Solution

TRIGONOMETRY MCQS

Class 11 - Mathematics

1.

(b) -1

Explanation: In quadrant IV, sin θ < 0.

$\sec\theta = \sqrt{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\therefore \sin^2\theta = \left(1 - \cos^2\theta\right) = \left(1 - \frac{1}{2}\right) = \frac{1}{2} \Rightarrow \sin\theta = \frac{-1}{\sqrt{2}}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = -1$, $\cot\theta = -1, \operatorname{cosec}\theta = -\sqrt{2}$

$\therefore \frac{(1+\tan\theta+\operatorname{cosec}\theta)}{(1+\cot\theta-\operatorname{cosec}\theta)} = \frac{(1-1-\sqrt{2})}{(1-1+\sqrt{2})} = \frac{-\sqrt{2}}{\sqrt{2}}$ = -1.

2.

(d) 75 cm

Explanation: $\theta = 42^\circ = \left(42 \times \frac{\pi}{180}\right)^c = \left(\frac{7\pi}{30}\right)^c$ and l = 55 cm.

$\therefore r = \frac{l}{\theta} = \left(55 \times \frac{30}{7\pi}\right) \text{cm} = \left(55 \times \frac{30}{7} \times \frac{7}{22}\right) \text{cm}$ = 75 cm.

3.

(b) $\frac{-1}{2}$

Explanation: cos 40^o + cos 80^o + cos 160^o + cos 240^o

= 2 cos $\left(\frac{40^\circ+80^\circ}{2}\right)$ cos $\left(\frac{40^\circ-80^\circ}{2}\right)$ +cos 160^o + cos (180^o + 60^o) [$\because$ cos A + cos B = 2 cos $\left(\frac{A+B}{2}\right)$ cos $\left(\frac{A-B}{2}\right)$]

= 2 cos 60^o cos (-20^o) + cos 160^o - $\frac{1}{2}$

= $2 \times \frac{1}{2}$ cos 20^o + cos 160^o - $\frac{1}{2}$

= cos (180^o - 20^o) + cos 20^o - $\frac{1}{2}$

= - cos 20^o + cos 20^0 -$\frac{1}{2}$

= -$\frac{1}{2}$

4.

(b) 9.5

Explanation: We have $\sin^2 5°$ + $\sin^2 10°$ + $\sin^2 15°$ + ... + $\sin^2 85°$ + $\sin^2 90°$

$\sin^2 5°$ + $\sin^2 10°$ + $\sin^2 15°$ + ... + $\sin^2 (90° - 10°)$ + $\sin^2 (90° - 5°)$ + $\sin^2 90°$

= $\sin^2 5°$ + $\sin^2 10°$ + $\sin^2 15°$ + ... + $\cos^2 10°$ + $\cos^2 5°$ + $\sin^2 90°$

= ($\sin^2 5°$ + $\cos^2 5°$) + ($\sin^2 10°$ + $\cos^2 10°$) + ($\sin^2 15°$ + $\cos^2 15°$)

+ ($\sin^2 20°$ + $\cos^2 20°$) + ($\sin^2 25°$ + $\cos^2 25°$) + ($\sin^2 30°$ + $\cos^2 30°$)

+ ($\sin^2 35°$ + $\cos^2 35°$) + ($\sin^2 40°$ + $\cos^2 40°$) + $\sin^2 45°$ + $\sin^2 90°$)

= 1 + 1 + 1 + 1 + 1 + 1 + 1 + $\left(\frac{1}{\sqrt{2}}\right)^2$ + $(1)^2$ $\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= 8 + \frac{1}{2} + 1$

= 9.5

5.

(c) 4

Explanation: Given to find the value of $\left(\cot\frac{x}{2} - \tan\frac{x}{2}\right)^2$(1 - 2 tan x cot 2x)

We will solve the expression in two parts,

Now solving 1^{st} term = $\left(\frac{1}{\tan\frac{x}{2}} - \tan\frac{x}{2}\right)^2$

= $\left(\frac{1}{\tan\frac{x}{2}} - \tan\frac{x}{2}\right)^2$

$= \left(\frac{1-\tan^2\frac{x}{2}}{\tan\frac{x}{2}}\right)^2$

If we multiply and divide the term by 2, we get,

$= \left(\frac{2\left(1-\tan^2\frac{x}{2}\right)}{2\tan\frac{x}{2}}\right)^2$

$= 2^2\left(\frac{1-\tan^2\frac{x}{2}}{2\tan\frac{x}{2}}\right)^2$ ... [using the formula for cot 2x = $\frac{1-\tan^2 x}{2\tan x}$ and cot x = $\frac{1}{\tan x}$]

$= 2^2\left(\frac{1}{\tan x}\right)^2$

$\left(\cot\frac{x}{2} - \tan\frac{x}{2}\right)^2 = \frac{4}{\tan^2 x}$ ... (i)

Solving the 2nd term

(1 - 2 tan x cot 2x) = 1 - 2 tan x$\left(\frac{1-\tan^2 x}{2\tan x}\right)$ ... [using the formula for cot 2x = $\left(\frac{1-\tan^2 x}{2\tan x}\right)$]

1 - 2 tan x cot 2x = 1 - (1 - $\tan^2$x)

= 1 - 1 + $\tan^2$x

1 - 2 tan x cot 2x = $\tan^2$ x ... (ii)

Now by combining (i) and (ii) we get,

$\left(\cot\frac{x}{2} - \tan\frac{x}{2}\right)^2$(1 - 2 tan x cot 2x) = $\frac{4}{\tan^2 x}$($\tan^2$x)

$\left(\cot\frac{x}{2} - \tan\frac{x}{2}\right)^2$(1 - 2 tan x cot 2x) = 4

6.

