CH-2

Relations and Functions

Notes:

- Ordered Pair: Two numbers 'a' and 'b' listed in a specific order and enclosed in parenthsises from an ordered pair (a, b).

Here; a is called as first member and b is called as second member

→ Equality of two ordered Pairs:

Ex: find a and b when; $(a-1, b+5) = (2,3)$

$a - 1 = 2$	$b + 5 = 3$
$a = 2+1$	$b = 3-5$
$a = 3$	$b = (-2)$

$\therefore a = 3;\ b = (-2)$

→ Cartesian Product of two sets:

Let A and B be two non-empty sets, the cartesian Product of A and B is the set denoted by $(A \times B)$, consisting of all ordered pairs (a, b)

$\therefore A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$.

If $A = \phi$ OR $B = \phi$, then: $A \times B = \phi$.

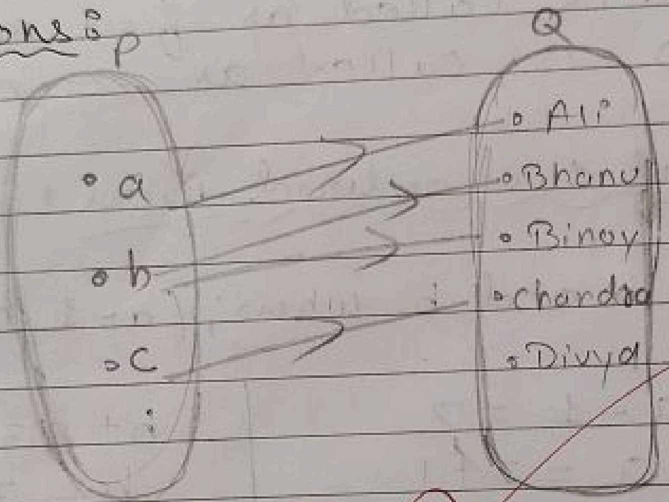
Note: 1. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and $n(B \times A) = pq$.

• **Ordred Triplet :** $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$ Here, (a, b, c) is called as orderd Triplet.

Ex: $P = \{1, 2\}$, form the set $P \times P \times P$.

$$P \times P \times P = \{(1,1,1), (1,1,2), (1,2,1), (1,2,2), (2,1,1), (2,1,2), (2,2,1), (2,2,2)\}$$

→ **Relations :**



Consider the two sets $P = \{a, b, c\}$ and $Q = \{$Ali, Bhanu, Binoy, chandra, Divya$\}$

$P \times Q = \{$(a, ali), (a, Bhanu), (a, Binoy) ....
(c, Divya)$\}$.

→ **Image :** A Relation R from a non-empty set A to a non-empty set B is a subset the cartesian product $A \times B$. The subset is derived by describing the relationship between the first element and the second element of ordred pairs in $A \times B$. The second element is called the image of the first element.

→ Domain: The set of all first elements of the ordered pairs in Relation R from a set A to B is called domain of the Relation R.

→ Co-Domain: The set of all second element in a Relation R from a set A to set B is called Range of the Relation. The whole set B is called the Co-Domain of the Relation R.

Note: 1. A relation may be represented algebrica-ally either by the Roster method or set-builder form.

2. An arrow diagram is a visual representation of a Relation.

→ The total number of relations that can be defined from a set A to set B is the number of possible subsets (A×B) if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and the total number of relations is 2^{pq}.

→ Functions: Let X and Y be two non-empty sets. Then, a relation 'f' from X to Y is called a function. if every element in X has a unique image in Y and we write $f: X \rightarrow Y$.

→ Equal functions: Two functions f and g are said to be equal if;

(i) dom (f) = dom(g)

(ii) co-domain of f = co-domain of g

(iii) $f(x) = g(x)$ for every x in their common domain.

→ Greatest integer function: The function $f: R \to R : f(x) = [x]$, where (x) denotes the greatest integer less than or equal to x, is called greatest integer function.

• Ex: Find the domain and range of the real function, $f(x) = \frac{1}{\sqrt{x+[x]}}$

we have, $f(x) = \frac{1}{\sqrt{x+[x]}}$

when;

$$\begin{cases} x+[x] > 0 & \text{for all } x > 0 \\ x+[x] = 0 & \text{for all } x = 0 \\ x+[x] \leq 0 & \text{for all } x < 0 \end{cases}$$

$\therefore f(x) = \frac{1}{\sqrt{x+[x]}}$ is defined only when $x+[x] > 0$.

