EXERCISE 5.4

1. Find the second order derivatives of the following functions :

(*i*) $x^2 + 3x + 2$ (*ii*) $x^3 - 5x^2 + 3x + 4$ (*iii*) $\log x$

(*iv*) $\log (\log x)$ (*v*) $\frac{\log x}{x}$ (*vi*) $\frac{2x+1}{2x+3}$ (*vii*) $\sqrt{1-x^2}$.

Hint. (*vi*) $\frac{2x+1}{2x+3} = 1 - \frac{2}{2x+3}$.

2. (*i*) If $y = ae^{mx} + be^{-mx}$, prove that $y_2 - m^2 y = 0$.

(*ii*) If $y = 500e^{7x} + 600e^{-7x}$, prove that $\frac{d^2y}{dx^2} = 49y$.

(*iii*) If $y = ae^{2x} + be^{-x}$, prove that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.

3. (*i*) If $y = \log\ (x + \sqrt{x^2 + 1})$, prove that $(x^2 + 1)\ \frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

(*ii*) If $y = \log\ (x + \sqrt{x^2 + a^2})$, prove that $(x^2 + a^2)\ y_2 + xy_1 = 0$.

4. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$.

5. If $x = \log t$ and $y = \frac{1}{t}$, prove that $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.

6. If $x = a\left(t + \frac{1}{t}\right), y = a\left(t - \frac{1}{t}\right)$, show that $y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.

7. If $x = at^2, y = 2at$, find $\frac{d^2y}{dx^2}$ at $t = 3$.

Answers

1. (*i*) 2 (*ii*) $6x - 10$ (*iii*) $-\frac{1}{x^2}$ (*iv*) $-\frac{1 + \log x}{(x \log x)^2}$

(*v*) $\frac{2\log x - 3}{x^3}$ (*vi*) $-\frac{16}{(2x + 3)^3}$ (*vii*) $-(1 - x^2)^{-3/2}$ 7. $-\frac{1}{54a}$