$\Rightarrow \; x < x^2 \Rightarrow 0 < x^2 - x$

$\Rightarrow \; x(x-1) > 0$ ...(1)

Mark the numbers 0 and 1 on the real line.

By the method of intervals, the inequality (1) is satisfied when

$x > 1$ or $x < 0$.

$\therefore$ The solution set is $(-\infty, 0) \cup (1, \infty)$.

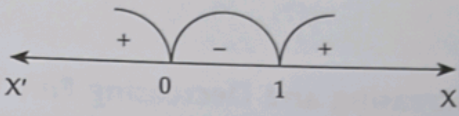

Fig. 6.10.

(*ii*) Given $\dfrac{x+3}{x-4} \geq 0$. First, we note that $x \neq 4$.

Since $(x-4)^2 > 0$ for all $x \in \mathbf{R}, x \neq 4$,

$\dfrac{x+3}{x-4} \geq 0 \Rightarrow (x+3)(x-4) \geq 0$ (multiplying by $(x-4)^2$)

$\Rightarrow \; (x-(-3))(x-4) \geq 0$ ...(1)

Mark the numbers -3 and 4 on the number line.

By the method of intervals, the inequality (1) is satisfied when $x \geq 4$ or $x \leq -3$ but $x \neq 4$.

$\therefore$ The solution set is $(-\infty, -3] \cup (4, \infty)$.

Fig. 6.11.

(*iii*) Given $\dfrac{2x-3}{(x-2)(x-4)} \leq 0$. First, we note that $x \neq 2, 4$.

Since $(x-2)^2 (x-4)^2 > 0$ for all $x \in \mathbf{R}, x \neq 2, 4$.

$\dfrac{2x-3}{(x-2)(x-4)} \leq 0 \Rightarrow (2x-3)(x-2)(x-4) \leq 0$

$\Rightarrow \; 2\left(x - \dfrac{3}{2}\right)(x-2)(x-4) \leq 0 \Rightarrow \left(x - \dfrac{3}{2}\right)(x-2)(x-4) \leq 0$ ...(1)

Mark the numbers $\dfrac{3}{2}$, 2 and 4 on the number line.

By the method of intervals, the inequality (1) is satisfied when $x \leq \dfrac{3}{2}$ or $2 \leq x \leq 4$ but $x \neq 2, 4$.

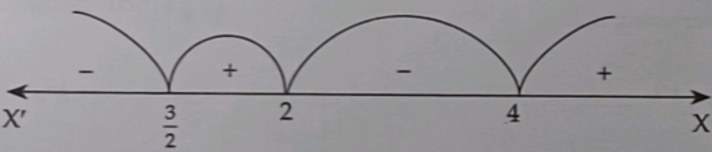

Fig. 6.12.

$\therefore$ The solution set is $\left(-\infty, \dfrac{3}{2}\right] \cup (2, 4)$.

EXERCISE 6.3

1. Solve for x :

   (*i*) $x(x-2)(x-5)(x+3) > 0$ (*ii*) $x^4 - 5x^2 + 4 \geq 0$.

2. Find all real values of x which satisfy

   (*i*) $x^3(x-1)(x-2) > 0$ (*ii*) $x^2(x-1)(x-2) \leq 0$.

3. Solve for x :

   (*i*) $\dfrac{1}{x-2} \leq 1$ (*ii*) $\dfrac{(x+1)(x-3)}{x+2} \geq 0$.