Let $y = f(x)$, then;

$y = \frac{1}{\sqrt{x+[x]}} \Rightarrow \sqrt{x+[x]} = \frac{1}{y}$

Now, $x > 0 \Rightarrow x + [x] > 0$

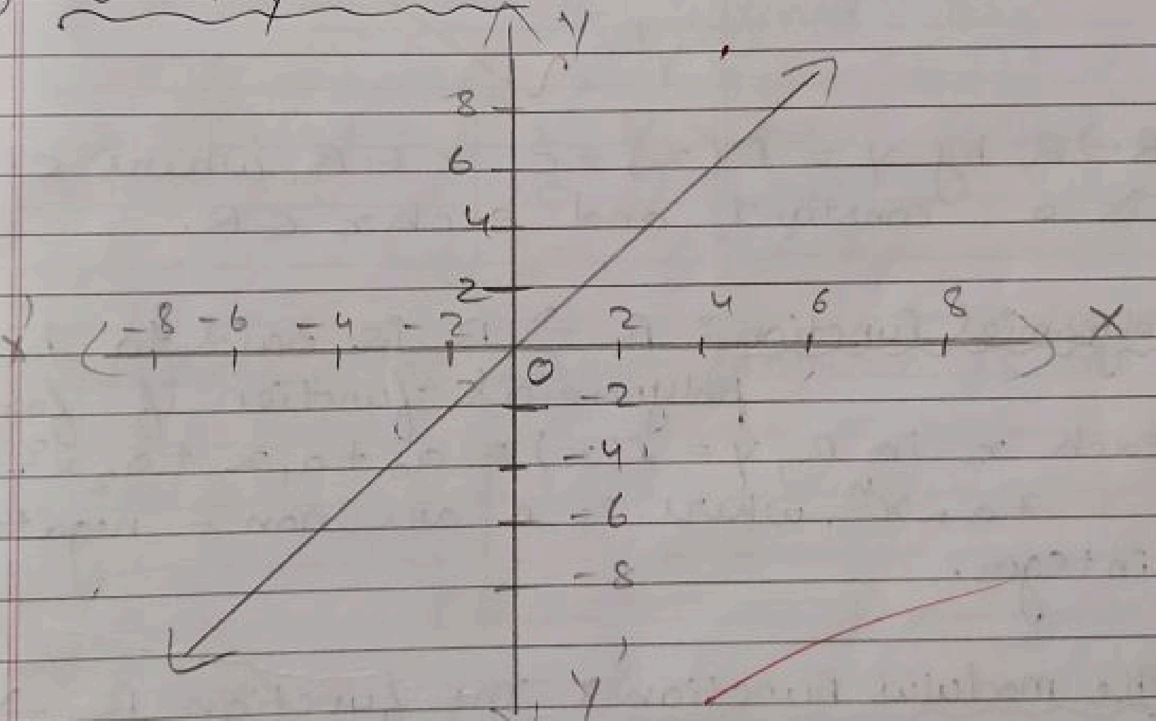
$\Rightarrow \sqrt{x+[x]} > 0 \Rightarrow \frac{1}{y} > 0 \Rightarrow y > 0$

$\therefore$ range $(f) = (0, \infty)$

Hence, range$(f) = (0, \infty)$ and range $(0, \infty)$.

→ Function's and their Graphs:

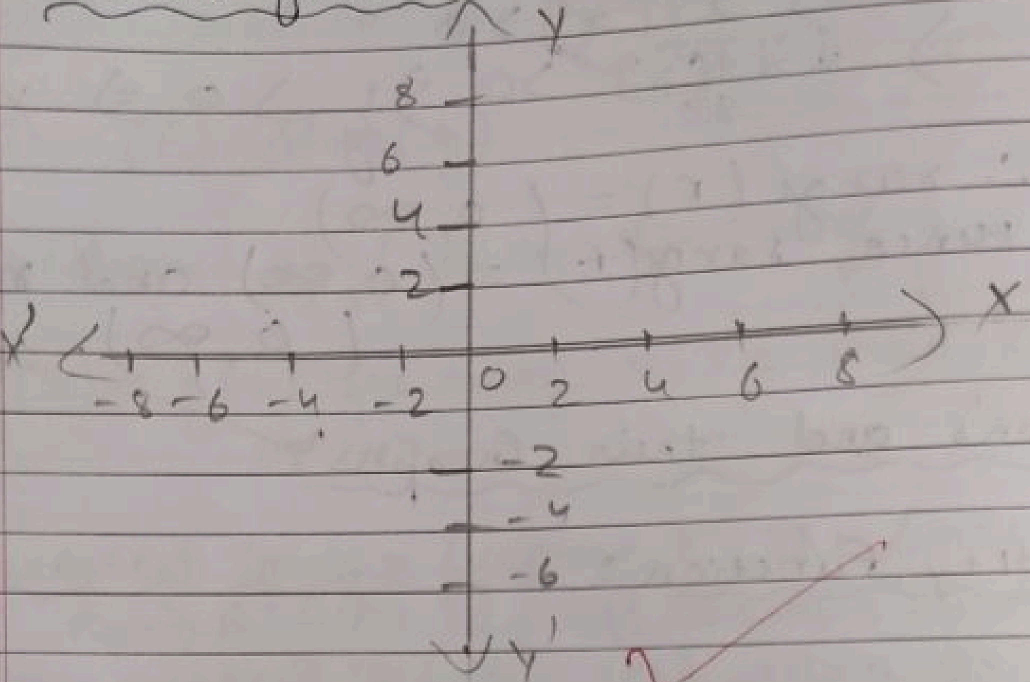
(i) Identity Function:



