

EXERCISE 5.2

Differentiate the following functions (1 to 2) w.r.t. x :

1. (i) $(x + 3)^2 (x + 4)^3 (x + 5)^4$

(ii) $\frac{x\sqrt{x^2 + 1}}{(x + 1)^{2/3}}, x > 0.$

2. (i) $(2x + 3)^{x-5}, x > -\frac{3}{2}$

(ii) $(\log x)^{\log x}, x > 1.$

3. (i) If $y = x^y$, prove that $x \frac{dy}{dx} = \frac{y^2}{1 - y \log x}.$

(ii) If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{x - y}{x \log x}.$

(iii) If $x^y = e^{x - y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(\log xe)^2}.$

(iv) If $x^{16}y^9 = (x^2 + y)^{17}$, prove that $\frac{dy}{dx} = \frac{2y}{x}.$

4. Find the derivative of $x^x + a^x + x^a + a^a$ for some fixed $a > 0, x > 0.$

5. Differentiate $x^{\log x} + (\log x)^x$ w.r.t. $x.$

6. Find $\frac{dy}{dx}$ when

(i) $x^y y^x = a^b$

(ii) $x^y + y^x = \log a$

(iii) $xy = e^{x-y}.$

7. If $y = x^{x^{x^{\dots \infty}}}$, prove that $\frac{dy}{dx} = \frac{y^2}{x(1 - y \log x)}.$

Answers

$$1. \quad (i) \quad (x+3)^2 (x+4)^3 (x+5)^4 \left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5} \right]$$

$$(ii) \quad \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}} \left(\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)} \right), x > 0$$

$$2. \quad (i) \quad (2x+3)^{x-5} \left(\frac{2(x-5)}{2x+3} + \log(2x+3) \right)$$

$$(ii) \quad (\log x)^{\log x} \cdot \frac{1}{x} (1 + \log(\log x))$$

$$4. \quad x^x (1 + \log x) + a^x \log a + ax^{a-1}$$

$$5. \quad x^{\log x} \cdot \frac{2 \log x}{x} + (\log x)^x \left(\log(\log x) + \frac{1}{\log x} \right)$$

$$6. \quad (i) \quad -\frac{y(x \log x + y)}{x(y \log x + x)}$$

$$(ii) \quad -\frac{y(x^{y-1} + y^{x-1} \log y)}{x(x^{y-1} \log x + y^{x-1})}$$

$$(iii) \quad \frac{y(x-1)}{x(y+1)}$$