(c) r^2

Explanation: $(x^2 + y^2 + z^2) = r^2\cos^2\alpha\cos^2\beta + r^2\ \cos^2\alpha\sin^2\beta + r^2\sin^2\alpha$

$= r^2\cos^2\alpha\left(\cos^2\beta + \ \sin^2\beta\right) + r^2\sin^2\alpha$

$= r^2\cos^2\alpha + r^2\sin^2\alpha\ = r^2\left(\cos^2\alpha + \sin^2\alpha\right) = r^2$

7. **(a)** sin x

Explanation: 8sin$\frac{x}{8}$cos$\frac{x}{2}$cos$\frac{x}{4}$cos$\frac{x}{8}$

4(2 sin$\frac{x}{8}$cos$\frac{x}{8}$)cos$\frac{x}{2}$cos$\frac{x}{4}$ [by rearranging terms]

4(2 sin$\frac{x}{8}$cos$\frac{x}{8}$)cos$\frac{x}{2}$cos$\frac{x}{4}$ [using the formula sin 2θ = 2 sinθcosθ]

= 4(sin$\frac{x}{4}$)cos$\frac{x}{2}$cos$\frac{x}{4}$

= 2(2 sin$\frac{x}{4}$cos$\frac{x}{4}$)cos$\frac{x}{2}$

= 2(sin$\frac{2x}{4}$)cos$\frac{x}{2}$

= (2 sin$\frac{x}{2}$cos$\frac{x}{2}$)

= sin x

Hence sin$\frac{x}{8}$cos$\frac{x}{2}$cos$\frac{x}{4}$cos$\frac{x}{8}$ = sin x

8.

(c) $\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$

Explanation: tan 15° = tan(45° - 30°) $= \frac{\tan 45^\circ - \tan 30^\circ}{1+\tan 45^\circ\ \tan 30^\circ} = \frac{\left(1-\frac{1}{\sqrt{3}}\right)}{\left(1+\frac{1}{\sqrt{3}}\right)} = \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$

9.

(d) 0

Explanation: Since cos90° = 0

Thus cos 1° cos 2° cos 3°… cos90°… cos 179° = 0

10.

(b) -$\frac{b}{a}$

Explanation: Given: sin α + sin β = a ......(i)

cos α - cos β = b .....(ii)

Dividing (i) by (ii):

$\Rightarrow \frac{\sin\alpha+\sin B}{\cos\alpha-\cos B} = \frac{a}{b}$

$\Rightarrow \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)} = \frac{a}{b}$ [∵ sin A + sin B = 2 sin $\left(\frac{A+B}{2}\right)$ cos $\left(\frac{A-B}{2}\right)$ and cos A + cos B = -2 sin $\left(\frac{A+B}{2}\right)$ sin $\left(\frac{A-B}{2}\right)$]

$\Rightarrow \frac{\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{-\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)} = \frac{a}{b}$

$\Rightarrow \cot\left(\frac{\alpha-\beta}{2}\right) = -\frac{a}{b}$

$\Rightarrow \frac{1}{\cot\left(\frac{\alpha-\beta}{2}\right)} = \frac{1}{-\frac{a}{b}}$

$\Rightarrow \tan\left(\frac{\alpha-\beta}{2}\right) = -\frac{b}{a}$

11. **(a)** sin 10x sin 2x

Explanation: $\sin^2 6x - \sin^2 4x = \sin(6x + 4x)\sin(6x - 4x)$ [∵ $\sin^2 x - \sin^2 y = \sin(x + y)\sin(x - y)$]
= sin 10x sin 2x.

12.

(b) 2

Explanation: $\alpha + \beta = \frac{\pi}{4}$

taking tan both sides,

$\Rightarrow \tan(\alpha + \beta) = \tan\frac{\pi}{4}$

$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} = 1$

$\Rightarrow \tan\alpha + \tan\beta = 1 - \tan\alpha . \tan\beta$

$\Rightarrow \tan\alpha + \tan\beta + \tan\alpha . \tan\beta = 1$

On adding 1 both sides, we get

$\Rightarrow 1 + \tan\alpha + \tan\beta + \tan\alpha . \tan\beta = 1 + 1$

$\Rightarrow 1(1 + \tan\alpha) + \tan\beta(1 + \tan\alpha) = 2$

$\Rightarrow (1 + \tan\alpha)(1 + \tan\beta) = 2$

13.

(b) $\frac{1}{2}\cos 2x$

Explanation: $\cos^2\left(\frac{\pi}{6} + x\right) - \sin^2\left(\frac{\pi}{6} - x\right)$

$= \cos\left(\frac{\pi}{6} + x + \frac{\pi}{6} - x\right)\cos\left(\frac{\pi}{6} + x - \frac{\pi}{6} + x\right)$ [Using cos(A + B) cos(A - B) = $\cos^2 A - \sin^2 B$]

$= \cos\frac{2\pi}{6}\cos 2x$

$= \frac{1}{2}\cos 2x\left[\text{ As } \cos\frac{\pi}{3} = \frac{1}{2}\right]$

14. **(a)** $\frac{(2+\sqrt{3})}{2}$

Explanation: $2\sin\frac{5\pi}{12}\cos\frac{\pi}{12}$

$= \sin\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) + \sin\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$ [using 2sinASin B= sin (A + B) + sin (A - B)]

$= \left(\sin\frac{\pi}{2} + \sin\frac{\pi}{3}\right) = \left(1 + \frac{\sqrt{3}}{2}\right) = \frac{(2+\sqrt{3})}{2}$

15.

(c) $\frac{(\sqrt{3}-1)}{2\sqrt{2}}$

Explanation: sin 15° = sin (45° - 30°) = (sin 45° cos 30° - cos 45° sin 30°)

$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right) = \frac{(\sqrt{3}-1)}{2\sqrt{2}}$

16.

