	I	II	III	IV
1. $\sin x$	+	+	-	-
2. $\cos x$	+	-	-	+
3. $\tan x$	+	-	+	-
4. $\operatorname{cosec} x$	+	+	-	-
5. $\sec x$	+	-	-	+
6. $\cot x$	+	-	+	-

Note: Radian measure = $\frac{\pi}{180}$ x Degree Measure

Degree measure = $\frac{180}{\pi}$ x Radian Measure.

- ↳
- $\cos^2 x + \sin^2 x = 1$
 - $1 + \tan^2 x = \sec^2 x$
 - $1 + \cot^2 x = \operatorname{cosec}^2 x$
 - $\cos(2n\pi + x) = \cos x$
 - $\sin(2n\pi + x) = \sin x$

- $\sin(-x) = -\sin x$
- $\cos(-x) = \cos x$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$

$$\hookrightarrow \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\hookrightarrow \sin\left(\frac{\pi}{2} - x\right) = \cos x$$

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$

- $* \cos\left(\frac{\pi}{2} + x\right) = -\sin x$

- $* \sin\left(\frac{\pi}{2} + x\right) = \cos x$

- $\cos(\pi - x) = -\cos x$

- $\sin(\pi - x) = \sin x$

- $\cos(\pi + x) = -\cos x$

- $\sin(\pi + x) = -\sin x$

- $\cos(2\pi - x) = \cos x$

- $\sin(2\pi - x) = -\sin x$

- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

- $\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$

- $\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$

• $\sin 2x = 2 \sin x \cos x = 2 \tan x$
 $\cos 2x = \cos^2 x - \sin^2 x$

• $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ ✓

• $\sin 3x = 3 \sin x - 4 \sin^3 x$

• $\cos 3x = 4 \cos^3 x - 3 \cos x$

• $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - \tan^2 x}$ ✓

* (i) $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$

(ii) $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$

(iii) $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$

(iv) $\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$

Notes: $1^\circ = 60'$, $1' = 60''$
 $\pi \text{ rad} = 180^\circ$ ✓ $1 \text{ radian} = 57^\circ 16' 21.8''$
 $\frac{22}{7} = 180'$

$1 \text{ rad} = \frac{180 \times 7}{22} = \frac{90 \times 7}{11} = 630 \frac{6}{11}$

$\begin{array}{r} 57 \\ 11 \overline{) 630} \\ \underline{55} \\ 80 \\ \underline{77} \\ 3 \end{array}$	$\begin{array}{r} 22 \quad 16 \\ 11 \overline{) 180} \\ \underline{11} \\ 70 \\ \underline{60} \\ 10 \\ \underline{11} \\ 1 \end{array}$	$\begin{array}{r} 21.8 \\ 11 \overline{) 240} \\ \underline{22} \\ 20 \\ \underline{11} \\ 9 \\ \underline{11} \\ 2 \end{array}$
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