CBSE Warm-up!

CLASS-XII

Chapterwise practice questions for CBSE Exams as per the latest pattern and syllabus by CBSE for the academic session 2024-25.

Series-2 Inverse Trigonometric Functions and Matrices

General Instructions : Read the following instructions very carefully and strictly follow them :

(i) This question paper contains 38 questions. All questions are compulsory.
(ii) This question paper is divided into five Sections – A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are long answer (LA) type questions, carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are case study based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
(ix) Use of calculators is not allowed.

Time Allowed : 3 hours **Maximum Marks :** 80

SECTION A

This section comprises multiple choice questions (MCQs)

1. $\sin^{-1}\left(\frac{-1}{2}\right)$ is equal to

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $-\frac{\pi}{6}$

2. $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)+4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{3\pi}{4}$

3. Domain of $\cos^{-1}[x]$ (where $[\cdot]$ denotes G.I.F.) is

(a) $[-1, 2]$ (b) $[-1, 2)$
(c) $(-1, 2]$ (d) None of these

4. If $6\sin^{-1}(x^2 - 6x + 8.5) = \pi$, then x is equal to

(a) 1 (b) 2 (c) 3 (d) 8

5. If $\tan^{-1}(\cot\theta) = 2\theta$, then θ is equal to

(a) $\pi/3$ (b) $\pi/4$
(c) $\pi/6$ (d) None of these

6. $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right)=$

(a) x (b) $1/x$ (c) $2x$ (d) $2/x$

7. If $\theta = \tan^{-1}a$, $\phi = \tan^{-1}b$ and $ab = -1$, then $|\theta - \phi|$ is equal to

(a) 0 (b) $\pi/4$
(c) $\pi/2$ (d) None of these

8. Find the value of $\sec^2(\tan^{-1}2) + \mathrm{cosec}^2(\cot^{-1}3)$.

(a) 12 (b) 5 (c) 15 (d) 9

9. If $\sin^{-1}(x^2 - 7x + 12) = n\pi$, $\forall\ n \in I$, then $x =$

(a) -2 (b) 4 (c) -3 (d) 5

10. If $A = [a_{ij}]_{2\times2}$, where $a_{ij} = i + j$, then A is equal to

(a) $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ (b) $\begin{bmatrix}2 & 3\\3 & 4\end{bmatrix}$ (c) $\begin{bmatrix}1 & 1\\2 & 2\end{bmatrix}$ (d) $\begin{bmatrix}1 & 2\\1 & 2\end{bmatrix}$

11. The order of the single matrix obtained from

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left\{ \begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 23 \\ 1 & 0 & 21 \end{bmatrix} \right\}$$ is

(a) 2×3 (b) 2×2 (c) 3×2 (d) 3×3

12. If $[1 \ x \ 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$, then $x =$

(a) -7 (b) -11 (c) -2 (d) 14

13. If $AB = A$ and $BA = B$, then

(a) $B = I$ (b) $A = I$ (c) $A^2 = A$ (d) $B^2 = I$

14. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then $(A - I)(A + I) = O$ for

(a) $a = b = 0$ only (b) $a = 0$ only
(c) $b = 0$ only (d) any a and b

15. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$, then $(AB)^T$ is equal to

(a) $\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 7 \\ 10 & 2 \end{bmatrix}$ (d) None of these

16. If $A = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$ is a symmetric matrix, then x is equal to

(a) 4 (b) 3 (c) -4 (d) -3

17. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then x equals

(a) 2 (b) $-\frac{1}{2}$ (c) 1 (d) $\frac{1}{2}$

18. If $A^3 = O$, then $A^2 + A + I =$

(a) $I - A$ (b) $(I - A)^{-1}$ (c) $(I + A)^{-1}$ (d) $I + A$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) options as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The value of

$\sin\left[\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is 1.

Reason (R) : $\tan^{-1}(-x) = -\tan^{-1}x$ and $\cos^{-1}(-x) = \cos^{-1}x$.

20. **Assertion (A) :** $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $(A + B)^2 = A^2 + B^2 + 2AB$.

Reason (R) : For the matrices A and B, $AB = BA$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$.

OR

Evaluate : $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$.

22. Find the value of $\cos^{-1}(\cos 350°) - \sin^{-1}(\sin 350°)$.

23. For what values of x and y, the following matrices are equal?

$$A = \begin{bmatrix} 2x+1 & 3y \\ 0 & y^2 - 5y \end{bmatrix} \text{ and } B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

24. If $A = [a_{ij}]$ is a matrix given by $\begin{bmatrix} 4 & -2 & 1 & 3 \\ 5 & 7 & 9 & 6 \\ 21 & 15 & 18 & -25 \end{bmatrix}$, then write the order of A.
Also, show that $a_{32} = a_{23} + a_{24}$.