(b) 6.6 cm

Explanation: Angle traced by the minute hand in 60 min $= (2\pi)^c$

Angle traced by the minute hand in 45 min $= \left(\frac{2\pi}{60} \times 45\right)^c = \left(\frac{3\pi}{2}\right)^r$

∴ r = 1.4 cm and $\theta = \left(\frac{3\pi}{2}\right)^c$

$\Rightarrow l = r\theta = \left(1.4 \times \frac{3\pi}{2}\right)$ cm $= \left(1.4 \times \frac{3}{2} \times \frac{22}{7}\right)$ cm = 6.6 cm

17. **(a)** $\frac{\left(b^2-a^2\right)}{\left(b^2+a^2\right)}$

Explanation: On dividing num. and denom. by cos θ, we get

the given exp. $= \frac{b\tan\theta-a}{b\tan\theta+a} = \frac{\left(b\times\frac{b}{a}\right)-a}{\left(b\times\frac{b}{a}\right)+a} = \frac{\left(b^2-a^2\right)}{\left(b^2+a^2\right)}$

18. **(a)** $\left(\frac{5\pi}{36}\right)^c$

Explanation: $180^\circ = \pi^c \Rightarrow 1^\circ = \left(\frac{\pi}{180}\right)^c \Rightarrow 25^\circ = \left(\frac{\pi}{180}\times 25\right)^c = \left(\frac{5\pi}{36}\right)^c$

19. **(a)** -2

Explanation: $180^\circ = \pi^c \Rightarrow 1110^\circ = \left(\frac{\pi}{180}\times 1110\right)^c = \left(\frac{37\pi}{6}\right)^c$

$\therefore$ cosec (-1110°) = -cosec 1110° = - cosec $\frac{37\pi}{6}$

$= -\operatorname{cosec}\left(6\pi+\frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2 \ [\because \operatorname{cosec}(2n\pi+\theta) = \operatorname{cosec}\theta]$

20. **(a)** 0

Explanation: We have, sin (45° + θ) - cos (45°- θ)

= sin (45° + θ) - sin (90° - (45° - θ))

= sin (45° + θ) - sin (45° + θ)

= 0.

21.

(c) $\frac{(\sqrt{5}+1)}{4}$

Explanation: cos 36° = (1 - 2 sin^2 18°) $= \left\{1 - 2\cdot\frac{(\sqrt{5}-1)^2}{16}\right\} = \left\{1 - \frac{(6-2\sqrt{5})}{8}\right\} = \frac{(\sqrt{5}+1)}{4}$

22.

(c) 3

Explanation: Let y = 3 cos x + 4 sin x + 8

$\Rightarrow$ y - 8 = 3 cos x + 4 sin x

We know that, minimum value of A cos θ + B sin θ = $-\sqrt{(A^2+B^2)}$

$\Rightarrow$ y - 8 = $-\sqrt{\left(3^2+4^2\right)}$ = -$\sqrt{25}$ = -5

$\Rightarrow$ y = -5 + 8

Thus y= 3

23.

(b) cos 24°

Explanation: cos 24° = cos (90° - 66°) = sin 66°.

In quadrant I, sin θ is increasing

$\therefore$ sin 66° > sin 24° $\Rightarrow$ cos 24° > sin 24°

24.

(b) tan 37°

Explanation: $\frac{\cos 8^\circ-\sin 8^\circ}{\cos 8^\circ+\sin 8^\circ} = \frac{1-\tan 8^\circ}{1+\tan 8^\circ} = \frac{\tan 45^\circ-\tan 8^\circ}{1+\tan 45^\circ\tan 8^\circ} [\because 1 = \tan 45^\circ]$

= tan (45° - 8°) = tan 37°

25.

(b) $\frac{\sqrt{2-\sqrt{3}}}{2}$

Explanation: Since x lies in quadrant IV, we have: cos x > 0

Now, cos^{2}x = (1 - sin^{2}x) $= \left(1-\frac{1}{4}\right) = \frac{3}{4} \Rightarrow \cos x = +\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Now, $\frac{3\pi}{2} < x < 2\pi \Rightarrow \frac{3\pi}{4} < \frac{x}{2} < \pi \Rightarrow \frac{x}{2}$ lies in quadrant II.

$\therefore \sin\frac{x}{2} > 0$

2 sin^2 $\frac{x}{2}$ = (1 - cos x) $= \left(1-\frac{\sqrt{3}}{2}\right) = \frac{(2-\sqrt{3})}{2}$

$\Rightarrow \sin^2\frac{x}{2} = \frac{(2-\sqrt{3})}{4} \Rightarrow \sin\frac{x}{2} = \frac{\sqrt{2-\sqrt{3}}}{2}$

26. **(a)** $\frac{\sqrt{3}}{2}$

Explanation: Using (2 $\cos^2\theta$ - 1) = cos 2θ, we get

(2 $\cos^2 15° = \cos(2 \times 15^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

27.

(c) 2

Explanation: Given exp. $= \frac{\sin\theta\left[\sin^2\theta+(1+\cos\theta)^2\right]}{\sin\theta(1+\cos\theta)} = \frac{\left(\sin^2\theta+\cos^2\theta+1+2\cos\theta\right)}{(1+\cos\theta)}$

$= \frac{(2+2\cos\theta)}{(1+\cos\theta)} = \frac{2(1+\cos\theta)}{(1+\cos\theta)} = 2$

28.

