Solution

LIMITS AND DERIVATIVES

Class 11 - Mathematics

1. **(a)** 100

   Explanation: $\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+\ldots+(x+100)^{10}}{x^{10}+10^{10}}$

   Dividing N^r and D^r by x^{10}

   $\Rightarrow \lim_{x\to\infty}\frac{\left(1+\frac{1}{x}\right)^{10}+\left(1+\frac{2}{x}\right)^{10}+\ldots+\left(1+\frac{100}{x}\right)^{10}}{1+\frac{10^{10}}{x^{10}}}$

   = 1 + 1 + 1 + ... + 100 times

   = 100

2. **(b)** 0

   Explanation: $\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\cos x}=\lim_{y\to 0}\frac{1-\sin(\frac{\pi}{2}-y)}{\cos(\frac{\pi}{2}-y)}\quad \text{taking } \frac{\pi}{2}-x=y$

   $=\lim_{y\to 0}\frac{1-\cos y}{\sin y}=\lim_{y\to 0}\frac{2\sin^2\frac{y}{2}}{2\sin\frac{y}{2}\cos\frac{y}{2}}$

   $=\lim_{y\to 0}\tan\frac{y}{2}=0$

3. **(d)** $\frac{-x}{\sqrt{1-x^2}}$

   Explanation: f(x) = $\sqrt{1-x^2}$

   f'(x) = $\frac{1}{2\sqrt{1-x^2}}-2x$ = $\frac{-x}{\sqrt{1-x^2}}$

4. **(c)** 1

   Explanation: We have;

   sin y = x and cos z = $\sqrt{1-x^2}$ ⇒ cos z = cos y

   ⇒ z = y

   ⇒ $\frac{dy}{dz}$ = 1

5. **(b)** -1

   Explanation: Given, $\lim_{x\to\pi}\frac{\sin x}{x-\pi}=\lim_{x\to\pi}\frac{\sin(\pi-x)}{-(\pi-x)}$

   $=-1\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1 \text{ and } \pi-x\to 0\Rightarrow x\to\pi\right]$

6. **(a)** $\frac{1}{2}$

   Explanation: Substitute x = $\frac{1}{t}$

   $\Rightarrow \lim_{t\to 0}\frac{\sqrt{t^2+t+1}-1}{t}$

   Using L' Hospital

   $\lim_{t\to 0}\frac{\frac{2t+1}{2\sqrt{t^2+t+1}}}{1}$

   = $\frac{1}{2}$

7. **(b)** $\frac{1}{2}$

   Explanation: $\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos x-1}{\cot x-1}$

   Rationalising the numerator, we get;

   $=\lim_{x\to\frac{\pi}{4}}\left(\frac{\sqrt{2}\cos x-1}{\cot x-1}\right)\times\left(\frac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x+1}\right)$

$= \lim_{x \to \frac{\pi}{4}} \frac{(2\cos^2 x - 1)}{(\cos x - \sin x)} \times \frac{\sin x}{(\sqrt{2}\cos x + 1)}$

$= \lim_{x \to \frac{\pi}{4}} \frac{(\cos^2 x - \sin^2 x)}{(\cos x - \sin x)} \times \frac{\sin x}{(\sqrt{2}\cos x + 1)}$

$= \lim_{x \to \frac{\pi}{4}} \frac{(\cos x + \sin x)\sin x}{(\sqrt{2}\cos x + 1)}$

$= \frac{\left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\sin\frac{\pi}{4}}{\left(\sqrt{2}\cos\frac{\pi}{4} + 1\right)}$

$= \frac{\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)}{\sqrt{2} \cdot \frac{1}{\sqrt{2}} + 1}$

$= \frac{\left(\frac{2}{\sqrt{2}}\right) \times \frac{1}{\sqrt{2}}}{2}$

$= \frac{1}{2}$

8.

(c) 0

Explanation: Given that $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}$

$\left(\frac{dy}{dx}\right)_{\text{at } x=1} = \frac{1}{2} - \frac{1}{2} = 0$

9.

(d) $\frac{1}{|x|\sqrt{x^2-1}}$ for |x| > 1

Explanation: y = $\sec^{-1}$ x = sec y = x

⇒ y' = cot y cos y

⇒ $\frac{1}{|x|\sqrt{x^2-1}}$ for |x| > 1

10.

(c) $\frac{-1}{\sqrt{24}}$

Explanation: The equation is in the form of $\frac{0}{0}$

Using L' Hospital rule we have $\frac{\frac{1}{2\sqrt{25-x^2}} \cdot (-2x)}{1}$

Substituting x = 1 we get $\frac{-1}{\sqrt{24}}$

11.

(b) $2\sqrt{a^2 - x^2}$

Explanation: y' = $(x) \cdot \left(\frac{-2x}{2\sqrt{a^2-x^2}}\right) + (\sqrt{a^2 - x^2})$ + (a^2) $\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \cdot \left(\frac{1}{a}\right)$

⇒ $2\sqrt{a^2 - x^2}$

12.

