Question Bank

LIMITS AND DERIVATIVES

Class 11 - Mathematics

1. The value of $\lim_{x\to\infty} \frac{(x+1)^{10}+(x+2)^{10}+\ldots+(x+100)^{10}}{x^{10}+10^{10}}$ is: **[1]**

   a) 100 b) 10

   c) 10^{10} d) None of these

2. $\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos x}$ is equal to **[1]**

   a) 1 b) 0

   c) -1 d) does not exit

3. If f (x) = $\sqrt{1-x^2}$, x ∈ (0, 1), then f'(x), is equal to **[1]**

   a) $\sqrt{1-x^2}$ b) $\sqrt{x^2-1}$

   c) $\frac{1}{\sqrt{1-x^2}}$ d) $\frac{-x}{\sqrt{1-x^2}}$

4. If y = $\sin^{-1}$ x and z = $\cos^{-1}\sqrt{1-x^2}$, then $\frac{dy}{dz}$ = **[1]**

   a) -1 b) $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$

   c) 1 d) 0

5. $\lim_{x\to\pi} \frac{\sin x}{x-\pi}$ is equal to **[1]**

   a) 1 b) -1

   c) 2 d) -2

6. $\lim_{x\to\infty} \left(\sqrt{x^2+x+1}-x\right)$ is equal to **[1]**

   a) $\frac{1}{2}$ b) 2

   c) 0 d) -1

7. $\lim_{x\to\pi/4} \frac{\sqrt{2}\cos x-1}{\cot x-1}$ is equal to **[1]**

   a) 1 b) $\frac{1}{2}$

   c) $\frac{1}{\sqrt{2}}$ d) $\frac{1}{2\sqrt{2}}$

8. If $y=\sqrt{x}+\frac{1}{\sqrt{x}}$, then $\frac{dy}{dx}$ at x = 1 is equal to **[1]**

   a) $\frac{1}{\sqrt{2}}$ b) 1

   c) 0 d) $\frac{1}{2}$

9. $\frac{d}{dx}\left(\sec^{-1}x\right)$ is equal to **[1]**

   a) $\frac{1}{1+x^2}$ b) $\frac{1}{3x\sqrt{x^2-x}}$ for |x| > 1

c) $\frac{-1}{x\sqrt{x^2-1}}$ for |x| > 1

d) $\frac{1}{|x|\sqrt{x^2-1}}$ for |x| > 1

10. If G(x) = $\sqrt{25-x^2}$ then $\lim_{x\to 1}\frac{G(x)-G(1)}{x-1}$ has the value **[1]**

a) $\frac{1}{24}$

b) $-\sqrt{24}$

c) $\frac{-1}{\sqrt{24}}$

d) $\frac{1}{5}$

11. $\frac{d}{dx}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}\left(\frac{x}{a}\right)\right)$ is equal to **[1]**

a) 1 + x^2

b) $2\sqrt{a^2-x^2}$

c) $\frac{1}{\log x}$

d) $\sqrt{a^2-x^2}$

12. $\lim_{x\to 0}\left(\frac{\tan x-x}{x}\right)\sin\left(\frac{1}{x}\right)$ is equal to **[1]**

a) 1

b) a real number other than 0 and 1

c) -1

d) 0

13. $\lim_{x\to 2}\frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$ is equal to **[1]**

a) $\frac{1}{8\sqrt{3}}$

b) $8\sqrt{3}$

c) $\sqrt{3}$

d) $\frac{1}{\sqrt{3}}$

14. $\lim_{n\to\infty}\frac{1+2+3+\ldots+n}{n^2}, n\in\mathbf{N}$, is equal to **[1]**

a) $\frac{1}{2}$

b) 0

c) $\frac{1}{4}$

d) 1

15. If f(x) = x^{100} + x^{99} ... + x + 1, then f'(1) is equal to: **[1]**

a) 5049

b) 50051

c) 5050

d) 5051

16. $\lim_{x\to\infty}\frac{\sin x}{x}$ = **[1]**

a) None of these

b) 1

c) ∞

d) 0

17. $\lim_{x\to 3}\frac{\sqrt{x^2+10}-\sqrt{19}}{x-3}$ is equal to **[1]**

a) 1

b) $\frac{6}{\sqrt{19}}$

c) $\frac{3}{\sqrt{19}}$

d) 0

18. If y = $\sqrt{x+\sqrt{x+\sqrt{x+\ldots+to\infty}\infty}}$ then $\frac{dy}{dx}$ = **[1]**

a) $\frac{1}{2y+1}$

b) $\frac{1}{2y-1}$

c) $\frac{x}{y+1}$

d) $\sqrt{\frac{x}{y+1}}$

19. Derivative of tan $\sqrt{x^2+1}$ w.r.t $\sqrt{x^2+1}$ is **[1]**

a) $\frac{2x}{\sqrt{1+x^2}}$

b) $\sec^2$ x

c) $\sec^2\sqrt{x^2+1}$

d) $\sec^2\left(\frac{x}{\sqrt{x^2+1}}\right)$

20. $\lim_{\theta \to 0} \frac{1-\cos 4\theta}{1-\cos 6\theta}$ is equal to **[1]**

