Sets

Notes:

* Def:- A set is well defined collection of objects.
  - Sets are usually denoted by letter like A, B, C, D etc...
  - The objects of a set is called elements.
  - If 'a' is an element of set A. notationally $a \in A$
  - Sets are written within { }

Example:-
1. Set of Natural Numbers = "N"
2. Set of Integers = "Z or I"
3. Set of Real Numbers = "R"
4. Set of Rational Number = "Q"

↳ Representation of Sets:-
Sets are Represented in two ways:
1) Roster Form (listing method)
2) Set-Bulider method

↳ Roster form (listing method -

- Elements are listed

Ex:-

i) $A = \{1, 2, 3, 4\}$

ii) Set of all divisors of 40

$A = \{1, 2, 4, 5, 8, 10, 20, 40\}$

iii) set of all integers between -12 & 12.

$B = \{-11, -10 \ldots, 11\}$

iv) set of all letters of MATHEMATICS

$D = \{M, A, T, H, E, I, S\}$

Note:- ① Order of element in a set is immaterial

② Elements are Repeated.

↳ Set - Builder form:-

- sets are represented by using the properties of elements

Format: $\{x \mid x$ has some property of $P\}$

" | or : " Known or called as <u>such that</u>.

i) Set of all integer between -12 & 12

$C = \{x \in Z \mid -12 < x < 12\}$

ii) Set of all natural numbers between 1 & 101.

$D = \{x \in N \mid 1 < x < 101\}$

Types of Set :-

1. Empty Set :- A set with no element is call empty set.

Denoted By : $\{\ \}$ OR $\emptyset$

Example :-

i) Set of all integer between -1 and 0.

$C = \{\ \}$ OR $C = \emptyset$

2. Singleton Set :- A set with one elemen

Ex : $A = \{a\}$

3. Finite Set :- A set with finite number o element is called finite

Ex : ① Set of ~~all~~ first n natural number

$A = \{1, 2, 3 \ldots, n\}$

Ex : ② $\{n \in N \mid 2n^2 + 3n + \frac{36}{n} \in N\}$ is A finite ?

It is "Finite" set.

$A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

4. Infinite set: A set which is not finite is called infinite set.

Ex: N, Z, R, Q.

~~5. Equal set:~~

↳ Open Interval :- Let $a, b \in R$, $a < b$. Then set of the $(a, b) = \{x \in R \mid a < x < b\}$

Ex: (i) $(0, 1) = \{x \in R \mid 0 < x < 1\}$

$R = (\infty, \infty)$

(ii) Set of all Positive Number $= (0, \infty) = R^+$

↳ Close Interval :- Let $a, b \in R$, $a < b$. Then set of the $[a, b] = \{x \in R \mid a \le x \le b\}$

Subsets: A set A is said to be a subset of a set B if every element of A is also an element of B.

Sign of subset: "$\subset$"
we read it as "A is a subset of B if a is an element of A.

Ex: $A = \{a, b, c\}$

$B = \{a, b, c, d, e, f\}$

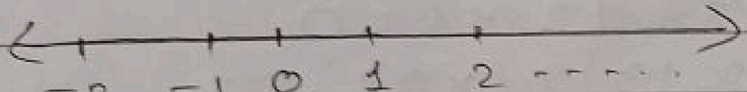
Here, A is subset of B Because B contains the elements of A.
i.e. $A \subset B$

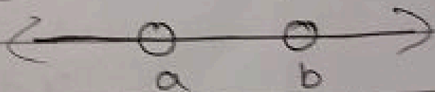
<u>Notes</u>: B is the superset of A.

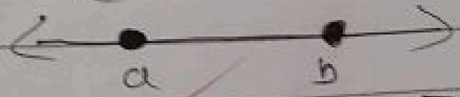
Q. How to Find the Number of sets?
Ex: $A = \{1, 2, 3, 4\}$ write all the subsets of A.