25. Find the matrix A such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

OR

If $A = \text{diag}[2 \ -1 \ 3]$ and $B = \text{diag}[3 \ 0 \ -1]$, then find $4A + 2B$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find the set of all 2×2 matrices which is commutative with the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ with respect to matrix multiplication.

27. Express $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

OR

If $A=\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then find α satisfying $0<\alpha<\frac{\pi}{2}$, when $A + A' = \sqrt{2}I_2$, where A' is transpose of A.

28. Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$.

29. If $\cot^{-1}\left(\frac{-1}{5}\right) = x$, then find the values of $\sin x$ and $\cos x$.

30. If $x, y, z \in [-1, 1]$ such that $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then find the value of $xy + yz + zx$.

OR

If $\operatorname{cosec}^{-1} x + \operatorname{cosec}^{-1} y + \operatorname{cosec}^{-1} z = -\frac{3\pi}{2}$, find the value of $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$.

31. Find the range of $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1}x$.

OR

Evaluate : $\frac{1}{\pi}\left\{216 \sin^{-1}\left(\sin\frac{7\pi}{6}\right) + 27 \cos^{-1}\left(\cos\frac{2\pi}{3}\right) + 28 \tan^{-1}\left(\tan\frac{5\pi}{4}\right) + 200 \cot^{-1}\left(\cot\frac{-\pi}{4}\right)\right\}$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. Three shopkeeper A, B and C go to a store to buy stationery. A purchased 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchase 10 dozen notebooks, 6 dozen pens and 7 dozens pencils. C purchase 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs ₹ 40, a pen costs ₹ 8.50 and a pencil costs ₹ 3.50. Use matrix multiplication to calculate each individuals bill.

OR

Three schools A, B and C organised a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold are given below.

Articles \ School	*A*	*B*	*C*
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the funds collected by each school separately by selling the above articles. Also, find the total funds collected for the purpose.

33. If $A=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying $AA^T = 9I_3$, then find the values of a and b.

34. Prove that $\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$.

OR

Evaluate : $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$.

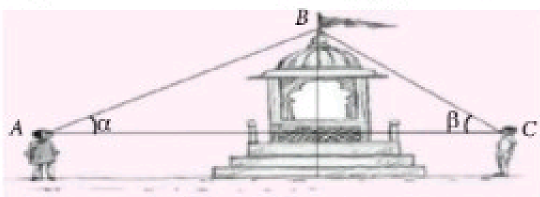
35. Prove that

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1} x^2.$$

SECTION E

Case Study 1

36. Two men on either side of a temple, which is 30 metres high above the stairs, observe its top at the angles of elevation α and β respectively (as shown in the figure below). The distance between the two men is $40\sqrt{3}$ metres and the distance between the first person A and the temple is $30\sqrt{3}$ metres.



Based on the above information, answer the following questions.

(i) Find $\angle BCA$. (ii) Find $\tan\alpha$.

(iii) Find $\angle CAB$ in terms of $\sin^{-1}$.

OR

Find $\angle CAB$ in terms of $\cos^{-1}$.

Case Study 2

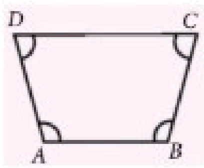
37. In a city there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories, type I, II and III. For boys the number of units of types I, II and III respectively are 80, 70 and 65 in factory A and 85, 65 and 72 are in factory B. For girls the number of units of types I, II and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

(i) If P represents the matrix of number of units of each type produced by factory A for both boys and girls and Q represents the matrix of number of units of each type produced by factory B for both boys and girls, then find P and Q.

(ii) Find the matrix to represents the total production of sports clothes of each type for boys.

Case Study 3

38. There are 4 friends A, B, C and D whose houses are situated at different places in a same colony and lines joining their houses forms a quadrilateral.



A mathematician measured angle between lines joining their houses as,

$$\angle A = \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right);$$

$$\angle B = \tan^{-1}(1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right);$$

$$\angle C = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right); \ \angle D = \cos^{-1}\left(\frac{1}{2}\right)$$

(i) Find the measure of angle C in degrees.

(ii) Find the measure of angle D in degrees .

(iii) Find the measure of angle A in terms of π.

OR

Find the measure of angle B in terms of π.