a) 4/9 b) 1/2

c) -1 d) -1/2

21. The function, f(x) = (x - a)^2 $\cos \frac{1}{x-a}$ for x $\neq$ 0 and f(a) = 0, is **[1]**

a) continuous but not derivable at x = 0 b) derivable at x = a

c) not continuous at x = a d) neither continuous nor derivable at x = a

22. $\frac{d}{dx}\left(\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right)\right)$ is equal to **[1]**

a) $\frac{1}{\sqrt{x^2+a^2}}$ b) $\sqrt{x^2+a^2}$

c) $\frac{1}{x+\sqrt{x^2+a^2}}$ d) $\sqrt{x^2-a^2}$

23. If α is a repeated root of ax^2 + bx + c = 0, then $\lim_{x \to \alpha} \frac{\tan(ax^2+bx+c)}{(x-\alpha)^2}$ is **[1]**

a) b b) c

c) 0 d) a

24. If f(x) = $\int \frac{x+\sin x}{x+\cos x}dx$, then $\lim_{x \to \infty} f'(x)$ = **[1]**

a) none of these b) 1

c) ∞ d) 0

25. $\lim_{x \to 3} \frac{\sum_{r=1}^{n} x^r - \sum_{r=1}^{n} 3^r}{x-3}$ is equal to **[1]**

a) $\frac{(2n-1)\times 3^n}{4}$ b) $\frac{(2n-1)\times 3^n-1}{4}$

c) (2n - 1) 3^n + 1 d) $\frac{(2n-1)\times 3^n+1}{4}$

26. The value of $\lim_{x \to \pi/2}(\sec x - \tan x)$ is **[1]**

a) -1 b) 2

c) 1 d) 0

27. $\frac{d}{dx}\left(\cos^{-1}\left(\sqrt{1-x^2}\right)\right)$ is equal to **[1]**

a) $\frac{1}{\sqrt{1-x^2}}$ for 0 < |x| < 1 b) $\frac{2}{\sqrt{4-(2x)^2}}$ for |x| < 1

c) $\frac{1}{\sqrt{1+x^2}}$ for |x| < 1 d) $\frac{x}{|x|\sqrt{1-x^2}}$ for 0 <|x| < 1

28. $\lim_{x \to 0} \frac{\tan x}{\log(1+x)}$ is equal to **[1]**

a) 1 b) does not exist

c) 0 d) log 2

29. $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$ is equal to: **[1]**

a) 0 b) 1

c) $\frac{1}{2}$ d) 2

30. $\lim_{x \to 1} \frac{(\sqrt{x}-1)(2x-3)}{2x^2+x-3}$ is equal to: **[1]**

a) 1 b) $\frac{1}{10}$

c) None of these d) $\frac{-1}{10}$

31. $\lim_{x\to 0} \frac{\sin x^n}{(\sin x)^m}$, n > m > 0 is equal to **[1]**

a) $\frac{m}{n}$ b) 0

c) 1 d) $\frac{n}{m}$

32. If $f(x) = \frac{x^n - a^n}{x-a}$ for some constant, a, then f'(a) is equal to **[1]**

a) 1/2 b) does not exist

c) 1 d) 0

33. $\lim_{x\to\infty} \frac{\sqrt{x^2-1}}{2x+1}$ is equal to: **[1]**

a) 1 b) $\frac{1}{2}$

c) 0 d) -1

34. $\lim_{x\to 0} \frac{|\sin x|}{x}$ is **[1]**

a) None of these b) -1

c) 1 d) 0

35. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at x = $\frac{1}{2}$ is **[1]**

a) 2 b) 4

c) 1 d) -2

36. If y = 1 + $\frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$ + ..., then $\frac{dy}{dx}$ = **[1]**

a) y^2 b) y + 1

c) y d) y - 1

37. f(x) = |[x] x| in -1 $\leq$ x $\leq$ 2 is **[1]**

a) continuous at x = 0 b) discontinuous at x = 0

c) continuous at x = 2 d) differentiable at x = 0

38. $\lim_{x\to\pi/4} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$ is equal to **[1]**

