

$$\text{Now, consider } I = \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$$

Put $\cos x + \sin x = t$ so that $(\cos x - \sin x) dx = dt$

$$\text{Therefore } I = \int \frac{dt}{t} = \log |t| + C_2 = \log |\cos x + \sin x| + C_2$$

Putting it in (1), we get

$$\begin{aligned} \int \frac{dx}{1 + \tan x} &= \frac{x}{2} + \frac{C_1}{2} + \frac{1}{2} \log |\cos x + \sin x| + \frac{C_2}{2} \\ &= \frac{x}{2} + \frac{1}{2} \log |\cos x + \sin x| + \frac{C_1}{2} + \frac{C_2}{2} \\ &= \frac{x}{2} + \frac{1}{2} \log |\cos x + \sin x| + C, \left(C = \frac{C_1}{2} + \frac{C_2}{2} \right) \end{aligned}$$

EXERCISE 7.2

Integrate the functions in Exercises 1 to 37:

- | | | |
|-------------------------------------|---------------------------------|--|
| 1. $\frac{2x}{1+x^2}$ | 2. $\frac{(\log x)^2}{x}$ | 3. $\frac{1}{x+x \log x}$ |
| 4. $\sin x \sin (\cos x)$ | 5. $\sin (ax+b) \cos (ax+b)$ | |
| 6. $\sqrt{ax+b}$ | 7. $x\sqrt{x+2}$ | 8. $x\sqrt{1+2x^2}$ |
| 9. $(4x+2)\sqrt{x^2+x+1}$ | 10. $\frac{1}{x-\sqrt{x}}$ | 11. $\frac{x}{\sqrt{x+4}}, x > 0$ |
| 12. $(x^3-1)^{\frac{1}{3}} x^5$ | 13. $\frac{x^2}{(2+3x^3)^3}$ | 14. $\frac{1}{x(\log x)^m}, x > 0, m \neq 1$ |
| 15. $\frac{x}{9-4x^2}$ | 16. e^{2x+3} | 17. $\frac{x}{e^{x^2}}$ |
| 18. $\frac{e^{\tan^{-1} x}}{1+x^2}$ | 19. $\frac{e^{2x}-1}{e^{2x}+1}$ | 20. $\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$ |

21. $\tan^2(2x - 3)$ 22. $\sec^2(7 - 4x)$ 23. $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$
24. $\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$ 25. $\frac{1}{\cos^2 x (1 - \tan x)^2}$ 26. $\frac{\cos \sqrt{x}}{\sqrt{x}}$
27. $\sqrt{\sin 2x} \cos 2x$ 28. $\frac{\cos x}{\sqrt{1 + \sin x}}$ 29. $\cot x \log \sin x$
30. $\frac{\sin x}{1 + \cos x}$ 31. $\frac{\sin x}{(1 + \cos x)^2}$ 32. $\frac{1}{1 + \cot x}$
33. $\frac{1}{1 - \tan x}$ 34. $\frac{\sqrt{\tan x}}{\sin x \cos x}$ 35. $\frac{(1 + \log x)^2}{x}$
36. $\frac{(x+1)(x + \log x)^2}{x}$ 37. $\frac{x^3 \sin(\tan^{-1} x^4)}{1 + x^8}$

Choose the correct answer in Exercises 38 and 39.

38. $\int \frac{10x^9 + 10^x \log_e 10 \, dx}{x^{10} + 10^x}$ equals
- (A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$
 (C) $(10^x - x^{10})^{-1} + C$ (D) $\log(10^x + x^{10}) + C$
39. $\int \frac{dx}{\sin^2 x \cos^2 x}$ equals
- (A) $\tan x + \cot x + C$ (B) $\tan x - \cot x + C$
 (C) $\tan x \cot x + C$ (D) $\tan x - \cot 2x + C$

7.3.2 Integration using trigonometric identities

When the integrand involves some trigonometric functions, we use some known identities to find the integral as illustrated through the following example.

Example 7 Find (i) $\int \cos^2 x \, dx$ (ii) $\int \sin 2x \cos 3x \, dx$ (iii) $\int \sin^3 x \, dx$