(d) $\frac{-1}{16}$

Explanation: $\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$

= $\frac{1}{2\sin\frac{\pi}{5}}\ 2\sin\frac{\pi}{5}\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$

= $\frac{1}{2\sin\frac{\pi}{5}}\ \sin\frac{2\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$ (since 2sinAcos A = 2sinA)

= $\frac{1}{4\sin\frac{\pi}{5}}\ \sin\frac{4\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$ (since $2sin\frac{2\pi}{5}\cos\frac{2\pi}{5} = \sin\frac{4\pi}{5}$)

= $\frac{1}{8\sin\frac{\pi}{5}}\sin\frac{8\pi}{5}\cos\frac{8\pi}{5}$ (since $2sin\frac{4\pi}{5}\cos\frac{4\pi}{5} = \sin\frac{8\pi}{5}$)

= $\frac{\sin\frac{16\pi}{5}}{16\sin\frac{\pi}{5}} = \frac{\sin\left(3\pi+\frac{\pi}{5}\right)}{16\sin\frac{\pi}{5}}$ (since $2sin\frac{8\pi}{5}\cos\frac{8\pi}{5} = \sin\frac{16\pi}{5}$)

= $\frac{-\sin\frac{\pi}{5}}{16\sin\frac{\pi}{5}}$

= $-\frac{1}{16}$

29.

(c) HP

Explanation: Given

sin (B + C - A), sin (C + A - B) and sin (A + B - C) are in AP.

⇒ sin (C + A - B) - sin (B + C - A) = sin (A + B - C) - sin(C + A - B)

⇒ 2 sin $\left(\frac{C+A-B-B-C+A}{2}\right)$ cos $\left(\frac{C+A-B+B+C-A}{2}\right)$ = 2 sin $\left(\frac{A+B-C-C-A+B}{2}\right)$ cos $\left(\frac{A+B-C+C+A-B}{2}\right)$

⇒ sin (A - B) cos C = sin (B - C) cos A

⇒ sin A cos B cos C - cos A sin B cos C = sin B cos C cos A - cos B sin C cos A

⇒ 2 sin B cos A cos C = sin A cos B cos C + cos A cos B sin C

Dividing both sides by cos A cos B cos C:

2 tan B = tan A + tan C

⇒ Hence, cot A, cot B and cot C are in HP.

30.

(d) $\frac{1}{\sqrt{2}}$

Explanation: cos 405° = cos(360° + 45°) = cos 45° $= \frac{1}{\sqrt{2}}$

31.

(b) 194

Explanation: We have:

tan A + cot A = 4

Squaring both the sides:

$(\tan A + \cot A)^2 = 4^2$

⇒ $\tan^2 A + \cot^2 A + 2(\tan A)(\cot A) = 16$

⇒ $\tan^2 A + \cot^2 A + 2 = 16$

⇒ $\tan^2 A + \cot^2 A = 14$

Squaring both the sides again:

$(\tan^2 A + \cot^2 A)^2 = 14^2$

⇒ $\tan^4 A + \cot^4 A + 2(\tan^2 A)(\cot^2 A) = 196$

$\Rightarrow$ $\tan^4 A + \cot^4 A + 2 = 196$

$\Rightarrow$ $\tan^4 A + \cot^4 A = 194$

32. **(a)** $\left(\frac{9\pi}{10}\right)^c$

Explanation: $180^\circ = \pi^c \Rightarrow 1^\circ = \left(\frac{\pi}{180}\right)^c \Rightarrow 162^\circ = \left(\frac{\pi}{180} \times 162\right)^c = \left(\frac{9\pi}{10}\right)^c$

33.

(d) $\frac{-1}{\sqrt{2}}$

Explanation: cos 135° = cos (90° + 45°) = - sin 45° $= \frac{-1}{\sqrt{2}}$

34.

(b) cos 4x

Explanation: $\cos^4 x + \sin^4 x - 6\cos^2 x \sin^2 x = \cos^4 x + \sin^4 x - 2\cos^2 x \sin^2 x - 4\cos^2 x \sin^2 x$

$=(\cos^2 x - \sin^2 x)^2 - (2 \sin x \cos x)^2$

$=\cos^2 2x - \sin^2 2x$

=cos 4x

35.

(b) $\frac{(\sqrt{5}-1)}{4}$

Explanation: cos 72° = cos (90° - 18°) = sin 18° $= \frac{(\sqrt{5}-1)}{4}$

36.

(c) $\frac{1}{2}$

Explanation: Given cos x = $\frac{1}{2}\left(a + \frac{1}{a}\right)$ and cos 3x = $\lambda\left(\mathrm{a}^3 + \frac{1}{\mathrm{a}^3}\right)$

Consider the equation cos 3x = $\lambda\left(\mathrm{a}^3 + \frac{1}{\mathrm{a}^3}\right)$

Now take the LHS of the equation,

cos 3x = $4\cos^3 x - 3\cos x$ ... [using the formula for cos 3x = $4\cos^3 x - 3\cos x$]

From the question we know, cos x = $\frac{1}{2}\left(a + \frac{1}{a}\right)$

Substituting the known cos x values in the cos 3x expansion,

cos 3x = $4\left[\frac{1}{2}\left(a + \frac{1}{a}\right)\right]^3 - 3\left[\frac{1}{2}\left(a + \frac{1}{a}\right)\right]$

$= 4\left[\frac{1}{8}\left(a^3 + \frac{1}{a^3} + 3a\frac{1}{a}\left(a + \frac{1}{a}\right)\right)\right] - 3\left[\frac{1}{2}\left(a + \frac{1}{a}\right)\right]$

$= 4\left[\frac{1}{8}\left(a^3 + \frac{1}{a^3}\right) + \frac{3}{8}\left(a + \frac{1}{a}\right)\right] - 3\left[\frac{1}{2}\left(a + \frac{1}{a}\right)\right]$

$= 4\left[\frac{1}{8}\left(a^3 + \frac{1}{a^3}\right)\right] + \frac{3\times 4}{8}\left(a + \frac{1}{a}\right) - 3\left[\frac{1}{2}\left(a + \frac{1}{a}\right)\right]$

$= 4\left[\frac{1}{8}\left(a^3 + \frac{1}{a^3}\right)\right] + \frac{3}{2}\left(a + \frac{1}{a}\right) - \frac{3}{2}\left(a + \frac{1}{a}\right)$

$= 4\left[\frac{1}{8}\left(a^3 + \frac{1}{a^3}\right)\right]$

cos 3x = $\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$ ... (i)

If we compare the RHS of the cos 3x equation with the now derived equation (i) we get,

$\lambda\left(\mathrm{a}^3 + \frac{1}{\mathrm{a}^3}\right) = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$

From the here we can clearly say that $\lambda = \frac{1}{2}$

37.