(d) 0

Explanation: $\lim_{x \to 0} \left(\frac{\tan x - x}{x}\right) \sin\left(\frac{1}{x}\right)$

⇒ (0) Finite number = 0

13. **(a)** $\frac{1}{8\sqrt{3}}$

Explanation: $\because \lim_{x \to 2} \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2} = \lim_{x \to 2} \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2} \times \frac{\sqrt{1+\sqrt{2+x}} + \sqrt{3}}{\sqrt{1+\sqrt{2+x}} + \sqrt{3}}$

$= \lim_{x \to 2} \frac{\sqrt{2+x} - 2}{(x-2)(\sqrt{1+\sqrt{2+x}} + \sqrt{3})}$

$= \lim_{x \to 2} \frac{\sqrt{2+x} - 2}{(x-2)(\sqrt{1+\sqrt{2+x}} + \sqrt{3})} \times \frac{\sqrt{2+x} + 2}{\sqrt{2+x} + 2}$

$= \lim_{x \to 2} \frac{(x-2)}{(x-2)(\sqrt{1+\sqrt{2+x}} + \sqrt{3})(\sqrt{2+x} + 2)}$

$= \lim_{x \to 2} \frac{1}{(\sqrt{1+\sqrt{2+x}}+\sqrt{3})(\sqrt{2+x}+2)}$

$= \frac{1}{(\sqrt{1+\sqrt{2+2}}+\sqrt{3})(\sqrt{2+2}+2)}$

$= \frac{1}{2\sqrt{3}\times 4}$

$= \frac{1}{8\sqrt{3}}$

14. **(a)** $\frac{1}{2}$

Explanation: $\lim_{n \to \infty} \frac{1+2+3+\ldots+n}{n^2}$

$= \lim_{n \to \infty} \frac{n(n+1)}{2n^2} = \lim_{n \to \infty} \frac{1}{2}\left(1+\frac{1}{n}\right) = \frac{1}{2}$

15.

(c) 5050

Explanation: Given, f(x) = x^{100} + x^{99} ... + x + 1

$\therefore$ f'(x) = $100x^{99}$ + $99x^{98}$ ... + x + 1

So, f'(1) = 100 + 99 + 98 + ... + 1

= $\frac{100}{2}$[2 $\times$ 100 + (100 - 1)(-1)]

= 50[200 - 99] = 50 $\times$ 101

= 5050

16.

(d) 0

Explanation: $\lim_{x \to \infty} \frac{\sin x}{x}$

Let $x = \frac{1}{y}$

$x \to \infty$

$\therefore y \to 0$

$= \lim_{y \to 0} \frac{\sin \frac{1}{y}}{\frac{1}{y}}$

$= \lim_{y \to 0} y \sin \frac{1}{y}$

$= \lim_{y \to 0} y \times \lim_{y \to 0} \sin \frac{1}{y}$

$= 0 \times \lim_{y \to 0} \sin \frac{1}{y}$

= 0

17.

(c) $\frac{3}{\sqrt{19}}$

Explanation: Using L'Hospital,

$\lim_{x \to 3} \frac{\frac{2x}{2\sqrt{x^2+10}}}{1}$

Substituting x = 3 in $\frac{\frac{2x}{2\sqrt{x^2+10}}}{1}$

We get $\frac{3}{\sqrt{19}}$

18.

(b) $\frac{1}{2y-1}$

Explanation: y = $\sqrt{(x+y)}$

y^2 = x + y

2yy' = 1 + y'

19.

(c) $\sec^2 \sqrt{x^2+1}$

Explanation: Let $\sqrt{x^2+1}$ = t

Now we have to find $\frac{d(\tan t)}{dt}$

$\Rightarrow$ $\sec^2$ t = $\sec^2 \sqrt{x^2+1}$

20. **(a)** 4/9

Explanation: Given, $\lim\limits_{\theta \to 0} \frac{1-\cos 4\theta}{1-\cos 6\theta} = \lim\limits_{\theta \to 0} \frac{2\sin^2 2\theta}{2\sin^2 3\theta}$ $\left[\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right]$

$= \lim\limits_{\theta \to 0} \frac{\sin^2 2\theta}{\sin^2 3\theta} = \lim\limits_{\theta \to 0} \left[\frac{\sin 2\theta}{\sin 3\theta}\right]^2$

$= \lim\limits_{\substack{\theta \to 0 \\ 2\theta \to 0 \\ 3\theta \to 0}} \left[\frac{\frac{\sin 2\theta}{2\theta} \times 2\theta}{\frac{\sin 3\theta}{3\theta} \times 3\theta}\right]^2 = \left[\frac{2\theta}{3\theta}\right]^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

21.

(b) derivable at x = a

Explanation: Substitute x - a = t; then the function will become

$\Rightarrow \lim\limits_{t \to 0} t^2 \cos\frac{1}{t}$

$\Rightarrow$ 0

Finite number = 0

f(a) = 0

22.

(b) $\sqrt{x^2+a^2}$

Explanation: y' = $\left(\frac{x}{2}\right) \cdot \left(\frac{2x}{2\sqrt{x^2+a^2}}\right) + \left(\sqrt{x^2+a^2}\right) \cdot \left(\frac{1}{2}\right) + \left(\frac{a^2}{2}\right) \left[\frac{1+\left(\frac{2x}{2\sqrt{x^2+a^2}}\right)}{x+\sqrt{x^2+a^2}}\right]$

$\Rightarrow \sqrt{x^2+a^2}$

23.