$f(x) = x$.

Let R be the set of real numbers; $f: R \rightarrow$ by $y = f(x) = x$ for each $x \in R$. Such function is known as identity function.

(ii) Constant function:

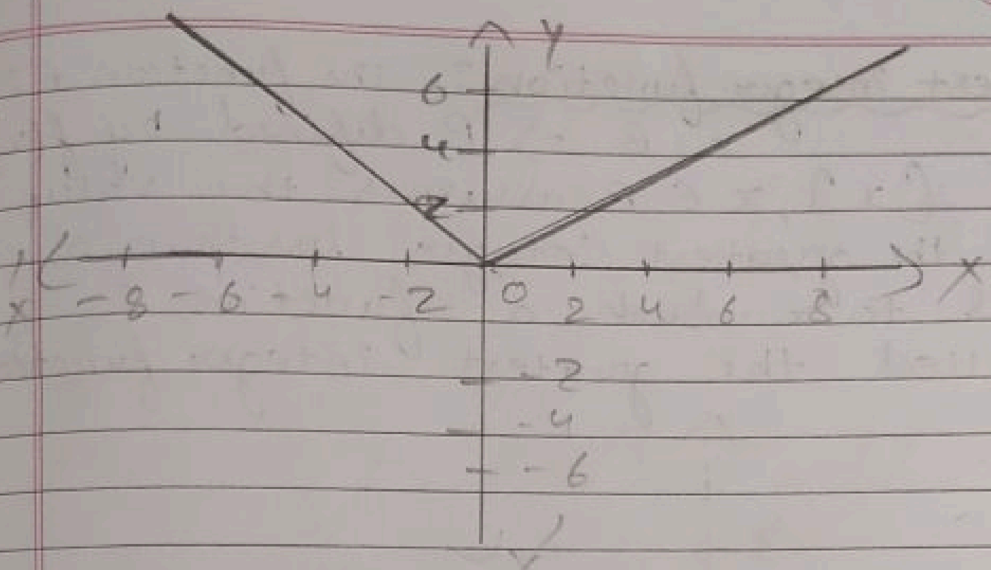


$R \to R$ by $y = f(x) = c$, $x \in R$, where c is a constant and each $x \in R$.

Note: Polynomial Function: $R \to R$ is said to be polynomial function if for each x in R, $y = f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$, where is n are non-negative integer.

(iii) The modulus Function: The function $R \to R$ is defined by $f(x) = [x]$ for each $x \in R$ is called modulus function.