SOLUTIONS

1. (d) : Let

$$\sin^{-1}\left(\frac{-1}{2}\right) = \theta \Rightarrow \sin\theta = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\frac{-\pi}{6}\right)$$

$$\Rightarrow \theta = \frac{-\pi}{6} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$\therefore$ Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $\left(\frac{-\pi}{6}\right)$.

2. (c) : We have,

$$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) + 4\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\pi}{3} + 2\cdot\frac{\pi}{6} + 4\cdot\frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{4\pi}{3}$$

3. (b) : Clearly, $-1 \le [x] \le 1 \Rightarrow -1 \le x < 2$

$\Rightarrow x \in [-1, 2)$

4. (b) : We have, $6\sin^{-1}(x^2 - 6x + 8.5) = \pi$

$$\Rightarrow \sin^{-1}(x^2 - 6x + 8.5) = \frac{\pi}{6}$$

$$\Rightarrow x^2 - 6x + 8.5 = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \Rightarrow x^2 - 6x + 8 = 0$$

$\Rightarrow (x-4)(x-2) = 0 \Rightarrow x = 4$ or $x = 2$

5. (c) : We have, $\tan^{-1}(\cot\theta) = 2\theta \Rightarrow \cot\theta = \tan 2\theta$

$$\Rightarrow \cot\theta = \cot\left(\frac{\pi}{2} - 2\theta\right) \Rightarrow \theta = \frac{\pi}{2} - 2\theta$$

$$\Rightarrow 3\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{6}$$

6. (d) : Let $x = \cos 2\theta$. Then

$$\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = \frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}$$

$$= \frac{2(1+\tan^2\theta)}{1-\tan^2\theta} = \frac{2}{\cos 2\theta} = \frac{2}{x}$$

7. (c) : Given that, $\theta = \tan^{-1} a$, $\phi = \tan^{-1} b$ and $ab = -1$.

$\therefore \tan\theta\tan\phi = ab = -1 \Rightarrow \tan\theta = -\cot\phi$

$$\Rightarrow \tan\theta = \tan\left(\frac{\pi}{2} + \phi\right) \Rightarrow \theta - \phi = \frac{\pi}{2}$$

8. (c) : We have, $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3)$

$= [\sec(\tan^{-1}2)]^2 + [\operatorname{cosec}(\cot^{-1}3)]^2$

$= [\sec(\sec^{-1}\sqrt{5})]^2 + [\operatorname{cosec}(\operatorname{cosec}^{-1}\sqrt{10})]^2$

$= (\sqrt{5})^2 + (\sqrt{10})^2 = 5 + 10 = 15$

9. (b) : We have, $\sin^{-1}(x^2 - 7x + 12) = n\pi$

$\Rightarrow x^2 - 7x + 12 = \sin(n\pi)$

$\Rightarrow x^2 - 7x + 12 = 0 \qquad (\because \sin(n\pi) = 0 \ \forall n \in I)$

$\Rightarrow (x-4)(x-3) = 0 \Rightarrow x = 4$ or $x = 3$

10. (b) : Here, $a_{11} = 1 + 1 = 2$, $a_{12} = 1 + 2 = 3$, $a_{21} = 2 + 1 = 3$ and $a_{22} = 2 + 2 = 4$

Hence, $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$.

11. (d) : When a 3×2 matrix is post multiplied by a 2×3 matrix, then the product is a 3×3 matrix.

12. (c) : We have, $[1 \quad x \quad 1]\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$

$\Rightarrow [1 \quad x \quad 1]\begin{bmatrix} 7+2x \\ 12+x \\ 21+2x \end{bmatrix} = O \Rightarrow 7+2x+12x+x^2+21+2x=0$

$\Rightarrow x^2 + 16x + 28 = 0 \Rightarrow x = -2$ or $x = -14$

13. (c) : We have, $A = AB = A(BA) = (AB)A = A \cdot A = A^2$ and $B = BA = B(AB) = (BA)B = B \cdot B = B^2$

14. (d) : We have, $(A - I)(A + I) = O \Rightarrow A^2 = I$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, for any a and b.

15. (b) : We have, $AB = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$

$\Rightarrow (AB)^T = \begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

16. (c) : Since, A is a symmetric matrix $\Rightarrow A^T = A$

$\Rightarrow \begin{bmatrix} 3 & 2x+3 \\ x-1 & x+2 \end{bmatrix} = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$

On equating the corresponding elements, we get

$\Rightarrow x - 1 = 2x + 3 \Rightarrow x = -4$

17. (d) : $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

We know that, $AA^{-1} = I$

$\Rightarrow \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On comparing, we get

$2x = 1 \Rightarrow x = \frac{1}{2}$

18. (b) : Since, $I - A^3 = I \Rightarrow (I - A)(I + A + A^2) = I$

$\therefore I + A + A^2 = (I - A)^{-1}$

19. (c) : We have, $\sin\left[\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$

$= \sin\left[-\tan^{-1}(\sqrt{3}) + \pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \sin\left[-\frac{\pi}{3} + \pi - \frac{\pi}{6}\right] = \sin\frac{\pi}{2} = 1$

Thus, Assertion (A) is true, but Reason(R) is false.

