158 MATHEMATICS

Interval	Sign of $f'(x)$	Nature of function
$\left[0,\frac{\pi}{4}\right)$	> 0	f is increasing
$\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$	< 0	f is decreasing
$\left[\left(\frac{5\pi}{4},2\pi\right]\right]$	> 0	f is increasing

EXERCISE 6.2

- 1. Show that the function given by f(x) = 3x + 17 is increasing on **R**.
- 2. Show that the function given by $f(x) = e^{2x}$ is increasing on **R**.
- 3. Show that the function given by $f(x) = \sin x$ is
 - (a) increasing in $\left(0, \frac{\pi}{2}\right)$ (b) decreasing in $\left(\frac{\pi}{2}, \pi\right)$
 - (c) neither increasing nor decreasing in $(0, \pi)$
- 4. Find the intervals in which the function f given by f(x) = 2x² 3x is
 (a) increasing
 (b) decreasing
- 5. Find the intervals in which the function f given by $f(x) = 2x^3 3x^2 36x + 7$ is (a) increasing (b) decreasing
- 6. Find the intervals in which the following functions are strictly increasing or decreasing:
 - (a) $x^2 + 2x 5$ (b) $10 6x 2x^2$

(c)
$$-2x^3 - 9x^2 - 12x + 1$$
 (d) $6 - 9x - x^2$

(e)
$$(x+1)^3 (x-3)^3$$

- 7. Show that $y = \log(1+x) \frac{2x}{2+x}$, x > -1, is an increasing function of x throughout its domain.
- 8. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

9. Prove that
$$y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$$
 is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

- **10.** Prove that the logarithmic function is increasing on $(0, \infty)$.
- Prove that the function f given by $f(x) = x^2 x + 1$ is neither strictly increasing 11. nor decreasing on (-1, 1).
- 12. Which of the following functions are decreasing on $0, \frac{\pi}{2}$?

(B) $\cos 2x$ (C) $\cos 3x$ (A) $\cos x$ (D) $\tan x$

13. On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ decreasing ?

(A) (0,1) (B)
$$\frac{\pi}{2}, \pi$$
 (C) $0, \frac{\pi}{2}$ (D) None of these

- 14. For what values of *a* the function *f* given by $f(x) = x^2 + ax + 1$ is increasing on [1, 2]?
- **15.** Let I be any interval disjoint from [-1, 1]. Prove that the function f given by

$$f(x) = x + \frac{1}{x}$$
 is increasing on I.

16. Prove that the function f given by $f(x) = \log \sin x$ is increasing on $\left(0, \frac{\pi}{2}\right)$ and

decreasing on $\left(\frac{\pi}{2},\pi\right)$.

- Prove that the function f given by $f(x) = \log |\cos x|$ is decreasing on $\left(0, \frac{\pi}{2}\right)$ and 17. increasing on $\left(\frac{3\pi}{2}, 2\pi\right)$.
- **18.** Prove that the function given by $f(x) = x^3 3x^2 + 3x 100$ is increasing in **R**.
- The interval in which $y = x^2 e^{-x}$ is increasing is 19.

(A) $(-\infty, \infty)$ (B) (-2, 0) (C) $(2, \infty)$ (D) (0,2)

6.4 Maxima and Minima

In this section, we will use the concept of derivatives to calculate the maximum or minimum values of various functions. In fact, we will find the 'turning points' of the graph of a function and thus find points at which the graph reaches its highest (or