(d) a

Explanation: $\lim\limits_{x \to \alpha} \left[\frac{\tan(ax^2+bx+c)}{(x-\alpha)^2}\right]$

$= \lim\limits_{x \to \alpha} \left[\frac{\tan\{a(x-\alpha)(x-\alpha)\}}{a(x-\alpha)^2}\right] \times a$

$= \lim\limits_{x \to \alpha} \left[\frac{\tan\left\{a(x-\alpha)^2\right\}}{a(x-\alpha)^2}\right] \times a$

$= 1 \times a \left[\lim\limits_{\theta \to 0} \left(\frac{\tan\theta}{\theta}\right) = 1\right]$

= a

24.

(b) 1

Explanation: Substitute x with $\frac{1}{t}$

$\Rightarrow \lim\limits_{t \to 0} \frac{1+t\sin\frac{1}{t}}{1+t\cos\frac{1}{t}}$

= 1

25.

(d) $\frac{(2n-1)\times 3^n+1}{4}$

Explanation: $\lim\limits_{x \to 3} \frac{\sum_{r=1}^{n} x^r - \sum_{r=1}^{n} 3^r}{x-3}$

$= \lim\limits_{x \to 3} \frac{x^1+x^2+x^3+\ldots\ldots+x^n-(3^1+3^2+3^3\ldots\ldots3^n)}{x-3}$

$= \lim\limits_{x \to 3} \frac{x-3}{x-3} + \frac{x^2-3^2}{x-3} + \frac{x^3-3^3}{x-3} + \ldots. \frac{x^n-3^n}{x-3}$

$= 1 + 2\times 3 + 3 \times 3^2 + \ldots\ldots + n3^{n-1} \left[\because \frac{x^n-a^n}{x-a} = na^{n-1}\right]$

This series is an AGP of the form given below:

S = 1 + 2r + 3r^2 .... nr^{n-1}

$S = \frac{1-r^n}{(1-r)^2} - \frac{nr^n}{1-r}$

r = 3, a = 1, d = 1

$= \frac{1-3^n+2n3^n}{4}$
$= \frac{3^n(2n-1)+1}{4}$

26.
(d) 0
Explanation: $\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$
$= \lim_{h \to 0} \left(\sec\left(\frac{\pi}{2} - h\right) - \tan\left(\frac{\pi}{2} - h\right)\right)$
$= \lim_{h \to 0} (\text{cosec h} - \cot h)$
$= \lim_{h \to 0} \frac{1-\cos h}{\sin h}$
$= \lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}}{\sin h}$
$= \lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}}{2\sin \frac{h}{2} \cos \frac{h}{2}}$
$= \lim_{h \to 0} \tan \frac{h}{2}$
= 0

27.
(d) $\frac{x}{|x|\sqrt{1-x^2}}$ for 0 <|x| < 1
Explanation: Substitute x = sin θ; $\frac{dx}{d\theta}$ = cos θ
$\Rightarrow$ y = θ
$\Rightarrow \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \sec \theta$
$\Rightarrow \frac{x}{|x|\sqrt{1-x^2}}$ for 0 < |x| < 1

28. **(a)** 1
Explanation: $\lim_{x \to 0} \frac{\tan x}{\log(1+x)} \cdot \frac{x}{x}$
$\Rightarrow \lim_{x \to 0} \frac{\tan x}{x} \cdot \frac{1}{\frac{\log(1+x)}{x}}$
$\Rightarrow$ 1·1 = 1

29.
(c) $\frac{1}{2}$
Explanation: $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$
= $\lim_{x \to 0} \frac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{(\sqrt{1+x}+1)x}$
= $\lim_{x \to 0} \frac{1+x-1}{x\sqrt{1+x+1}}$
= $\lim_{x \to 0} \frac{x}{x(\sqrt{1+x}+1)}$
= $\frac{1}{2}$

30.
(d) $\frac{-1}{10}$
Explanation: Given, $\lim_{x \to 1} \frac{(\sqrt{x}-1)(2x-3)}{2x^2+3x-2x-3}$
= $\lim_{x \to 1} \frac{(\sqrt{x}-1)(2x-3)}{x(2x+3)-1(2x+3)} = \lim_{x \to 1} \frac{(\sqrt{x}-1)(2x-3)}{(x-1)(2x+3)}$
= $\lim_{x \to 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)(2x-3)}{(x-1)(\sqrt{x}+1)(2x+3)} = \lim_{x \to 1} \frac{(x-1)(2x-3)}{(x-1)(\sqrt{x}+1)(2x+3)}$
= $\lim_{x \to 1} \frac{2x-3}{(\sqrt{x}+1)(2x+3)}$
Taking limit we have
= $\frac{2(1)-3}{(\sqrt{1}+1)(2\times1+3)} = \frac{-1}{2\times5} = \frac{-1}{10}$

31.
(b) 0
Explanation: $\lim_{x \to 0} \frac{\sin x^n}{(\sin x)^m} \cdot \frac{x^{m+n}}{x^{m+n}}$

$\Rightarrow \lim_{x\to 0} \frac{\sin x^n}{x^n} \cdot \frac{x^m}{(\sin x)^m} \cdot \frac{x^n}{x^m}$
$\Rightarrow 1.1^m . x^{n-m}$
$\Rightarrow$ 1(0) = 0

32.