20. (c) : $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$AB = AI = A$ and $BA = IA = A \Rightarrow AB = BA$

Consequently, $(A + B)^2 = (A + B)(A + B)$

$= A(A + B) + B(A + B) = A^2 + AB + BA + B^2$

$= A^2 + AB + AB + B^2 = A^2 + 2AB + B^2$

Thus, Assertion (A) is true, but Reaction (R) is false.

21. We have,

$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \tan^{-1}\left[2\sin\left(2 \cdot \frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\sin\frac{\pi}{3}\right] = \tan^{-1}\left[2 \cdot \frac{\sqrt{3}}{2}\right] = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

OR

We have,

$\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right] = \sin\left[\frac{\pi}{3} - \sin^{-1}\left(\sin\left(\frac{-\pi}{6}\right)\right)\right]$

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\frac{\pi}{2} = 1$

22. We have, $\cos^{-1}(\cos 350°) - \sin^{-1}(\sin 350°)$

$\Rightarrow \cos^{-1}(\cos(360° - 10°)) - \sin^{-1}(\sin(360° - 10°))$

$\Rightarrow \cos^{-1}(\cos(10°)) - \sin^{-1}(\sin(-10°)) = 10° - (-10°)$

$= 20°$

23. By definition of equal matrices, we have

$2x + 1 = x + 3 \Rightarrow x = 2;$

$3y = y^2 + 2$

$\Rightarrow y^2 - 3y + 2 = 0 \Rightarrow (y - 1)(y - 2) = 0 \Rightarrow y = 1, 2;$

$y^2 - 5y = -6 \Rightarrow y^2 - 5y + 6 = 0$

$\Rightarrow (y - 2)(y - 3) = 0 \Rightarrow y = 2, 3$

$\therefore x = 2, y = 1, 2$ and $y = 2, 3$

$\Rightarrow x = 2$ and $y = 2$

24. We observe that there are 3 rows and 4 columns in matrix A.

$\therefore$ It is of order 3×4.

Here, $a_{32} = 15$, $a_{23} = 9$ and $a_{24} = 6$

$\therefore a_{23} + a_{24} = 9 + 6 = 15 = a_{32}$, which is true.

25. Given, $2A - 3B + 5C = O$

$\Rightarrow 2A = 3B - 5C$

$\Rightarrow A = \frac{1}{2}[3B - 5C]$ ...(i)

Now, $3B-5C=3\begin{bmatrix}-2 & 2 & 0\\ 3 & 1 & 4\end{bmatrix}-5\begin{bmatrix}2 & 0 & -2\\ 7 & 1 & 6\end{bmatrix}$

$=\begin{bmatrix}-6 & 6 & 0\\ 9 & 3 & 12\end{bmatrix}-\begin{bmatrix}10 & 0 & -10\\ 35 & 5 & 30\end{bmatrix}=\begin{bmatrix}-16 & 6 & 10\\ -26 & -2 & -18\end{bmatrix}$

From (i), we get

$A=\frac{1}{2}\begin{bmatrix}-16 & 6 & 10\\ -26 & -2 & -18\end{bmatrix}=\begin{bmatrix}-8 & 3 & 5\\ -13 & -1 & -9\end{bmatrix}$

OR

We have, $A=\begin{bmatrix}2 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$

Now, $4A+2B=4\begin{bmatrix}2 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 3\end{bmatrix}+2\begin{bmatrix}3 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$

$=\begin{bmatrix}8 & 0 & 0\\ 0 & -4 & 0\\ 0 & 0 & 12\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -2\end{bmatrix}=\begin{bmatrix}14 & 0 & 0\\ 0 & -4 & 0\\ 0 & 0 & 10\end{bmatrix}$

$\therefore\ 4A+2B=\text{diag}\ [14\ \ -4\ \ 10]$

26. Let $A=\begin{bmatrix}x & y\\ z & t\end{bmatrix}$ be a matrix, which commutes with matrix $B=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

Then, $AB=BA$

$\Rightarrow\ \begin{bmatrix}x & y\\ z & t\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x & y\\ z & t\end{bmatrix}$

$\Rightarrow\ \begin{bmatrix}x+y & x\\ z+t & z\end{bmatrix}=\begin{bmatrix}x+z & y+t\\ x & y\end{bmatrix}$

Here, both matrices are equal. So, we equate the corresponding elements.