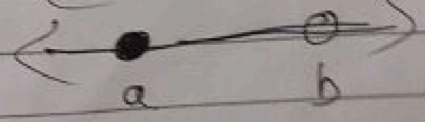
Subsets of A are: $\phi, \{1\}, \{2\}, \{3\}, \{4\}$
$\{1,2\}, \{2,3\}, \{3,4\}, \{4,1\}$ $\{4,2\}$ $\{4,4\}$,
$\{2,4\}, \{1,3\}, \{1,4\}, \{4,3\}, \{1,2,3,4\}$
By using Formula $= 2^n$ [where n is the number of elements in set]

- <u>Intervals</u>:

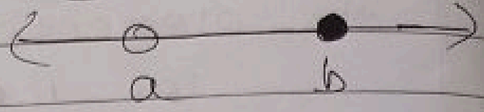


- $x \in (0,1)$ = open Interval $[0 < x < 1]$
Figure to illustrate:


- $x \in [0,1]$ = closed Interval $[0 \leq x \leq 1]$
Figure to illustrate:


- $x \in [0,1)$ = semi opened $[0 \leq x < 1]$
Figure to illustrate:


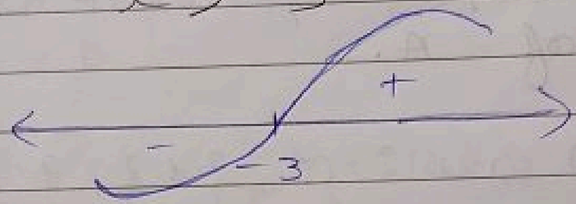
- $x \in (0, 1]$ = semi opened $[\, 0 < x \leq 1 \,]$
  figure to illustrate:



- Wavy curve method:

Ex: 1. $x + 3 > 0$

$x > -3$



$x \in (-3, \infty)$

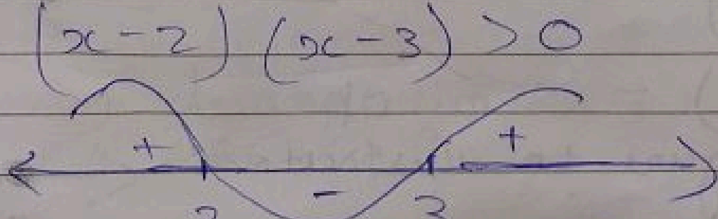
2. $x^2 (-5x + 6) > 0$

$x^2 + 5x + 6 = 0$
$x^2 + (3+2)x + 6 = 0$
$x^2 + 3x + 2x + 6 = 0$
$x(x+3) 2(x+3) = 0$

$(x+2) \quad (x+3)$

$x + 2 = 0 \qquad x + 3 = 0$
$x = (-2) \qquad x = (-3)$

$(x-2)(x-3) > 0$



$x \ (-\infty, 2)$

- Operation on Sets :

- Union of sets : Symbol used = "$\cup$"

Example : $A = \{2, 4, 6, 8\}$
$B = \{6, 8, 10, 12\}$
Find $A \cup B$.

$A \cup B = \{2, 4, 6, 8, 10, 12\}$
Here, the repeated elements are taken once only.

- Intersection of sets : Symbol used = "$\cap$"

Example : $A = \{2, 4, 6, 8, 10, 12, 14\}$
$B = \{8, 10, 12, 14, 16\}$
Find $A \cap B$.

$A \cap B = \{8, 10, 12, 14\}$
Here, we only take the common element which are common in both the sets.

Properties of Union :-

- $A \cup B = B \cup A$ [Commutative law]
- $(A \cup B) \cup C = A \cup (B \cup C)$ [Associative law]
- $A \cup \emptyset = A$ [Law of identity element]
- $A \cup A = A$ [Idempotent law]
- $U \cup A = U$ [Law of U]

Properties of intersection:

- $A \cap B = B \cap A$ — commutative law
- $(A \cap B) \cap C = A \cap (B \cap C)$ — Associative law
- $\phi \cap A = \phi$, $U \cap A = A$ — Law of ϕ and U
- $A \cap A = A$ — Idempotent law
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ — Distributive law
- Complement of set: Symbol used = " A' "

Example: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{1, 3, 5, 7, 9\}$

Find A'.

$A' = \{2, 4, 6, 8, 10\}$
Here, we write all the elements of U which doesn't belong in the set A.