(b) does not exist

Explanation: Given $f(x) = \frac{x^n - a^n}{x-a}$

$f'(x) = \frac{(x-a)(n \cdot x^{n-1}) - (x^n - a^n) \cdot 1}{(x-a)^2}$

$\therefore \quad f'(a) = \frac{(a-a)(n \cdot a^{n-1}) - (a^n - a^n)}{(a-a)^2}$

So $f'(a) = \frac{0}{0}$ = does not exist

33.

(b) $\frac{1}{2}$

Explanation: $\lim_{x\to\infty} \frac{\sqrt{x^2-1}}{2x+1}$

= $\lim_{x\to\infty} \frac{\sqrt{1-\frac{1}{x^2}}}{2+\frac{1}{x}}$

Dividing the numerator and the denominator by x, we get;

= $\frac{\sqrt{1-0}}{2+0}$

= $\frac{1}{2}$

34. **(a)** None of these

Explanation: We have,

$|\sin x| = \begin{cases} \sin x, & 0 \le x \le \frac{\pi}{2} \\ -\sin x, & -\frac{\pi}{2} \le x < 0 \end{cases}$

Now,

$\lim_{x\to 0^-} \frac{|\sin x|}{x} = \lim_{x\to 0} \frac{-\sin x}{x} = -\lim_{x\to 0} \frac{\sin x}{x} = -1$

$\lim_{x\to 0^+} \frac{|\sin x|}{x} = \lim_{x\to 0} \frac{\sin x}{x} = 1$

Clearly,

$\lim_{x\to 0^-} \frac{|\sin x|}{x} \ne \lim_{x\to 0^+} \frac{|\sin x|}{x}$

$\therefore \lim_{x\to 0} \frac{|\sin x|}{x}$ does not exist.

35.

(b) 4

Explanation: Substitute x = cos θ

$\Rightarrow \frac{d(2\theta)}{d(\sin\theta)} = \frac{\frac{d(2\theta)}{dt}}{\frac{d(\sin\theta)}{dt}}$

$\Rightarrow$ 2 sec θ

$\Rightarrow$ 2(2) = 4

36.

(c) y

Explanation: y = 1 + $\frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$ + ...

Differentiating both sides with respect to x, we get $\frac{dy}{dx} = \frac{d}{dx}\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^2}{3!} + \dots\right)$

= $\frac{d}{dx}(1) + \frac{d}{dw}\left(\frac{x}{11}\right) + \frac{d}{dw}\left(\frac{x^2}{2!}\right) + \frac{d}{dw}\left(\frac{x^3}{3!}\right) + \frac{d}{dx}\left(\frac{x^4}{4!}\right)$ + ...

= $\frac{d}{dx}(1) + \frac{1}{1!}\frac{d}{dx}(x) + \frac{1}{2!}\frac{d}{dw}(x^2) + \frac{1}{3!}\frac{d}{dw}(x^3) + \frac{1}{4!}\frac{d}{dw}(x^4)$ + ...

= 0 + $\frac{1}{1!} \times$ 1 + $\frac{1}{2!} \times 2\alpha + \frac{1}{3!} \times 3\alpha^2 + \frac{1}{4!} \times 4\alpha^3$ + ... (y = $\alpha^2 \Rightarrow \frac{dy}{\partial\alpha}$ = nα^{n-1})

= 1 + $\frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$ + ... $\left[\frac{x}{n!} = \frac{1}{(n-1)!}\right]$

= y

$\therefore \frac{dy}{dx}$ = y

37. **(a)** continuous at x = 0

Explanation: f(x) = $\begin{cases} -x & -1 \le x < 0 \\ 0 & 0 \le x < 1 \\ x & 1 \le x \le 2 \end{cases}$

Function is continuous at x = 0 but discontinuous at x = 1

38.

(b) $5\sqrt{2}$

Explanation: $\lim\limits_{x \to \frac{\pi}{4}} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$

$= \lim\limits_{x \to \frac{\pi}{4}} \frac{2^{\frac{5}{2}}-\left((\cos x+\sin x)^2\right)^{\frac{5}{2}}}{2-(1+\sin 2x)}$

$= \lim\limits_{x \to \frac{\pi}{4}} \frac{2^{\frac{5}{2}}-\left((\cos x+\sin x)^2\right)^{\frac{5}{2}}}{2-(\cos x+\sin x)^2}$

Let t= (cos x + sin x)^2

$x \to \frac{\pi}{4}$

$\therefore t = \left(\cos \frac{\pi}{4} + \sin \frac{\pi}{4}\right)^2 \to (\sqrt{2})^2 = 2$

$\lim\limits_{x \to \frac{\pi}{4}} \frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x} = \lim\limits_{t \to 2} \frac{2^{\frac{5}{2}}-(t)^{\frac{5}{2}}}{2-(t)}$

$= \frac{5}{2}(2)^{\frac{3}{2}} \left[\because \lim_{x \to a} \frac{x^n-a^n}{x-a} = na^{n-1}\right]$

$= 5\sqrt{2}$

39.

(b) does not exist

Explanation: $\underset{x \to 0^+}{Lt} \frac{sin\ [x]}{[x]}$ = 1

$\Rightarrow \underset{x \to 0^-}{Lt} \frac{sin\ [x]}{[x]}$ = sin 1

$\Rightarrow$ L.H.L.$\neq$ R.H.L.

hence limit does not exist.