$\therefore\ x+y=x+z\Rightarrow y=z$

$z+t=x\Rightarrow t=x-z=x-y$

$\therefore$ Required matrix is $\begin{bmatrix}x & y\\ y & x-y\end{bmatrix}$

27. We have, $A=\begin{bmatrix}p & q\\ r & s\end{bmatrix}\Rightarrow A'=\begin{bmatrix}p & r\\ q & s\end{bmatrix}$

$\therefore\ A+A'=\begin{bmatrix}p & q\\ r & s\end{bmatrix}+\begin{bmatrix}p & r\\ q & s\end{bmatrix}=\begin{bmatrix}2p & q+r\\ q+r & 2s\end{bmatrix}$

$\Rightarrow\ \frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}2p & q+r\\ q+r & 2s\end{bmatrix}=\begin{bmatrix}p & \frac{q+r}{2}\\ \frac{q+r}{2} & s\end{bmatrix}$

Also, $A-A'=\begin{bmatrix}p & q\\ r & s\end{bmatrix}-\begin{bmatrix}p & r\\ q & s\end{bmatrix}=\begin{bmatrix}0 & q-r\\ r-q & 0\end{bmatrix}$

$\Rightarrow\ \frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0 & q-r\\ r-q & 0\end{bmatrix}=\begin{bmatrix}0 & \frac{q-r}{2}\\ \frac{r-q}{2} & 0\end{bmatrix}$

$\therefore\ A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$

$=\begin{bmatrix}p & \frac{q+r}{2}\\ \frac{q+r}{2} & s\end{bmatrix}+\begin{bmatrix}0 & \frac{q-r}{2}\\ \frac{r-q}{2} & 0\end{bmatrix}$

OR

Given $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

Also, $A+A'=\sqrt{2}\ I_2$

$\Rightarrow\ \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}'=\sqrt{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow\ \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}\sqrt{2} & 0\\ 0 & \sqrt{2}\end{bmatrix}$

$\Rightarrow\ \begin{bmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{bmatrix}=\begin{bmatrix}\sqrt{2} & 0\\ 0 & \sqrt{2}\end{bmatrix}$

On equating the corresponding elements, we get

$2\cos\alpha=\sqrt{2}\Rightarrow\cos\alpha=\frac{1}{\sqrt{2}}\Rightarrow\alpha=\frac{\pi}{4}$

28. Let $A=[x\ \ y\ \ z]$

Given, $\begin{bmatrix}4\\ 1\\ 3\end{bmatrix}_{3\times1} A=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3}$

$\therefore\ \begin{bmatrix}4\\ 1\\ 3\end{bmatrix}_{3\times1}\begin{bmatrix}x & y & z\end{bmatrix}_{1\times3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3}$

$\Rightarrow\ \begin{bmatrix}4x & 4y & 4z\\ x & y & z\\ 3x & 3y & 3z\end{bmatrix}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}$

On comparing the corresponding elements of two matrices, we get

$4x = -4 \Rightarrow x = -1;\ 4y = 8 \Rightarrow y = 2$

$4z = 4 \Rightarrow z = 1$

Hence, $A = [-1\ \ 2\ \ 1]$

29. Given, $\cot^{-1}\left(\frac{-1}{5}\right) = x \Rightarrow \cot x = \frac{-1}{5}, x \in [\pi/2, \pi)$

Now, $\operatorname{cosec} x = \sqrt{1+\cot^2 x} = \sqrt{1+\left(\frac{-1}{5}\right)^2} = \sqrt{\frac{26}{25}} = \frac{\sqrt{26}}{5}$

[cosec x is +ve since, $0 < x < \pi$]

$\Rightarrow \sin x = \frac{5}{\sqrt{26}}$

Also, $\cos x = \frac{\cos x}{\sin x} \cdot \sin x = \cot x \cdot \sin x = \left(\frac{-1}{5}\right)\left(\frac{5}{\sqrt{26}}\right)$

$= \frac{-1}{\sqrt{26}}$

30. We have, $x, y, z \in [-1, 1]$

$\Rightarrow -1 \le x \le 1, -1 \le y \le 1, -1 \le z \le 1$

$\Rightarrow 0 \le \cos^{-1} x \le \pi, 0 \le \cos^{-1} y \le \pi, 0 \le \cos^{-1} z \le \pi$

Given, $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$

$\Rightarrow \cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi + \pi + \pi$

$\Rightarrow \cos^{-1}x = \pi, \cos^{-1}y = \pi, \cos^{-1}z = \pi$

$\Rightarrow x = -1, y = -1, z = -1$

$\therefore xy + yz + zx = (-1) \times (-1) + (-1) \times (-1) + (-1) \times (-1)$

$= 1 + 1 + 1 = 3$

OR

We know that the minimum value of $\operatorname{cosec}^{-1} x$ is $-\frac{\pi}{2}$ which is attained at $x = -1$.