a) None of these b) $5\sqrt{2}$

c) $3\sqrt{2}$ d) $\sqrt{2}$

39. If $\begin{matrix} \frac{\sin[x]}{[x]}, [x] \neq 0 \\ 0 \quad , [x] = 0 \end{matrix}$, where [.] denotes the greatest integer function, then $\lim_{x\to 0} f(x)$ is equal to **[1]**

a) is equal to -1 b) does not exist

c) is equal to 0 d) is equal to 1

40. $\lim_{x\to 0+} \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$ is equal to **[1]**

a) does not exist b) 2

c) 0 d) 8

41. $\lim_{x\to 0}\frac{8}{x^8}\left\{1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right\}$ is equal to **[1]**

a) $-\frac{1}{32}$ b) $\frac{1}{32}$

c) $-\frac{1}{16}$ d) $\frac{1}{16}$

42. $\lim_{x\to 0}\frac{\sin x^0}{x}$ is equal to **[1]**

a) x b) $\frac{\pi}{180}$

c) 1 d) π

43. If $f(x)=\begin{cases}\frac{\sin[x]}{[x]}, & [x]\neq 0\\ 0 & ,[x]=0\end{cases}$, where $[\cdot]$ denotes the greatest integer function, then $\lim_{x\to 0} f(x)$ is equal to **[1]**

a) None of these b) -1

c) 0 d) 1

44. $\lim_{x\to 0}\frac{1-\cos 2x}{x}$ is **[1]**

a) 0 b) 1

c) 4 d) 2

45. $\frac{d^2}{dx^2}\left(\cos^{-1}(1-x)\right)$ is equal to **[1]**

a) $\frac{1}{1-x^2}$ b) $\frac{1-x}{(2x-x^2)^{3/2}}$

c) $\frac{x-1}{(2x-x^2)^{3/2}}$ d) $\frac{1}{2(2x-x^2)^{3/2}}$

46. $\lim_{x\to\pi/3}\frac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x-1}$ is equal to **[1]**

a) $\sqrt{3}$ b) $\frac{1}{2}$

c) $\frac{1}{\sqrt{3}}$ d) $\sqrt{5}$

47. $\lim_{x\to 0}\frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$ is equal to **[1]**

a) 1 b) 0

c) 2 d) -1

48. $\frac{d}{dx}\left(\tan^{-1}\left(\frac{2}{x^{-1}-x}\right)\right)$ is equal to **[1]**

a) $\sqrt{1-x^2}$ b) $\frac{2}{1+x^2}, x\neq 0,\pm 1$

c) $\frac{1}{1+x^2}, x\neq 0,\pm 1$ d) $\frac{-2}{\sqrt{1-x^2}}$

49. $\lim_{x\to 0}\frac{\sin x}{x(1+\cos x)}$ is equal to **[1]**

a) $\frac{1}{2}$ b) 0

c) 1 d) -1

50. If y = log x, then y_n = **[1]**

a) $\frac{(-1)^n\cdot n!}{x^n}$ b) $\frac{(-1)^n\cdot n!}{x^{n+1}}$

c) $\frac{(-1)^{n-1}(n-1)!}{x^n}$ d) $\frac{(-1)^n(n-1)!}{x^n}$

51. If k be an integer, then $\lim_{x\to k^-}(x-[x])$ is equal to **[1]**

a) 0

b) 1

c) k

d) -1

52. $\lim_{n\to\infty} \frac{1^2+2^2+3^2+\ldots+n^2}{n^3}$ is equal to: **[1]**

a) 0

b) $\frac{1}{3}$

c) $\frac{1}{4}$

d) $\frac{1}{2}$

53. $\lim_{n\to\infty} \left\{\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots+\frac{1}{(2n+1)(2n+3)}\right\}$ is equal to **[1]**

a) $\frac{1}{2}$

b) 0

c) 2

d) $-\frac{1}{2}$

54. $\lim_{x\to 1} \frac{\sin \pi x}{x-1}$ is equal to **[1]**

a) $\frac{1}{\pi}$

b) π

c) $-\pi$

d) $-\frac{1}{\pi}$

55. $\lim_{x\to 0} \frac{x^2 \cos x}{1-\cos x}$ is equal to **[1]**

a) 2

b) -3/2

c) 3/2

d) 1

56. The value of $\lim_{n\to\infty} \frac{n!}{(n+1)!-n!}$ is **[1]**

a) None of these

b) -1

c) 0

d) 1

57. The value of $\lim_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$ is: **[1]**

a) 0

b) 1

c) None of these

d) -1

58. $\begin{cases} x & , \ 0 \le x < 1 \\ 3-x & , \ 1 \le x \le 2 \end{cases}$, then at x = 1, f (x) is **[1]**