40.

(d) 8

Explanation: $\lim\limits_{x \to 0^+} \frac{1}{\frac{\sqrt{16+\sqrt{z}}-\sqrt{16}}{(16+\sqrt{z})-(16)}}$

$\Rightarrow \lim\limits_{x \to 0^+} lim2(\sqrt{16+\sqrt{x}})$

= 8

41.

(b) $\frac{1}{32}$

Explanation: $\lim\limits_{x \to 0} \frac{8}{x^8}\left[1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4}\right]$

$= \lim\limits_{x \to 0} \frac{8}{x^8}\left[\left(1 - \cos \frac{x^2}{4}\right) - \cos \frac{x^2}{2}\left(1 - \cos \frac{x^2}{4}\right)\right]$

$= \lim\limits_{x \to 0} \frac{8}{x^8}\left[\left(1 - \cos \frac{x^2}{4}\right)\left(1 - \cos \frac{x^2}{2}\right)\right]$

$= \lim\limits_{x \to 0} \frac{8}{x^8}\left[\left(2\sin^2 \frac{x^2}{8}\right)\left(2\sin^2 \frac{x^2}{4}\right)\right]$

$= \lim\limits_{x \to 0} 4 \times 8 \frac{\left(\sin^2 \frac{x^2}{8}\right)}{\left(64 \times \frac{x^4}{64}\right)} \frac{\left(\sin^2 \frac{x^2}{4}\right)}{16\left(\frac{x^4}{16}\right)}$

$= \frac{32}{64 \times 16}$

$= \frac{1}{32}$

42.

(b) $\frac{\pi}{180}$

Explanation: $\lim_{x\to 0} \frac{\sin x^0}{x}$

$= \lim_{x\to 0} \frac{\sin \frac{\pi}{180} x}{x}$

$= \lim_{x\to 0} \frac{\sin\left(\frac{\pi}{180} x\right)}{\left(\frac{\pi}{180} x\right)} \times \frac{\pi}{180}$

$= \frac{\pi}{180} \times 1 = \frac{\pi}{180}$

43. **(a)** None of these

Explanation: We have,

$[x] = \begin{cases} 0, & 0 \leq x < 1 \\ -1, & -1 \leq x < 0 \end{cases}$

$\therefore f(x) = \begin{cases} \frac{\sin(-1)}{-1}, & -1 \leq x < 0 \\ 0, & 0 \leq x < 1 \end{cases}$

$\Rightarrow f(x) = \begin{cases} \sin 1, & -1 \leq x < 0 \\ 0, & 0 \leq x < 1 \end{cases}$

Now,

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} \sin 1 = \sin 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} 0 = 0$

Clearly,

$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

Thus,

$\lim_{x\to 0} f(x)$ does not exist.

44. **(a)** 0

Explanation: $\lim_{x\to 0} \frac{1-\cos 2x}{x}$

$= \lim_{x\to 0} \frac{2\sin^2 x}{x}$

$= \lim_{x\to 0} 2x \times \frac{\sin^2 x}{x^2}$

= 0

45.

(c) $\frac{x-1}{(2x-x^2)^{3/2}}$

Explanation: y = $\cos^{-1}$ (1 - x)

⇒ cos y = 1 - x

⇒ (-sin y)y' = -1

⇒ y'' = -$(y')^2$ (cot y) = $\frac{(-\cos y)}{(\sin y)^3}$

⇒ $\frac{x-1}{(2x-x^2)^{\frac{3}{2}}}$

46.

(c) $\frac{1}{\sqrt{3}}$

Explanation: $\lim_{x\to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x-1}$

$= \lim_{h\to 0} \frac{\sin \frac{\pi}{3} - \left(\frac{\pi}{3}-h\right)}{2\cos\left(\frac{\pi}{3}-h\right)-1}$

$= \lim_{h\to 0} \frac{\sin h}{2\left[\cos \frac{\pi}{3}\cos h+\sin \frac{\pi}{3}\sin h\right]-1}$

$= \lim_{h\to 0} \frac{\sin h}{2\left[\frac{1}{2}\cos h+\frac{\sqrt{3}}{2}\sin h\right]-1}$

$= \lim_{h\to 0} \frac{\sin h}{\cos h+\sqrt{3}\sin h-1}$

$= \lim_{h\to 0} \frac{\sin h}{-2\sin^2 \frac{h}{2}+\sqrt{3}\sin h}$

Dividing N^r and D^r by h

$= \lim_{h \to 0} \frac{\frac{\sin h}{h}}{-\left(2 \times \frac{h}{4}\right)\left(\frac{\sin^2 \frac{h}{2}}{\frac{h}{4}}\right) + \frac{\sqrt{3}\sin h}{h}}$

$= \frac{1}{\sqrt{3}}$

47. **(a)** 1

Explanation: Given, $\lim_{x \to 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{1-x}}$

$= \lim_{x \to 0} \frac{\sin x[\sqrt{x+1} + \sqrt{1-x}]}{(\sqrt{x+1} - \sqrt{1-x})(\sqrt{x+1} + \sqrt{1-x})}$

$= \lim_{x \to 0} \frac{\sin x[\sqrt{x+1} + \sqrt{1-x}]}{x+1-1+x}$

$= \lim_{x \to 0} \frac{\sin x[\sqrt{x+1} + \sqrt{1-x}]}{2x} = \frac{1}{2} \cdot \lim_{x \to 0} \frac{\sin x}{x}[\sqrt{x+1} + \sqrt{1-x}]$

Taking limits, we get

$= \frac{1}{2} \times 1 \times [\sqrt{0+1} + \sqrt{1-0}] = \frac{1}{2} \times 1 \times 2 = 1$

48.