Answers

1. (i) $(-\infty, -3) \cup (0, 2) \cup (5, \infty)$ (ii) $(-\infty, -2] \cup [-1, 1] \cup [2, \infty)$
2. (i) $(0, 1) \cup (2, \infty)$ (ii) $\{0\} \cup [1, 2]$
3. (i) $(-\infty, 2) \cup [3, \infty)$ (ii) $(-2, -1] \cup [3, \infty)$

Increasing and Decreasing Functions

Let f be a real valued function defined in an interval D (a subset of **R***), then f is called an* ***increasing function*** *in an interval D_1 (a subset of D) iff for all $x_1, x_2 \in D_1$, $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$ and f is called a* ***strictly increasing function*** *in D_1 iff for all $x_1, x_2 \in D_1$, $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$.*

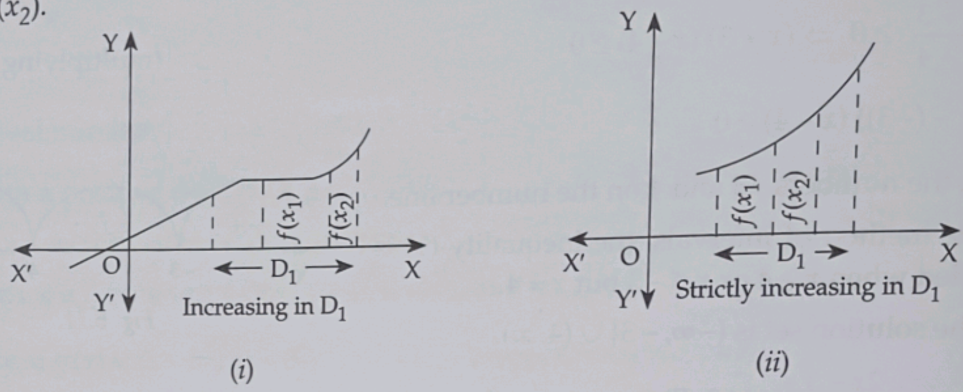

Fig. 6.13.

Analogously, f is called a ***decreasing function*** *in an interval D_2 (a subset of D) iff for all $x_1, x_2 \in D_2$, $x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$ and f is called a* ***strictly decreasing function*** *in D_2 iff for all $x_1, x_2 \in D_2$, $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$.*

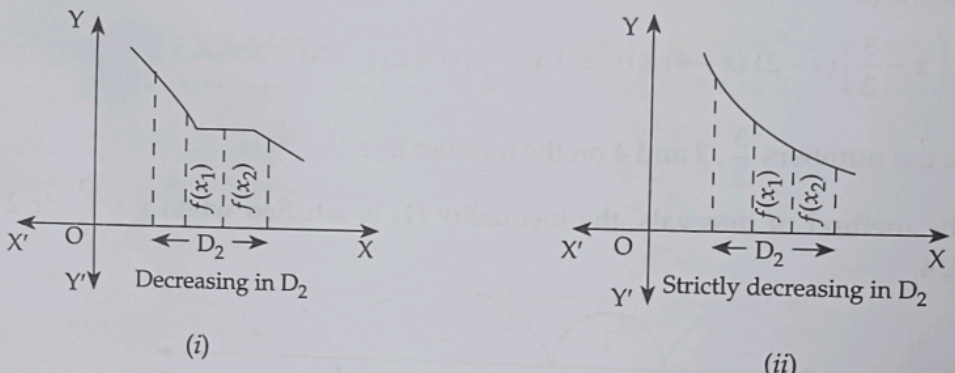

Fig. 6.14.

In particular, if $D_1 = D$ then f is called an **increasing function** iff for all $x_1, x_2 \in D$, $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$; and f is called **strictly increasing function** iff for all $x_1, x_2 \in D$, $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$. Analogously, if $D_2 = D$ then f is called a **decreasing function** iff for all $x_1, x_2 \in D$, $x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$; and f is called **strictly decreasing function** iff for all $x_1, x_2 \in D$, $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$.

A function which is either (strictly) increasing or (strictly) decreasing is called a ***(strictly) monotonic function.***

Conditions for an Increasing or a Decreasing Function

Now we shall see how to use derivative of a function to determine where it is increasing and w... it is decreasing.

In fact, we have:

(i) *If a function f is increasing in D_1 (a subset of D_f), then $f'(x) \ge 0$ for all $x \in D_1$.*

(ii) *If a function f is decreasing in D_2 (a subset of D_f), then $f'(x) \le 0$ for all $x \in D_2$.*