(d) 1

Explanation: $\cot\left(\frac{29\pi}{4}\right) = \cot\left(7\pi + \frac{\pi}{4}\right) = \cot\frac{\pi}{4} = 1$ $[\because \cot(n\pi + \theta) = \cot\theta]$

38.

(c) 2

Explanation: $\sec\left(\frac{-19\pi}{3}\right) = \sec\frac{19\pi}{3}$ $[\because \sec(-\theta) = \sec\theta]$

$= \sec\left(6\pi + \frac{\pi}{3}\right) = \sec\frac{\pi}{3} = 2$ $[\because \sec(2n\pi + \theta) = \sec\theta]$

39.

(d) $\frac{220}{221}$

Explanation: $\sin\theta = \frac{15}{17} \Rightarrow \cos^2\theta = \left(1 - \sin^2\theta\right) = \left(1 - \frac{225}{289}\right) = \frac{64}{289}$
$\Rightarrow \cos\theta = \sqrt{\frac{64}{289}} = \frac{8}{17}$
$\cos\varphi = \frac{12}{13} \Rightarrow \sin^2\phi = \left(1 - \cos^2\phi\right) = \left(1 - \frac{144}{169}\right) = \frac{25}{169}$
$\Rightarrow \sin\phi = \sqrt{\frac{25}{169}} = \frac{5}{13}$
$\therefore \sin(\theta + \theta) = \sin\theta\cos\phi + \cos\theta\sin\phi$
$= \left(\frac{15}{17} \times \frac{12}{13}\right) + \left(\frac{8}{17} \times \frac{5}{13}\right) = \left(\frac{180}{221} + \frac{40}{221}\right) = \frac{(180+40)}{221} = \frac{220}{221}$

40. **(a)** 20 cm
Explanation: Here, arc length l = 15π cm
Angle $\theta = \frac{3\pi}{4}$
We know, angle subtended by the arc is given by,
$\theta = \frac{\text{length of arc}}{\text{radius}}$
$\therefore$ radius $= \frac{l}{\theta}$
$= \frac{15\pi}{3\pi} \times 4$
= 20 cm

41.
(c) [-1, 3]
Explanation: A = 2 $\sin^2$x - cos 2x
$= 2\sin^2 x - \left(1 - 2\sin^2 x\right)$
4$\sin^2$x - 1
$\because 0 \leq \sin^2 x \leq 1$
$\Rightarrow 4 \times 0 \leq 4 \times \sin^2 x \leq 4 \times 1$
$\Rightarrow 0 \leq 4\sin^2 x \leq 4$
$\Rightarrow 0 - 1 \leq 4\sin^2 x - 1 \leq 4 - 1$
$\Rightarrow$ -1 $\leq$ 4$\sin^2$x - 1 $\leq$ 3
$\Rightarrow$ -1 $\leq$ A $\leq$ 3
$\Rightarrow$ A $\in$ [-1, 3]

42.
(b) $-\frac{1}{2}$
Explanation: tan 135^o = tan(90^o + 45^o)
= -tan 45^o
= -1
or, tan(69^o + 66^o) $= \frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ}$
$\Rightarrow -1 = \frac{tan69^o + tan66^o}{1 - tan69^o tan66^o}$
$\Rightarrow$ tan 69^o + tan66^o - tan 69^o tan 66^o = -1
Therefore,
2k = -1
$\Rightarrow k = \frac{-1}{2}$

43.
(b) cot x
Explanation: Using cos C + cos D $= 2\cos\frac{(C+D)}{2}\cos\frac{(C-D)}{2}$
and sin C - sin D $= 2\cos\frac{(C+D)}{2}\ \sin\frac{(C-D)}{2}$, we get:
$\frac{\cos 6x + \cos 4x}{\sin 6x - \sin 4x} = \frac{2\cos\left(\frac{10x}{2}\right)\cos\left(\frac{2x}{2}\right)}{2\cos\left(\frac{10x}{2}\right)\sin\left(\frac{2x}{2}\right)} = \frac{\cos 5x\cos x}{\cos 5x\sin x} = \frac{\cos x}{\sin x}$ = cot x

44.
(b) 0
Explanation: We have, sin 50°- sin 70° + sin 10°

$= 2\cos\left(\frac{50^\circ+70^\circ}{2}\right)\sin\left(\frac{50^\circ-70^\circ}{2}\right)+\sin 10^\circ$

=2 cos 60° (-sin 10°) + sin 10°

$= -2 \times \frac{1}{2}\sin 10^\circ$ + sin 10^o

= -sin 10° + sin 10°

= 0

45. **(a)** 0

Explanation: cos 52^o + cos 68^o + cos 172^o

= 2 cos $\left(\frac{52^\circ+68^\circ}{2}\right)$ cos $\left(\frac{52^\circ-68^\circ}{2}\right)$ + cos 172^o [∵ cos A + cos B = 2 cos $\left(\frac{A+B}{2}\right)$ cos $\left(\frac{A-B}{2}\right)$]

= 2 cos 60^o cos(-8^o) + cos 172^o

$= 2 \times \frac{1}{2}$ cos 8^o + cos 172^o

= cos 8^o + cos 172^o

= 2 cos $\left(\frac{8^\circ+172^\circ}{2}\right)$ cos $\left(\frac{8^\circ-172^\circ}{2}\right)$

= 2 cos 90^o cos 82^o

= 0

46.