$\therefore \operatorname{cosec}^{-1}x + \operatorname{cosec}^{-1}y + \operatorname{cosec}^{-1}z = -\frac{3\pi}{2}$

$\Rightarrow \operatorname{cosec}^{-1}x+\operatorname{cosec}^{-1}y+\operatorname{cosec}^{-1}z=\left(-\frac{\pi}{2}\right)+\left(-\frac{\pi}{2}\right)+\left(-\frac{\pi}{2}\right)$

$\Rightarrow \operatorname{cosec}^{-1}x = -\frac{\pi}{2}, \operatorname{cosec}^{-1}y = -\frac{\pi}{2}, \operatorname{cosec}^{-1}z = -\frac{\pi}{2}$

$\Rightarrow x = -1, y = -1, z = -1$

$\therefore \frac{x}{y}+\frac{y}{z}+\frac{z}{x} = \frac{(-1)}{(-1)}+\frac{(-1)}{(-1)}+\frac{(-1)}{(-1)} = 3$

31. Given, $f(x) = \sin^{-1}x + \tan^{-1}x + \sec^{-1}x$

Domain of $\sin^{-1}x = [-1, 1]$

Domain of $\tan^{-1}x = (-\infty, \infty)$

Domain of $\sec^{-1}x = (-\infty, \infty) - (-1, 1)$

Domain of $f(x) = [-1, 1] \cap (-\infty, \infty) \cap [(-\infty, \infty) - (-1, 1)]$

$= \{-1, 1\}$

Now, $f(-1) = \sin^{-1}(-1) + \tan^{-1}(-1) + \sec^{-1}(-1)$

$= -\frac{\pi}{2} - \frac{\pi}{4} + \pi = \frac{\pi}{4}$

and $f(1) = \sin^{-1}(1) + \tan^{-1}(1) + \sec^{-1}(1)$

$= \frac{\pi}{2} + \frac{\pi}{4} + 0 = \frac{3\pi}{4}$

Range of $f(x) = \left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

OR

Let, $\sin^{-1}\left(\sin\frac{7\pi}{6}\right) = x$

$\Rightarrow x = \sin^{-1}\left(\sin\left(\pi + \frac{\pi}{6}\right)\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$= \sin^{-1}\left(\sin\left(\frac{-\pi}{6}\right)\right) \Rightarrow \sin x = \sin\left(\frac{-\pi}{6}\right) \Rightarrow x = \frac{-\pi}{6}$

Let $y = \cos^{-1}\left(\cos\frac{2\pi}{3}\right) \Rightarrow \cos y = \cos\frac{2\pi}{3} \Rightarrow y = \frac{2\pi}{3}$

and $z = \tan^{-1}\left(\tan\frac{5\pi}{4}\right) \Rightarrow z = \tan^{-1}\left(\tan\left(\pi + \frac{\pi}{4}\right)\right)$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}\right)\right] \Rightarrow \tan z = \tan\left(\frac{\pi}{4}\right) \Rightarrow z = \frac{\pi}{4}$

and $m = \cot^{-1}\left\{\cot\left(-\frac{\pi}{4}\right)\right\}$

$\Rightarrow \cot m = \cos\left(\pi - \frac{\pi}{4}\right) \Rightarrow m = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Hence, required value is

$\frac{1}{\pi}\left\{216 \times -\frac{\pi}{6} + 27 \times \frac{2\pi}{3} + 28 \times \frac{\pi}{4} + 200 \times \frac{3\pi}{4}\right\}$

$= -36 + 18 + 7 + 150 = 139$

32. A purchases 144 notebooks, 60 pens and 72 pencils, B purchases 120 notebooks, 72 pens and 84 pencils, C purchases 132 notebooks, 156 pens and 96 pencils.

Let $D = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix}$ be the matrix of purchases.

Let E be the price matrix.