- Properties of Complement of sets:
- Complement laws:
(i) $A \cup A' = U$ (ii) $A \cap A' = \phi$
- De morgan's law:
(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

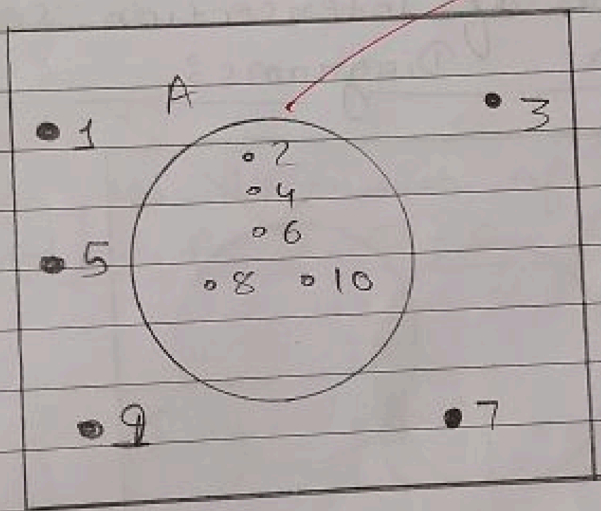
- law of double Complementation :

$$(A')' = A$$

- law of empty set and Universal set :

$$\phi' = U \quad \text{and} \quad U' = \phi$$

- Venn Diagram :-

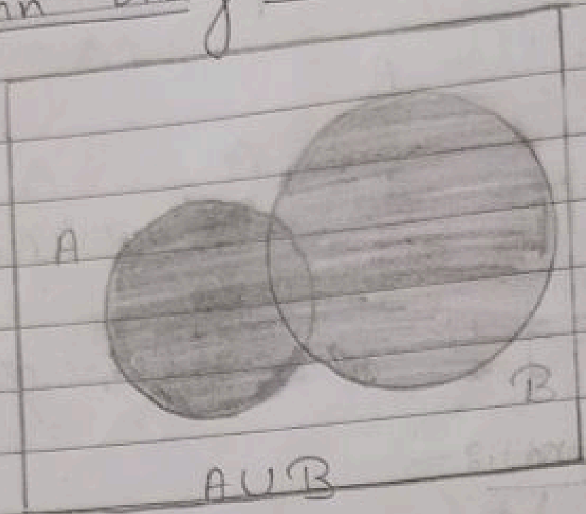


- Relationship between sets can be represented by means of diagrams known as Venn Diagram [Named after John Venn]

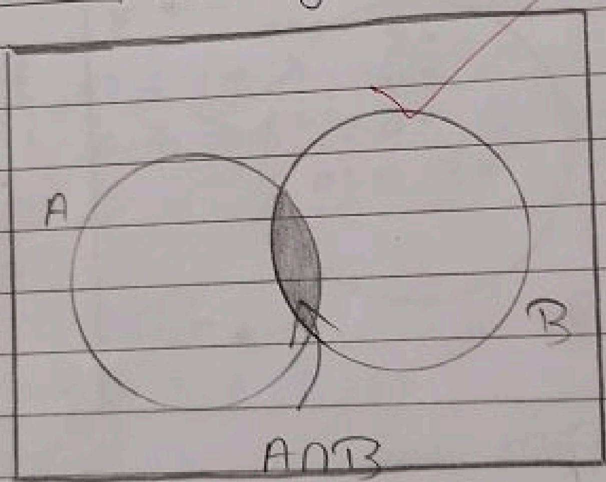
- These Diagrams consists of rectangles and closed curves usually circles.

- The Universal set is Represented usually by rectangle and its subsets by Circle.

- Reprenstation of Union sets by using Venn Diagrams:



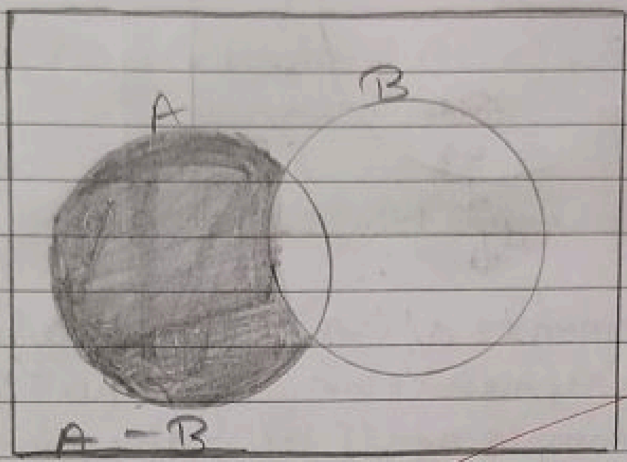
- Repsenstation of Intersection sets by using Venn Diagrams:



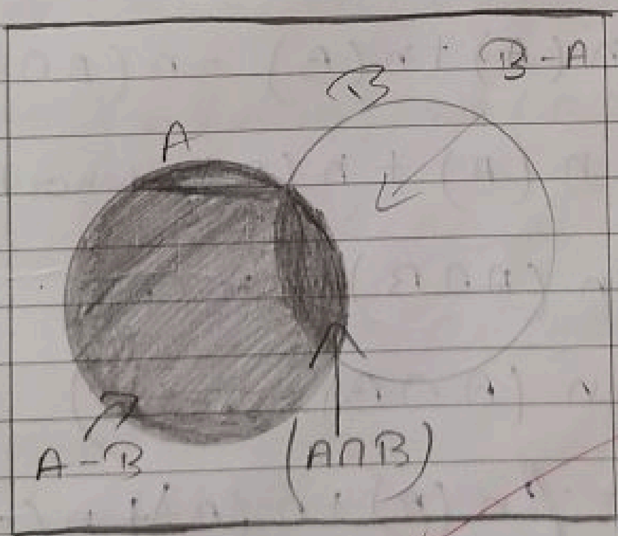
- **Difference of sets:** The difference of the set is defined as when there are two sets A and B if elements of A doesn't belong in B then;

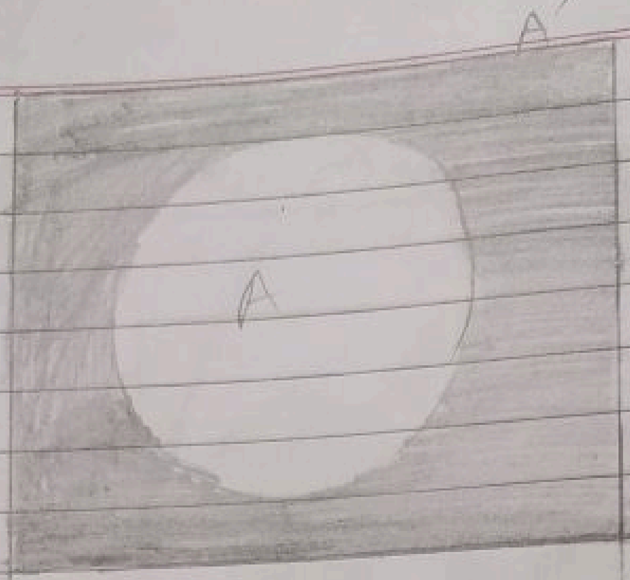
$$\boxed{A - B}$$

- Repsensting the differunce's of set's using venn Diagrams :



- Repsensting the differences of set's using venn Diagrams i.e. A - B , B - A.





The Complement of the Union of two sets is the intersection of their Complement and the Complement of the intersection of two sets is the Union of their Complements.

Named after :- De Morgan's law

Note:- If $A \cap B = \phi$ then, A and B are disjoint.

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
- $n(A \cup B) = n(A) + n(B)$; when $A \cap B = \phi$
- $n(A - B) + n(A \cap B) = n(A)$
- $n(B - A) + n(A \cap B) = n(B)$
- $n(A \cup B \cup C) = [n(A) + n(B) + n(C) + n(A \cap B \cap C)] - [n(A \cap B) + n(B \cap C) + n(A \cap C)]$

Q. For any sets A, B, C prove that;

$$n(A \cup B \cup C) = [n(A) + n(B) + n(C) + n(A \cap B \cap C)] - [n(A \cap B) + n(B \cap C) + n(A \cap C)].$$

Proof: we have;

$$n(A \cup B \cup C) = n[(A \cup B) \cup C]$$

$$= n(A \cup B) + n(C) - n[(A \cup B) \cap C]$$

$$= \{n(A) + n(B) - n(A \cap B)\} + n(C) - n[(A \cap C) \cup (B \cap C)]$$

$$= \{n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$= \{n(A) + n(B) + n(C) + n(A \cap B \cap C)\} - \{n(A \cap B) + n(B \cap C) + n(A \cap C)\}$$

∴ Proved.