

EXERCISE 5.1

Find $\frac{dy}{dx}$ in the following questions (1 to 5):

1. $y^3 - 3xy^2 = x^3 + 3x^2y$

2. $(x + y)^2 = 2axy$

3. $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

4. $3x - 2y = \log(3x + 2y)$

5. $e^{x-y} = \log\left(\frac{x}{y}\right)$

6. If $xy^3 = 1$, prove that $3\frac{dy}{dx} + y^4 = 0$.

7. If $\sqrt{y+x} + \sqrt{y-x} = c$, show that $\frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}$.

8. Use implicit differentiation to verify that $\frac{dy}{dx} \cdot \frac{dx}{dy} = 1$, when

(i) $y^2 = 4ax$

(ii) $x^3 + y^3 = 3axy$.

9. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$, prove that $(2y - 1) \frac{dy}{dx} = 1$.

Answers

1. $\frac{(x+y)^2}{y^2 - 2xy - x^2}$

2. $\frac{ay - x - y}{x + y - ax}$

3. $-\frac{(ax + hy + g)}{(hx + by + f)}$

4. $\frac{3(3x + 2y - 1)}{2(3x + 2y + 1)}$

5. $\frac{y(xe^{x-y} - 1)}{x(ye^{x-y} - 1)}$