$\therefore E = \begin{bmatrix} 40 \\ 8.50 \\ 3.50 \end{bmatrix}$

Now,

$DE = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix}\begin{bmatrix} 40 \\ 8.50 \\ 3.50 \end{bmatrix} = \begin{bmatrix} 5760+510+252 \\ 4800+612+294 \\ 5280+1326+336 \end{bmatrix} = \begin{bmatrix} 6522 \\ 5706 \\ 6942 \end{bmatrix}$

$\therefore$ Bill of A, B and C are ₹ 6522, ₹ 5706 and ₹ 6942 respectively.

OR

Three items sold by three schools can be written in matrix form as $X=\begin{bmatrix}40 & 25 & 35\\50 & 40 & 50\\20 & 30 & 40\end{bmatrix}$

The cost of each item can be written in matrix form as $Y=[25 \quad 100 \quad 50]$

The fund collected by each school is given by

$$YX=\begin{bmatrix}25 & 100 & 50\end{bmatrix}\begin{bmatrix}40 & 25 & 35\\50 & 40 & 50\\20 & 30 & 40\end{bmatrix}=\begin{bmatrix}7000 & 6125 & 7875\end{bmatrix}$$

$\therefore$ Funds collected by schools A, B and C are ₹ 7000, ₹ 6125 and ₹ 7875 respectively.

Thus, total fund collected = ₹(7000 + 6125 + 7875)
= ₹21000.

33. We have, $A=\begin{bmatrix}1 & 2 & 2\\2 & 1 & -2\\a & 2 & b\end{bmatrix}\Rightarrow A^T=\begin{bmatrix}1 & 2 & a\\2 & 1 & 2\\2 & -2 & b\end{bmatrix}$

Also, $AA^T=9I_3$

$$\Rightarrow \begin{bmatrix}1 & 2 & 2\\2 & 1 & -2\\a & 2 & b\end{bmatrix}\begin{bmatrix}1 & 2 & a\\2 & 1 & 2\\2 & -2 & b\end{bmatrix}=9\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1+4+4 & 2+2-4 & a+2b+4\\2+2-4 & 4+1+4 & 2a+2-2b\\a+4+2b & 2a+2-2b & a^2+4+b^2\end{bmatrix}=\begin{bmatrix}9 & 0 & 0\\0 & 9 & 0\\0 & 0 & 9\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}9 & 0 & a+2b+4\\0 & 9 & 2a+2-2b\\a+4+2b & 2a+2-2b & a^2+4+b^2\end{bmatrix}=\begin{bmatrix}9 & 0 & 0\\0 & 9 & 0\\0 & 0 & 9\end{bmatrix}$$

On equating the corresponding elements, we get

$\Rightarrow a+2b+4=0$ ...(i)

$2a+2-2b=0 \Rightarrow a+1-b=0$ ...(ii)

and $a^2+4+b^2=9 \Rightarrow a^2+b^2=5$ ...(iii)

Solving (i) and (ii), we get

$a=-2, b=-1$

34. Let $x=\cos^{-1}\left(\frac{12}{13}\right)$ and $y=\sin^{-1}\left(\frac{3}{5}\right)$

$\Rightarrow \cos x=\frac{12}{13}$ and $\sin y=\frac{3}{5}$

Now, $\sin x=\sqrt{1-\cos^2 x}$ and $\cos y=\sqrt{1-\sin^2 y}$

$\Rightarrow \sin x=\sqrt{1-\frac{144}{169}}$ and $\cos y=\sqrt{1-\frac{9}{25}}$

$\Rightarrow \sin x=\frac{5}{13}$ and $\cos y=\frac{4}{5}$

We know that, $\sin(x+y)=\sin x\cos y+\cos x\sin y$

$=\frac{5}{13}\times\frac{4}{5}+\frac{12}{13}\times\frac{3}{5}=\frac{20}{65}+\frac{36}{65}=\frac{56}{65}$

$\Rightarrow x+y=\sin^{-1}\left(\frac{56}{65}\right)$

$\Rightarrow \cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$

Hence proved.