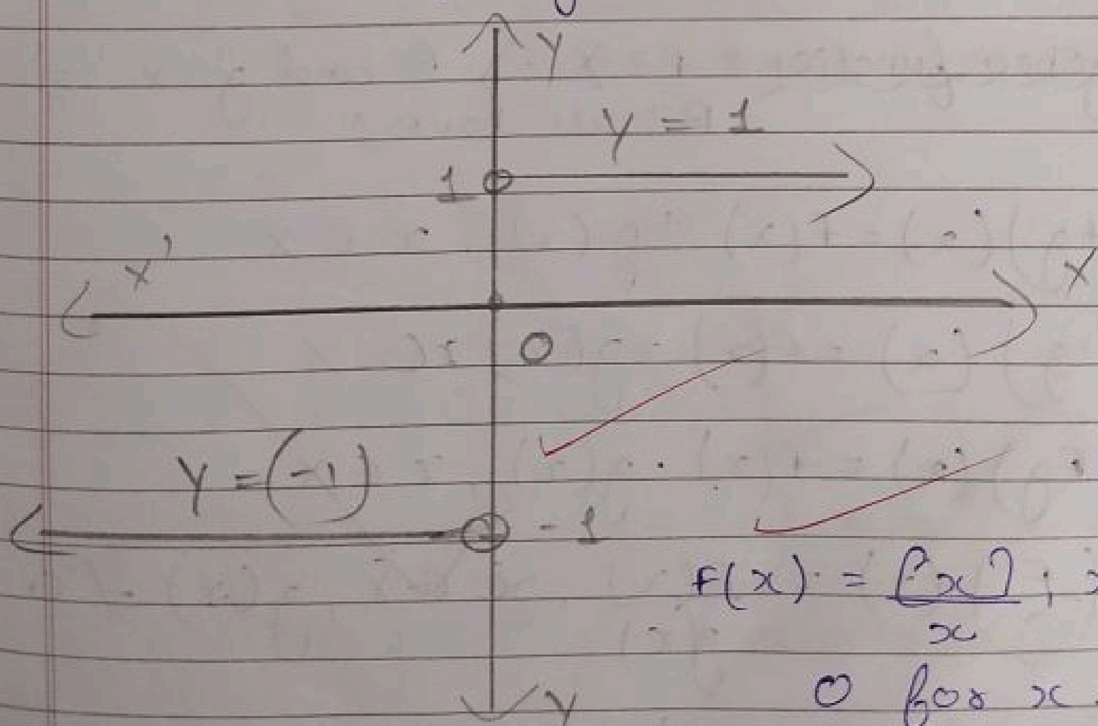
$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



(iv) Signum function: The function $f: R \to R$ is defined as.

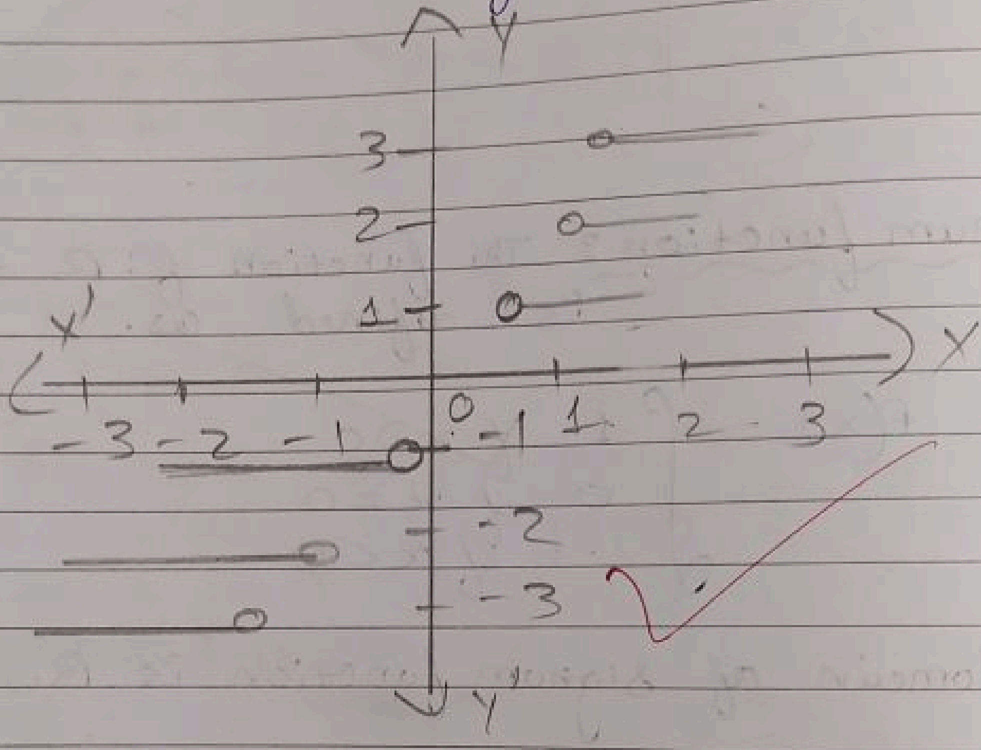
$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

→ The domain of signum function is R.



$F(x) = \frac{[x]}{x}$; x' 0 and 0 for $x = 0$.

(v) Greatest integer function: The function $f: R \to R$ defined by $f(x) = [x], x \in R$ assumes the value of the greatest integer less than or equal to x. Such a function is called the greatest integer function.



→ Algebra function: $f: X \to R$ and $g: X \to R$; we have:

- $(f+g)(x) = f(x) + g(x), x \in X$
- $(f-g)(x) = f(x) - g(x)$ $x \in X$
- $(f.g)(x) = f(x) \cdot g(x), x \in X$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, x \in X, g(x) \neq 0$
- $(kf)(x) = k(f(x)), x \in X$, where $k \in R$