a) continuous on left and discontinuous on right

b) continuous

c) continuous but not derivable

d) not continuous

59. $\lim_{x\to 0} x \sin \frac{1}{x}$ is equals to **[1]**

a) 1

b) does not exist

c) 1/2

d) 0

60. $\lim_{x\to \frac{\pi}{2}} (\sec x)^{\cot x}$ equals: **[1]**

a) 0

b) e^2

c) 1

d) -1

61. $\lim_{x\to 0} \frac{\tan 2x-x}{3x-\sin x}$ is equal to **[1]**

a) $\frac{1}{2}$

b) 2

c) $\frac{1}{4}$

d) $-\frac{1}{2}$

62. $\lim_{n\to\infty} \frac{n!}{(n+1)!+n!}$ is equal to **[1]**

a) 1

b) 2

c) 0

d) $\frac{1}{2}$

63. Maximum value of x^3 - 3x + 2 in [0, 2] is **[1]**

a) 32

b) 4

c) 1

d) 2

64. If y = log $\left(x+\sqrt{1+x^2}\right)$ then $\frac{d^2y}{dx^2}$ = **[1]**

a) $\frac{x}{(x^2+1)^{3/2}}$

b) $\frac{1}{\sqrt{x^2+1}}$

c) $\frac{-2x}{(x^2+1)^{3/2}}$

d) $\frac{-x}{(x^2+1)^{3/2}}$

65. $\frac{d}{dx}\left(\sin^{-1}\left(\sqrt{1-x^2}\right)\right)$ is equal to **[1]**

a) $-\frac{x}{\sqrt{1-x^2}}$ for 0 < |x| < 1

b) $-\frac{1}{\sqrt{1-x^2}}$ for |x| < 1

c) $1 + x^2$

d) $-\frac{x}{|x|\sqrt{1-x^2}}$ for 0 < |x| < 1

66. $\lim_{x\to\frac{\pi}{4}} \frac{\sec x-\sqrt{2}}{x-\frac{\pi}{4}}$ is equal to **[1]**

a) -1

b) 0

c) $\sqrt{2}$

d) $\sqrt{3}$

67. $\frac{d}{dx}\left(\cos^{-1}\left(\frac{x^{-1}-x}{x^{-1}+x}\right)\right)$ is equal to **[1]**

a) $\frac{4}{1+x^2}$, x > 0

b) $\frac{2}{1+x^2}$, x > 0

c) $\frac{-2}{\sqrt{1+x^2}}$

d) $1 - x^2$

68. If f(x) = x - [x], $\in$ R then $f'\left(\frac{1}{2}\right)$ is equal to **[1]**

a) -1

b) 1

c) $\frac{3}{2}$

d) 0

69. $\lim_{x\to\frac{\pi}{4}} \frac{\tan x-1}{x-\frac{\pi}{4}}$ is equal to **[1]**

a) 1

b) $\frac{1}{2}$

c) 0

d) 2

70. If y = $\frac{1-x}{1+x}$ then y_6 = **[1]**

a) $\frac{6!}{(1+x)^7}$

b) $\frac{(-2).(7!)}{(1+x)^6}$

c) $\frac{(-2).(6!)}{(1+x)^7}$

d) $\frac{(2).(6!)}{(1+x)^7}$

71. f(x) = $\begin{cases} x^3, & |x| \le 1 \\ x\ , & |x| > 1 \end{cases}$ then f(x) is **[1]**

a) not continuous at -1 and 1

b) not continuous at x = 0

c) derivable at all x $\in$ R

d) not derivable at – 1 and 1

72. The value of $\lim_{x\to\infty} \frac{\sqrt{1+x^4}+(1+x^2)}{x^2}$ is: **[1]**

a) 2 b) -1

c) None of these d) 1

73. If $y = \frac{\sin x+\cos x}{\sin x-\cos x}$, then $\frac{dy}{dx}$ at x = 0 is equal to **[1]**

a) 0 b) -2

c) $\frac{1}{2}$ d) Does not exist

74. $\lim_{x\to 0} \frac{x}{\tan x}$ = **[1]**

a) 4 b) 0

c) 1 d) None of these

75. $\lim_{x\to 0} \frac{(1-\cos 2x)\sin 5x}{x^2 \sin 3x}$ **[1]**

a) $\frac{5}{6}$ b) $\frac{3}{10}$

c) $\frac{6}{5}$ d) $\frac{10}{3}$