EXERCISE 6.1

1. Find the rate of change of the area of a circle with respect to its radius when the radius is 6 cm.
2. If the radius of a circle is increasing at the rate of 0.7 cm/sec, at what rate is its circumference increasing?
3. If the radius of a circle is increasing at the rate of 3 cm/sec, at what rate is its area increasing when its radius is 10 cm?
4. If the sides of a square are decreasing at the rate of 1.5 cm/sec, at what rate is its perimeter decreasing?
5. If the sides of a square are decreasing at the rate of 1.5 cm/sec, at what rate is its area decreasing when its side is 8 cm?
6. Find the rate of change of the volume of a cube with respect to its edge when the edge is 5 cm.
7. If an edge of a variable cube is increasing at the rate of 3 cm/sec, at what rate is its volume increasing when its edge is 10 cm?
8. A balloon which always remains spherical has a variable radius. Find the rate at which its volume is increasing with respect to radius when the radius is 10 cm.
9. If the radius of a balloon which always remains spherical is increasing at the rate 1.5 cm/sec, at what rate is its surface area increasing when its radius is 12 cm?
10. If the radius of a soap bubble is increasing at the rate of $\frac{1}{2}$ cm/sec, at what rate is its volume increasing when the radius is 1 cm?
11. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Find the rate at which the depth of the wheat is increasing, take $\pi = 3.14$.
12. Find the rate of change of the whole surface of a closed circular cylinder of radius r and height h with respect to change in radius.
13. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/sec. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?
14. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When $x = 8$ cm and $y = 6$ cm, find the rate of change of
    (*i*) the perimeter (*ii*) the area of the rectangle.
15. The volume of a cube is increasing at the rate of 9 cubic centimetres per second. How fast is the surface area increasing when the length of the edge is 10 centimetres?
16. A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the points on the curve at which the y-coordinate is changing twice as fast as the x-coordinate.
17. A man of height 2 metres walks at a uniform speed of 5 km/h away from a lamp post which is 6 metres high. Find the rate at which the length of shadow increases.
18. A kite is 120 m high and 130 m of string is out. If the kite is moving horizontally at the rate of 5.2 m/sec, find the rate at which the string is being paid out at that instant.
19. Sand is pouring from a pipe at the rate of 12 cm^3/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?
20. A conical vessel whose height is 4 metres and of base radius 2 metres is being filled with water at the rate of 0.75 cubic metres per minute. Find the rate at which the level of the water is rising when the depth of water is 1.5 metres.

Answers

1. 12π cm^2/cm 2. 1.4π cm/sec 3. 60π cm^2/sec 4. 6 cm/sec
5. 24 cm^2/sec 6. 75 cm^3/cm 7. 900 cm^3/sec 8. 400π cm^3/cm
9. 144π cm^2/sec 10. 2π cm^3/sec 11. 1 m/h
12. $2\pi(2r+h)$ unit^2/unit 13. 80π cm^2/sec 14. (*i*) -2 cm/min (*ii*) 2 cm^2/min
15. 3.6 cm^2/sec 16. $\left(1, \frac{5}{3}\right), \left(-1, \frac{1}{3}\right)$ 17. $\frac{5}{2}$ km/h 18. 2 m/sec
19. $\frac{1}{48\pi}$ cm/sec 20. $\frac{4}{3\pi}$ m/min

TANGENT AND NORMALS

In class XI, we have learnt to find the gradient and equation of tangent to the curve $y = f(x)$ at a given point. Here, we shall revise the formulae and solve some more problems of finding equation of tangent to the curve. We shall also learn to find the gradient and equation of normal to the curve $y = f(x)$ at a given point.

We have learned that $\frac{dy}{dx}$ (if it exists) *geometrically* represents the slope of the tangent to the curve $y = f(x)$ at any point $P(x, y)$. Thus, if $\psi \left(\neq \frac{\pi}{2}\right)$ is the angle which the tangent to the curve at P makes with the positive direction of x-axis, then the slope of the tangent to the curve $y = f(x)$ at the point $P = \tan\psi = \left(\frac{dy}{dx}\right)_P$.

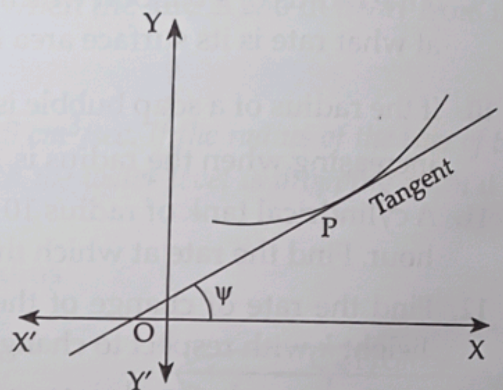

Fig. 6.5.

If the tangent to the curve $y = f(x)$ at the point $P(x, y)$ is parallel to x-axis, then $\psi = 0$ $\Rightarrow \tan\psi = 0 \Rightarrow \left(\frac{dy}{dx}\right)_P = 0$.

*The **gradient** of a curve at a point is defined as the slope of the tangent to the curve at that point.*

To find the equation of the tangent to the curve $y = f(x)$ at a given point

The given curve is $y = f(x)$ ...(*i*)

Let $P(x_1, y_1)$ be any point on the curve (*i*), then the slope of the tangent to the curve $y = f(x)$ at the point $P(x_1, y_1)$ is the value of $\frac{dy}{dx}$ at P.

So, the slope of tangent to the curve (*i*) at P is $\left(\frac{dy}{dx}\right)_{\substack{x=x_1 \\ y=y_1}}$.

Therefore, by coordinate geometry, the equation of the tangent to the given curve $y = f(x)$ at the point $P(x_1, y_1)$ is

$$y - y_1 = \left(\frac{dy}{dx}\right)_{\substack{x=x_1 \\ y=y_1}} (x - x_1).$$