(b) $\frac{2}{1+x^2}, x \neq 0, \pm 1$

Explanation: Substitute x = tan θ; $\frac{dx}{d\theta} = \sec^2 \theta$

$\Rightarrow$ y = 2θ

$\Rightarrow \frac{dy}{dx} = 2\cos^2 \theta$

$\Rightarrow \frac{2}{1+x^2}$

49. **(a)** $\frac{1}{2}$

Explanation: We have

$\lim_{x \to 0} \frac{\sin x}{x(1+\cos x)} = \lim_{x \to 0} \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{x \quad 2\cos^2 \frac{x}{2}}$

$= \frac{1}{2} \lim_{x \to 0} \frac{\tan \frac{x}{2}}{\frac{x}{2}} = \frac{1}{2}$

50.

(c) $\frac{(-1)^{n-1}(n-1)!}{x^n}$

Explanation: y = log x

y' = $\frac{1}{x}$

y'' = $-\frac{1}{x^2}$

y''' = $\frac{1.2}{x^3}$

y'''' = $-\frac{1.2.3}{x^4}$ ....

y'n = $\frac{(-1)^{n-1}(n-1)!}{x^n}$

51.

(b) 1

Explanation: $\underset{x \to k^-}{lim}(x - [x])$

$\Rightarrow \underset{h \to 0}{lim}(k - h - [k - h])$

$\Rightarrow \lim_{h \to 0}(k - h - k + 1)$

= 1

52.

(b) $\frac{1}{3}$

Explanation: $\lim_{n \to \infty} \frac{1^2 + 2^2 + 3^2 \cdot \dots \cdot n^2}{n^3}$

= $\lim_{n \to \infty} \frac{\Sigma n^2}{n^3}$

= $\lim_{n \to \infty} \frac{n(n+1)(2n+1)}{6n^3}$

= $\lim_{n \to 0} \frac{(n+1)(2n+1)}{6n^2}$

Dividing the numerator and the denominator by n^2, we get

$\lim\limits_{n \to \infty} \frac{\frac{(n+1)}{n} \times \frac{(2n+1)}{n}}{6}$

$= \lim\limits_{n \to \infty} \frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}$

$= \frac{2}{6} = \frac{1}{3}$

53. **(a)** $\frac{1}{2}$

Explanation: $\lim\limits_{n \to \infty} \left[\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} \cdots \frac{1}{(2n+1)(2n+3)}\right]$

Here, $T_n = \frac{1}{(2n-1)(2n+1)}$

$\Rightarrow T_n = \frac{A}{(2n-1)} + \frac{B}{(2n+1)}$

On equating A = $\frac{1}{2}$ and B = -$\frac{1}{2}$

$T_n = \frac{1}{2(2n-1)} - \frac{1}{2(2n+1)}$

$\Rightarrow T_1 = \frac{1}{2}\left[1 - \frac{1}{3}\right]$

$\Rightarrow T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{5}\right]$

$\Rightarrow T_{n-1} = \frac{1}{2}\left[\frac{1}{2n-1} - \frac{1}{2n-1}\right]$

$\Rightarrow T_n = \frac{1}{2}\left[\frac{1}{2n-1} - \frac{1}{2n+1}\right]$

$\Rightarrow T_1 + T_2 + T_3 \ldots T_n = \frac{1}{2}\left[1 - \frac{1}{2n+1}\right]$

$\Rightarrow T_1 + T_2 + T_3 \ldots T_n = \frac{1}{2}\left[\frac{2n}{2n+1}\right]$

$\Rightarrow T_1 + T_2 + T_3 \ldots T_n = \frac{n}{2n+1}$

$\therefore \lim\limits_{n \to \infty} \left[\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} \cdots \frac{1}{(2n+1)(2n+3)}\right]$

$= \lim\limits_{n \to \infty} \left[\sum\limits_{n=1}^{n} \frac{1}{(2n-1)(2n+1)}\right]$

$= \lim\limits_{n \to \infty} \left(\frac{n}{2n+1}\right)$

$= \lim\limits_{n \to \infty} \left(\frac{1}{2+\frac{1}{n}}\right)$ [Dividing N^r and D^r by n]

$= \frac{1}{2}$

54.