(c) $\sin(\alpha-\beta)=0$

Explanation: Given $\sin\alpha=\sin\beta$ and $\cos\alpha=\cos\beta$

Now $\sin\alpha\cdot\cos\beta$ = $\cos\alpha\cdot\sin\beta$

$\Rightarrow \sin\alpha\cdot\cos\beta-\cos\alpha\cdot\sin\beta$ = 0

$\Rightarrow \sin(\alpha-\beta)$ = 0

47.

(b) $\frac{23}{27}$

Explanation: sin 3x = (3 sin x - 4 $\sin^3$x) $=\left(3\times\frac{1}{3}-4\times\frac{1}{27}\right)=\left(1-\frac{4}{27}\right)=\frac{23}{27}$

48.

(c) $\frac{1}{2}$

Explanation: sin x cos x $=\frac{1}{2}\cdot$2sin x cos x = $\frac{1}{2}\cdot$ sin 2x

But the maximum value of sin 2x is 1.

So the maximum value of sin x cos x = $\frac{1}{2}$

49. **(a)** 1

Explanation: $(2^n+1)\theta=\pi$ Given)

$\Rightarrow 2^n\theta+\theta=\pi$

$\Rightarrow 2^n\theta=\pi-\theta$

$\Rightarrow \sin 2^n\theta=\sin(\pi-\theta)$

$\Rightarrow \sin 2^n\theta=\sin\theta$ .....(1)

$2^n\cos\theta\cos 2\theta\cos 2^2\theta\ldots\cos 2^{n-1}\theta = 2^n \times \frac{\sin 2^n\theta}{2^n sin\theta}$

$= \frac{\sin 2^n\theta}{\sin\theta}$

$= \frac{\sin\theta}{\sin\theta}$

= 1

50.

(b) $\frac{3}{16}$

Explanation: sin 20° sin 40° sin 60° sin 80°.

$= \frac{\sqrt{3}}{2}$ sin 20° sin (60° - 20°) sin (60° + 20°) (since sin 60° = $\frac{\sqrt{3}}{2}$)

$= \frac{\sqrt{3}}{2}$ sin 20° [$\sin^2$ 60° - $\sin^2$ 20°] { sin A + sin B = 2sin$\frac{A+B}{2}$ cos $\frac{A-B}{2}$ and sin A - Sin B = 2sin$\frac{A+B}{2}$ cos$\frac{A-B}{2}$

= $\sin^2$ 60° - $\sin^2$ 20° = (sin 60° + sin 20°)(sin 60° - sin 20°)

= (2sin40°cos20°)(2sin20°cos40°)

= (2sin40°cos40°) (2sin20°cos 20°) = sin80° sin 40°}

$= \frac{\sqrt{3}}{2}$ sin 20° [$\frac{3}{4}$ - $\sin^2$ 20°]

$= \frac{\sqrt{3}}{2} \times \frac{1}{4}$ [3sin 20° - 4 $\sin^3$ 20°]
$= \frac{\sqrt{3}}{2} \times \frac{1}{4}$ (sin 60°) (since sin 3θ = 3sin θ - 4 $\sin^3\theta$)
$= \frac{\sqrt{3}}{2} \times \frac{1}{4} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

51. **(a)** $(6\pi)^c$

Explanation: Number of revolutions made in 1 second $= \frac{180}{60}$ = 3

Angle turned in 1 revolution $= (2\pi)^c$

Angle turned in 3 revolutions $= (3 \times 2\pi)^c = (6\pi)^c$

52.

(c) $\frac{-15}{2}$

Explanation: In quadrant II, sin θ > 0, cos θ < 0. So, cosec θ > 0 and sec θ < 0.

$\operatorname{cosec}^2\theta = (1 + \cot^2\theta) = \left(1 + \frac{144}{25}\right) = \frac{169}{25} \Rightarrow \operatorname{cosec}\theta = \sqrt{\frac{169}{25}} = \frac{13}{5}$

$\cos^2\theta = (1 - \sin^2\theta) = \left(1 - \frac{25}{169}\right) = \frac{144}{169} \Rightarrow \cos\theta = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$

$\therefore \sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{-12}{13}$

$\therefore \frac{(1+\sin\theta-\cos\theta)}{(1-\sin\theta+\cos\theta)} = \frac{\left(1+\frac{5}{13}+\frac{12}{13}\right)}{\left(1-\frac{5}{13}-\frac{12}{13}\right)} = \left(\frac{30}{-4}\right) = \frac{-15}{2}$

53.

(b) 210°

Explanation: Here, radius of circular wire is r = 7 cm

So, length of wire = 2 $\times$ π $\times$ r

= 2 $\times$ π $\times$ 7

= 14 $\times$ π

Wire is cut and bent again into an arc of a circle of radius 12 cm.

So, length of arc=length of wire=14$\times$ π

We know, angle subtended by the arc is given by,

$\theta = \frac{\text{length of arc}}{\text{radius}}$

$= \frac{14\pi}{12}$

$= \frac{14\pi}{12} \times \frac{180^\circ}{\pi}$

= 210°

54.