OR

Let $\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)=x \Rightarrow \cos^{-1}\frac{\sqrt{5}}{3}=2x, 0\le 2x\le\pi$

$\Rightarrow \cos 2x=\frac{\sqrt{5}}{3}, 0\le x\le\frac{\pi}{2} \quad \Rightarrow \frac{1-\tan^2 x}{1+\tan^2 x}=\frac{\sqrt{5}}{3}$

$\Rightarrow \sqrt{5}+\sqrt{5}\tan^2 x=3-3\tan^2 x$

$\Rightarrow (3+\sqrt{5})\tan^2 x=3-\sqrt{5} \Rightarrow \tan^2 x=\frac{3-\sqrt{5}}{3+\sqrt{5}}$

$\Rightarrow \tan x=\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} \qquad (\because \text{ For } 0\le x\le\frac{\pi}{2}, \tan x\ge 0)$

$\Rightarrow \tan x=\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}}=\frac{3-\sqrt{5}}{2}$

$\Rightarrow \tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)=\frac{3-\sqrt{5}}{2}$

35. We have to prove that,

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}(x^2)$$

Consider, $\text{L.H.S.}=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

Put $x^2=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1}(x^2)$ ...(i)

$\therefore$ L.H.S. $=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$

$$\tan^{-1}\left(\frac{\sqrt{1+2\cos^2\theta-1}+\sqrt{1-1+2\sin^2\theta}}{\sqrt{1+2\cos^2\theta-1}-\sqrt{1-1+2\sin^2\theta}}\right)$$

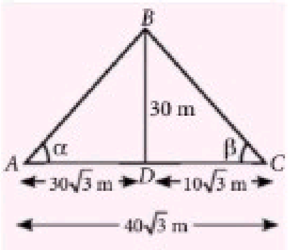
$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}\right) = \tan^{-1}\left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right)$

$= \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right) = \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]$

$= \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2) = \text{R.H.S.}$ [Using (i)]

36. (i) Clearly, $CD = AC - AD$

$= 40\sqrt{3} - 30\sqrt{3} = 10\sqrt{3}$ m



$\therefore \tan\beta = \frac{BD}{CD} = \frac{30}{10\sqrt{3}}$

$= \frac{3}{\sqrt{3}} = \sqrt{3}$

$\Rightarrow \angle BCA = \beta = \tan^{-1}(\sqrt{3})$

$= 60°$

(ii) In ΔABD

$\tan\alpha = \frac{BD}{AD} = \frac{30}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$

(iii) We have, $BD = 30$ m,

$AC = 40\sqrt{3}$ m and $AD = 30\sqrt{3}$ m

Clearly, $BD \perp AC$

$\therefore$ In right ΔADB, $AB^2 = AD^2 + BD^2$

$= (30\sqrt{3})^2 + (30)^2 = 3600 \Rightarrow AB = 60$ m

Now, $\sin\alpha = \frac{BD}{AB} = \frac{30}{60} = \frac{1}{2}$

$\Rightarrow \alpha = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \angle CAB = \alpha = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

OR

In right ΔADB, $\cos\alpha = \frac{AD}{AB} = \frac{30\sqrt{3}}{60} = \frac{\sqrt{3}}{2}$

$\Rightarrow \angle CAB = \alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$

37. In factory A, number of units of types I, II and III for boys are 80, 70 and 65 respectively and for girls number of units of types I, II and III are 80, 75 and 90 respectively.

$$\therefore P = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{array}{c} \text{Boys} \quad \text{Girls} \\ \begin{bmatrix} 80 & 80 \\ 70 & 75 \\ 65 & 90 \end{bmatrix} \end{array}$$

In factory B, number of units of types I, II and III for boys are 85, 65 and 72 respectively and for girls number of units of types I, II and III are 50, 55 and 80 respectively.

$$\therefore Q = \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{array}{c} \text{Boys} \quad \text{Girls} \\ \begin{bmatrix} 85 & 50 \\ 65 & 55 \\ 72 & 80 \end{bmatrix} \end{array}$$

(ii) Let X be the matrix that represent the number of units of each type produced by factory A for boys, and Y be the matrix that represent the number of units of each type produced by factory B for boys.

Then, $X = \begin{array}{c} \text{I} \quad \text{II} \quad \text{III} \\ [80 \quad 70 \quad 65] \end{array}$ and $Y = \begin{array}{c} \text{I} \quad \text{II} \quad \text{III} \\ [85 \quad 65 \quad 72] \end{array}$

Now, required matrix $= X + Y = [80\ 70\ 65] + [85\ 65\ 72]$
$= [165\ 135\ 137]$

38. (i) We know, $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

So, the value of $\angle C = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} = 45°$

(ii) We know, $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \in [0, \pi]$

So, the value of $\angle D = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} = 60°$.

(iii) We know, $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \in [0, \pi]$ and

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{2\pi}{3}$

So, the value of $\angle A = \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3}$.

OR

$\tan^{-1}(1) = \frac{\pi}{4} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$; $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \in [0, \pi]$

and $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \tan^{-1}(1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{4} + \frac{\pi}{3} + \frac{\pi}{6} = \frac{3\pi}{4}$