(c) $-\pi$

Explanation: $\lim\limits_{x \to 1} \frac{\sin \pi x}{x-1}$

$= \lim\limits_{h \to 0} \frac{\sin \pi(1+h)}{(1+h)-1}$

$= \lim\limits_{h \to 0} \frac{\sin(\pi+\pi h)}{h}$

$= \lim\limits_{h \to 0} \frac{-\sin \pi h}{h}$

$= \lim\limits_{h \to 0} -\left(\frac{\sin \pi h}{\pi h}\right)\pi$

$= -\pi$

55. **(a)** 2

Explanation: Given $\lim\limits_{x \to 0} \frac{x^2 \cos x}{1-\cos x} = \lim\limits_{x \to 0} \frac{x^2 \cos x}{2\sin^2 \frac{x}{2}} \left[\because 1 - \cos x = 2\sin^2 \frac{x}{2}\right]$

$= \lim\limits_{x \to 0} \frac{\frac{x^2}{4} \times 4\cos x}{2\sin^2 \frac{x}{2}} = \lim\limits_{\frac{x}{2} \to 0} \frac{\left(\frac{x}{2}\right)^2 \cdot 2\cos x}{\sin^2 \frac{x}{2}}$

$= \lim\limits_{\frac{x}{2} \to 0} \left(\frac{\frac{x}{2}}{\sin \frac{x}{2}}\right)^2 \cdot 2\cos x$

= 2 cos 0 = 2 × 1 = 2 $\left[\because \lim\limits_{x \to 0} \frac{x}{\sin x} = 1\right]$

56.

(c) 0

Explanation: $\lim\limits_{n \to \infty} \frac{n!}{(n+1)!-n!}$

Dividing N^r and D^r by n!;

$= \lim_{n\to\infty} \frac{1}{\frac{(n+1)!}{n!}-1}$

$= \lim_{n\to\infty} \frac{1}{\frac{(n+1)n!}{n!}-1}$

$= \lim_{n\to\infty} \frac{1}{n+1-1}$

$= \lim_{n\to\infty} \frac{1}{n} = 0$

57.

(b) 1

Explanation: $\lim_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$

Dividing Nr & Dr by (n+1)!

$= \lim_{n\to\infty} \frac{\frac{(n+2)(n+1)!}{(n+1)!}+1}{\frac{(n+2)(n+1)!}{(n+1)!}-1}$

$= \lim_{n\to\infty} \frac{n+2+1}{n+2-1}$

$= \lim_{n\to\infty} \frac{n+3}{n+1}$

$= \lim_{n\to\infty} \frac{1+\frac{3}{n}}{1+\frac{1}{n}}$

= 1

58.

(d) not continuous

Explanation: At x = 1,

L.H.L. $\neq$R.H.L.

Therefore function is not continuous at x = 1 which is why it is not derivable at x = 1.

59.

(d) 0

Explanation: $\lim_{x\to 0} x = 0$ and $-1 \leq \sin\frac{1}{x} \leq 1$, by Sandwitch Theorem, we have

$\lim_{x\to 0} x \sin\frac{1}{x} = 0$

60.

(c) 1

Explanation: Let Y = $\lim_{x\to\frac{\pi}{2}} (\sec x)^{\cot x}$

Taking log on both sides,

log y = $\lim_{x\to\frac{\pi}{2}}$ cot x log sec x

log y = 0 $\times$ log sec x ($\because$ cot $\frac{\pi}{2}$ = 0)

log y = 0

y = 10^0 = 1

61. **(a)** $\frac{1}{2}$

Explanation: Given, $\lim_{x\to 0} \frac{\tan 2x-x}{3x-\sin x} = \lim_{x\to 0} \frac{x\left[\frac{\tan 2x}{x}-1\right]}{x\left[3-\frac{\sin x}{x}\right]}$

$\lim_{x\to 0} \frac{\frac{\tan 2x}{2x}\times 2-1}{3-\frac{sinx}{x}} = \frac{1.2-1}{3-1} = \frac{2-1}{2} = \frac{1}{2}$

62.

(c) 0

Explanation: $\lim_{n\to\infty} \left[\frac{n!}{(n+1)!+n!}\right]$

$= \lim_{n\to\infty} \left[\frac{n!}{(n+1)\times n!+n!}\right]$

$= \lim_{n\to\infty} \left[\frac{n!}{n!(n+1+1)}\right]$

$= \lim_{n\to\infty}\left[\frac{1}{n+2}\right]$
= 0

63.

(b) 4

Explanation: Polynomial functions are continuous and derivable into their entire domain f'(x) > 0. for |x| > 0.

So it will be an increasing function in [0, 2]

⇒ f(0) = 2 and f(2) = 4

So, f(2) will be maximum.

64.

(d) $\frac{-x}{\left(x^2+1\right)^{3/2}}$

Explanation: Substitute x = tan θ

Then y = log (sin θ + 1) - log (cos θ)

y'(θ) = $\left(\frac{\cos\theta}{1+\sin\theta}\right) - \left(\frac{-\sin\theta}{\cos\theta}\right)$

y'(θ) = sec θ

y'(x) = $\frac{1}{\sqrt{\left(1+x^2\right)}}$

y''(x) = $\frac{-x}{\left(x^2+1\right)^{3/2}}$

65.

(d) $-\frac{x}{|x|\sqrt{1-x^2}}$ for 0 < |x| < 1

Explanation: Substitute x = cos θ

⇒ $\frac{dx}{d\theta}$ = -sin θ

⇒ $\frac{dy}{d\theta}$ = 1

⇒ $\frac{dy}{dx} = -\frac{x}{|x|\sqrt{1-x^2}}$; for 0 < |x| < 1

66.