(b) 2

Explanation: We have:

$\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9}$

$= \sin^2\frac{\pi}{18} + \sin^2\frac{2\pi}{18} + \sin^2\frac{7\pi}{18} + \sin^2\frac{8\pi}{18}$

$= \sin^2\frac{\pi}{18} + \sin^2\frac{2\pi}{18} + \sin^2\left(\frac{7\pi}{18}\right) + \sin^2\left(\frac{8\pi}{18}\right)$

$= \sin^2\frac{\pi}{18} + \sin^2\frac{2\pi}{18} + \sin^2\left(\frac{\pi}{2} - \frac{2\pi}{18}\right) + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{18}\right)$

$= \sin^2\frac{\pi}{18} + \sin^2\frac{2\pi}{18} + \cos^2\frac{2\pi}{18} + \cos^2\frac{\pi}{18}$

$= \sin^2\frac{\pi}{18} + \cos^2\frac{\pi}{18} + \sin^2\frac{2\pi}{18} + \cos^2\frac{2\pi}{18}$

= 1 + 1

= 2

55.

(b) $\frac{1}{2}$

Explanation: cos 50° cos 10° - sin 50° sin 10° = cos (50° + 10°) [$\because$ cos A cos B - sin A sin B = cos (A + B)]

= cos 60° $= \frac{1}{2}$

56.

(b) $-\sqrt{2}$

Explanation: $\operatorname{cosec}\left(\frac{-33\pi}{4}\right)=-\operatorname{cosec}\ \frac{33\pi}{4}=-\operatorname{cosec}\left(8\pi+\frac{\pi}{4}\right)$
$=-\operatorname{cosec}\frac{\pi}{4}=-\sqrt{2}\ \ [\because \operatorname{cosec}(2n\pi+\theta)=\operatorname{cosec}\theta]$

57. **(a)** 22 :13

Explanation: Let radius of two circles be the r_1 and r_2

Let θ_1 and θ_2 be the subtend angles of arcs of two circles

i.e. $\theta_1 = 65^o$ and $\theta_2 = 110^o$

We know that arc length,

$l = r \times \theta$

Here, arc lengths of two circles are same.

$\therefore r_1 \times \theta_1 = r_2 \times \theta_2$

$\therefore \frac{r_1}{r_2}=\frac{\theta_2}{\theta_1}=\frac{110}{65}$

$\therefore \frac{r_1}{r_2}=\frac{11\times 2}{13}$

$\therefore r_1 : r_2 = 22:13$

58.

(b) 75^o

Explanation: We know, in clock 1 rotation gives 360^o

i.e. 60 minutes = 360^o and 12 hours = 360^o

So, 1 minute = 6^o and 1 hour = 30^o

Now, For hour hand:

8 hours = $8 \times 30^o = 240^o$ and for another 30 minute (which is half of hour) = $\frac{30^\circ}{2}$ = 15^o

i.e. angle traced by hour hand is $240^o + 15^o = 255^o$

Now, For minute hand:

30 minute = $30 \times 6^o = 180^o$

i.e. angle traced by minute hand is 180^o.

So, the angle between hour hand and minute hand = $255^o - 180^o$

$= 75^o$

59.

(c) $\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

Explanation: $\sqrt{\frac{1+\sin x}{1-\sin x}}=\sqrt{\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{1+\cos\left(\frac{\pi}{2}+x\right)}}=\left\{\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right\}^{\frac{1}{2}}=\left\{\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\right\}^{\frac{1}{2}}=\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

60.

(b) $\frac{1}{4}$

Explanation: $\sin^2 x = (1 - \cos^2 x) = \left(1-\frac{15}{16}\right)=\frac{1}{16}$

When $\frac{\pi}{2}<x<\pi$, then x lies in quadrant II.

And sin x > 0 in quadrant II.

$\therefore \sin x=\sqrt{\frac{1}{16}}=\frac{1}{4}$

61.

(d) $\frac{-56}{65}$

Explanation: In quadrant IV, $\cos\theta>0, \sin\theta<0$, $\cos\phi>0$ and $\sin\phi<0$

Now, $\cos\theta=\frac{4}{5}\Rightarrow \sin^2\theta=\left(1-\cos^2\theta\right)=\left(1-\frac{16}{25}\right)=\frac{9}{25}$

$\Rightarrow \sin\theta=-\sqrt{\frac{9}{25}}=\frac{-3}{5}$

$\cos\phi=\frac{12}{13}\Rightarrow \sin^2\theta=\left(1-\cos^2\phi\right)=\left(1-\frac{144}{169}\right)=\frac{25}{169}$

$\Rightarrow \sin\phi=-\sqrt{\frac{25}{169}}=\frac{-5}{13}$

$\therefore \sin(\theta+\phi)=\sin\theta\cos\phi+\cos\theta\sin\phi$
$=\left(\frac{-3}{5}\times\frac{12}{13}\right)+\left(\frac{-5}{13}\times\frac{4}{5}\right)=\left(\frac{-36}{65}-\frac{20}{65}\right)=\frac{-56}{65}$

62. **(a)** 2
Explanation: Let C = 90°. Then, B = (90° - A).
$\sin^2A + \sin^2B + \sin^2C = \sin^2A + \sin^2(90° - A) + \sin^2 90°$
$= (\sin^2 A + \cos^2 A + 1) = 2.$

63.
(c) None of these
Explanation: Given ABC is a triangle, so ∠A + ∠B + ∠C = 180°
Now applying tan on both sides
tan (A + B + C) = tan (180°)
tan (A + B + C) = 0 ... (i)
Also given tan A + tan B + tan C = 0 ... (ii)
As per the formula of tan (A+B+C)
$\tan(A + B + C) = \frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$
Now,
$\tan(A + B + C) = \frac{0-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$ ... [from equation (i)]
$0 = \frac{-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$ ... [from equation (ii)]
By cross multiplying, we get
tan A tan B tan C = 0
$\implies \frac{1}{\tan A\tan B\tan C} = 0$
Hence cot A cot B cot C = 0

