Example 5 The total cost C(x) in Rupees, associated with the production of x units of an item is given by

$$C(x) = 0.005\, x^3 - 0.02\, x^2 + 30x + 5000$$

Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

Solution Since marginal cost is the rate of change of total cost with respect to the output, we have

Marginal cost (MC) $= \dfrac{dC}{dx} = 0.005(3x^2) - 0.02(2x) + 30$

When $x = 3$, MC $= 0.015(3^2) - 0.04(3) + 30$

$= 0.135 - 0.12 + 30 = 30.015$

Hence, the required marginal cost is ₹ 30.02 (nearly).

Example 6 The total revenue in Rupees received from the sale of x units of a product is given by R(x) = $3x^2 + 36x + 5$. Find the marginal revenue, when $x = 5$, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.

Solution Since marginal revenue is the rate of change of total revenue with respect to the number of units sold, we have

Marginal Revenue (MR) $= \dfrac{dR}{dx} = 6x + 36$

When $x = 5$, MR $= 6(5) + 36 = 66$

Hence, the required marginal revenue is ₹ 66.

EXERCISE 6.1

1. Find the rate of change of the area of a circle with respect to its radius r when
   (a) $r = 3$ cm (b) $r = 4$ cm
2. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of an edge is 12 cm?
3. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.
4. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?
5. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

6. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

7. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When $x = 8$cm and $y = 6$cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle.

8. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

9. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm.

10. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

11. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

12. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of the bubble increasing when the radius is 1 cm?

13. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

14. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

15. The total cost C (x) in Rupees associated with the production of x units of an item is given by

$$C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000.$$

Find the marginal cost when 17 units are produced.

16. The total revenue in Rupees received from the sale of x units of a product is given by

$$R(x) = 13x^2 + 26x + 15.$$

Find the marginal revenue when $x = 7$.

Choose the correct answer for questions 17 and 18.

17. The rate of change of the area of a circle with respect to its radius r at $r = 6$ cm is

(A) 10π (B) 12π (C) 8π (D) 11π

18. The total revenue in Rupees received from the sale of x units of a product is given by
$R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$ is
(A) 116 (B) 96 (C) 90 (D) 126

6.3 Increasing and Decreasing Functions

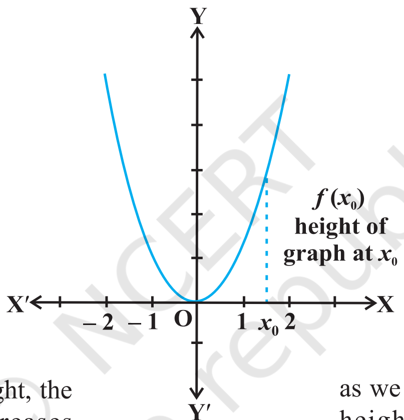
In this section, we will use differentiation to find out whether a function is increasing or decreasing or none.

Consider the function f given by $f(x) = x^2$, $x \in \mathbf{R}$. The graph of this function is a parabola as given in Fig 6.1.

Values left to origin

x	$f(x) = x^2$
-2	4
$-\frac{3}{2}$	$\frac{9}{4}$
-1	1
$-\frac{1}{2}$	$\frac{1}{4}$
0	0

as we move from left to right, the height of the graph decreases



Values right to origin

x	$f(x) = x^2$
0	0
$\frac{1}{2}$	$\frac{1}{4}$
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4

as we move from left to right, the height of the graph increases

Fig 6.1

First consider the graph (Fig 6.1) to the right of the origin. Observe that as we move from left to right along the graph, the height of the graph continuously increases. For this reason, the function is said to be increasing for the real numbers $x > 0$.

Now consider the graph to the left of the origin and observe here that as we move from left to right along the graph, the height of the graph continuously decreases. Consequently, the function is said to be decreasing for the real numbers $x < 0$.

We shall now give the following analytical definitions for a function which is increasing or decreasing on an interval.

Definition 1 Let I be an interval contained in the domain of a real valued function f. Then f is said to be

(i) increasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ I.

(ii) decreasing on I, if x_1, x_2 in I $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ I.

(iii) constant on I, if $f(x) = c$ for all $x \in$ I, where c is a constant.