(c) $\sqrt{2}$

Explanation: Using L' Hospital rule

$\lim_{x\to\frac{\pi}{4}}\frac{\sec x-\sqrt{2}}{x-\frac{\pi}{4}}$

⇒ $\sqrt{2}$

67.

(b) $\frac{2}{1+x^2}$, x > 0

Explanation: Substitute x = tan θ; $\frac{dx}{d\theta} = \sec^2\theta$

⇒ y = 2θ

⇒ $\frac{dy}{dx} = 2\cos^2\theta$

⇒ $\frac{2}{1+x^2}$, x > 0

68.

(b) 1

Explanation: Given f(x) = x - [x]

we have to first check for differentiability of f(x) at x = 1/2

$\therefore Lf'\left(\frac{1}{2}\right) = \text{LHD} = \lim_{h\to0}\frac{f\left[\frac{1}{2}-h\right]-f\left[\frac{1}{2}\right]}{-h}$

$= \lim_{h\to0}\frac{\left(\frac{1}{2}-h\right)-\left[\frac{1}{2}-h\right]-\frac{1}{2}+\left[\frac{1}{2}\right]}{-h}$

$= \lim_{h\to0}\frac{\frac{1}{2}-h-0-\frac{1}{2}+0}{-h} = \frac{-h}{-h} = 1$

$Rf'\left(\frac{1}{2}\right) = RHD = \lim_{h\to0}\frac{f\left(\frac{1}{2}+h\right)-f\left(\frac{1}{2}\right)}{h}$

$= \lim_{h\to0}\frac{\left(\frac{1}{2}+h\right)-\left[\frac{1}{2}+h\right]-\frac{1}{2}+\left[\frac{1}{2}\right]}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{2}+h-0-\frac{1}{2}+0}{h} = \frac{h}{h} = 1$

Since LHD = RHD

$\therefore \quad f'\left(\frac{1}{2}\right) = 1$

69.

(d) 2

Explanation: Let $x - \frac{\pi}{4}$ = t

$\Rightarrow \lim_{t \to 0} \frac{\tan\left(\frac{\pi}{4}+t\right)-1}{t}$

$\Rightarrow \lim_{t \to 0} \frac{2\tan t}{(1-\tan t)(t)}$

= 2

70.

(d) $\frac{(2).(6!)}{(1+x)^7}$

Explanation: $y_1 = \frac{-2}{(1+x)^2}$

$y_2 = \frac{(-2)(-2)}{(1+x)^3}$

$y_3 = \frac{(-2)(-2)(-3)}{(1+x)^4}$

$y_4 = \frac{(-2)(-2)(-3)(-4)}{(1+x)^5}$

$y_5 = \frac{(-2)(-2)(-3)(-4)(-5)}{(1+x)^6}$

$y_6 = \frac{(-2)(-2)(-3)(-4)(-5)(-6)}{(1+x)^7}$

$= \frac{(2).(6!)}{(1+x)^7}$

71.

(d) not derivable at – 1 and 1

Explanation: f(x) = $\begin{cases} x & x < -1 \\ x^3 & -1 \le x \le 1 \\ x & x > 1 \end{cases}$

The function is continuous at x = -1 and 1 as at x = 1 $\underset{x \to -1}{lim} f(x)$ and $\underset{x \to 1}{lim} f(x)$ equals to (-1) and f(1) respectively.

72. **(a)** 2

Explanation: $\lim_{x \to \infty} \frac{\sqrt{1+x^4}+(1+x^2)}{x^2}$

$= \lim_{x \to \infty} \sqrt{\frac{1}{x^4}+1} + \frac{1}{x^2} + 1$

= 2

73.

(b) -2

Explanation: Given, $y = \frac{\sin x+\cos x}{\sin x-\cos x}$

$\frac{dy}{dx} = \frac{(sinx-cosx)(cosx-sinx)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}$

$= \frac{-(\sin x-\cos x)^2-(\sin x+\cos x)^2}{(\sin x-\cos x)^2}$

$= \frac{-\left[\sin^2 x+\cos^2 x-2\sin x\cos x+\sin^2 x+\cos^2 x+2\sin x\cos x\right]}{(\sin x-\cos x)^2}$

$= \frac{-2}{(\sin x-\cos x)^2}$

$\therefore \left(\frac{dy}{dx}\right)_{\text{at } x=0} = \frac{-2}{(\sin 0-\cos 0)^2} = \frac{-2}{(-1)^2} = -2$

74.

(c) 1

Explanation: $\lim_{x \to 0} \frac{x}{\tan x}$

$= \lim_{x \to 0} \frac{1}{\frac{\tan x}{x}}$

$= \frac{1}{1}$

= 1

75.

(d) $\frac{10}{3}$

Explanation: $\lim\limits_{x \to 0} \frac{(1-\cos 2x)\sin 5x}{x^2 \sin 3x}$

$= \lim\limits_{x \to 0} \frac{2\sin^2 x \sin 5x}{x^2 \sin 3x}$

$= \lim_{x \to 0} \frac{\frac{2\sin^2 x}{x^2}\left(\frac{\sin 5x}{5x}\right) \times 5}{\left(\frac{\sin 3x}{3x}\right) 3}$

$= \frac{10 \times 1^2 \times 1}{1 \times 3}$

$= \frac{10}{3}$