64.
(d) $\frac{1}{6}$
Explanation: In quadrant III, $\sin\theta < 0$
$\sin^2\theta=\left(1-\cos^2\theta\right)=\left(1-\frac{9}{25}\right)=\frac{16}{25}\Rightarrow\sin\theta=-\sqrt{\frac{16}{25}}=-\frac{4}{5}$
$\therefore\tan\theta=\left(\frac{-4}{5}\times\frac{5}{-3}\right)=\frac{4}{3}$ and $\cot\theta=\frac{3}{4}$
$\operatorname{cosec}\theta=\frac{-5}{4}$ and $\sec\theta=\frac{-5}{3}$
$\therefore\frac{(\operatorname{cosec}\theta+\cot\theta)}{(\sec\theta-\tan\theta)}=\frac{\left(\frac{-5}{4}+\frac{3}{4}\right)}{\left(\frac{-5}{3}-\frac{4}{3}\right)}=\frac{\left(\frac{-2}{4}\right)}{\left(\frac{-9}{3}\right)}=\frac{-2}{4}\times\frac{3}{-9}=\frac{1}{6}$

65.
(b) $\frac{1}{\sqrt{2}}$
Explanation: $\tan\theta+\cot\theta=2\Rightarrow\tan\theta+\frac{1}{\tan\theta}=2$
$\Rightarrow\tan^2\theta+1=2\tan\theta\Rightarrow1+\tan^2\theta-2\tan\theta=0$
$\Rightarrow(1-\tan\theta)^2=0\Rightarrow1-\tan\theta=0\Rightarrow\tan\theta=1\Rightarrow\theta=\frac{\pi}{4}$
$\therefore\sin\theta=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

66.
(c) 75°
Explanation: Angle traced by the hour hand in 12 hours = 360°
Angle traced by the hour hand in 3.5 hours $=\left(\frac{360}{12}\times3.5\right)^\circ$ = 105 °.
Angle traced by the minute hand in 60 min = 360°.
Angle traced by the minute hand in 30 min $=\left(\frac{360}{60}\times30\right)^\circ$ = 180°.
Angle between the two hands = (180° - 105°) = 75°.

67.
(b) 630°
Explanation: $\pi^c=180^\circ\Rightarrow1^c=\left(\frac{180}{\pi}\right)^\circ\Rightarrow11^c=\left(\frac{180}{\pi}\times11\right)^\circ=\left(180\times\frac{7}{22}\times11\right)^\circ$ = 630°.

68.

(b) $\frac{1}{2\sqrt{6}}$

Explanation: we know that $\cos^2 x$ = (1 - $\sin^2 x$) $= \left(1 - \frac{24}{25}\right) = \frac{1}{25} \Rightarrow \cos x = \frac{-1}{5}$ [In quadrant III, cos x is negative]

$\therefore \cot x = \frac{\cos x}{\sin x} = \frac{-1}{5} \times \frac{5}{-2\sqrt{6}} = \frac{1}{2\sqrt{6}}$

69.

(b) - 1

Explanation: $\sin(180 + \phi)(180 - \phi) \operatorname{cosec}^2 \phi = -\sin\phi \cdot \sin\phi \operatorname{cosec}^2 \phi = -\sin^2 \phi \operatorname{cosec}^2 \phi$ = -1

70.

(d) $\frac{-(\sqrt{3}+1)}{2\sqrt{2}}$

Explanation: $\cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4} = \cos\left(\pi - \frac{\pi}{3}\right)\cos\frac{\pi}{4} - \sin\left(\pi - \frac{\pi}{3}\right)\sin\frac{\pi}{4}$

$= -\cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(-\frac{1}{2} \times \frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right) = \frac{-(\sqrt{3}+1)}{2\sqrt{2}}$

71.

(c) $\frac{\sqrt{5}+1}{8}$

Explanation: Given that, $\cos^2 48°$ - $\sin^2 12°$ = cos (48° + 12°) cos (48° - 12°)

[$\because$ $\cos^2$ A - $\sin^2$ B = cos (A + B) cos (A – B)]

= cos 60° cos 36°

= $\frac{1}{2} \times \frac{\sqrt{5}+1}{4}$

= $\frac{\sqrt{5}+1}{8}$

72.

(b) $\frac{(\sqrt{3}+1)}{2\sqrt{2}}$

Explanation: cos 15° = cos (45° - 30°) = cos 45°cos 30° + sin 45° sin 30°

$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right) = \frac{(\sqrt{3}+1)}{2\sqrt{2}}$

73.

(c) $-\sqrt{3}$

Explanation: $\tan\left(\frac{-16\pi}{3}\right) = -\tan\frac{16\pi}{3}$ [$\because \tan(-\theta) = -\tan\theta$]

$= -\tan\left(5\pi + \frac{\pi}{3}\right) = -\tan\frac{\pi}{3} = -\sqrt{3}$ [$\because \tan(n\pi + \theta) = \tan\theta$]

74.

(c) 1

Explanation: Given exp. $= \left(\tan\frac{\pi}{20}\tan\frac{9\pi}{20}\right)\left(\tan\frac{3\pi}{20}\tan\frac{7\pi}{20}\right)\tan\frac{\pi}{4}$

$= \tan\frac{\pi}{20}\tan\left(\frac{\pi}{2} - \frac{\pi}{20}\right)\tan\frac{3\pi}{20}\tan\left(\frac{\pi}{2} - \frac{3\pi}{20}\right)\tan\frac{\pi}{4}$

$= \tan\frac{\pi}{20}\cot\frac{\pi}{20}\tan\frac{3\pi}{20}\cot\frac{3\pi}{20} \times 1 = 1$.

75.

(d) $-\sqrt{3}$

Explanation: $\tan\left(\frac{-25\pi}{3}\right) = -\tan\frac{25\pi}{3} = -\tan\left(8\pi + \frac{\pi}{3}\right)$

$= -\tan\frac{\pi}{3} = -\sqrt{3}$ [$\because \tan(2n\pi + \theta) = \tan\theta$]