Therefore, $\int \frac{x+3}{\sqrt{5-4x-x^2}}\,dx = -\frac{1}{2}\int \frac{(-4-2x)\,dx}{\sqrt{5-4x-x^2}} + \int \frac{dx}{\sqrt{5-4x-x^2}}$

$$= -\frac{1}{2}\,\mathrm{I}_1 + \mathrm{I}_2 \qquad \ldots (1)$$

In I_1, put $5-4x-x^2 = t$, so that $(-4-2x)\,dx = dt$.

Therefore, $\mathrm{I}_1 = \int \frac{(-4-2x)\,dx}{\sqrt{5-4x-x^2}} = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + \mathrm{C}_1$

$$= 2\sqrt{5-4x-x^2} + \mathrm{C}_1 \qquad \ldots (2)$$

Now consider $\mathrm{I}_2 = \int \frac{dx}{\sqrt{5-4x-x^2}} = \int \frac{dx}{\sqrt{9-(x+2)^2}}$

Put $x + 2 = t$, so that $dx = dt$.

Therefore, $\mathrm{I}_2 = \int \frac{dt}{\sqrt{3^2-t^2}} = \sin^{-1}\frac{t}{3} + \mathrm{C}_2$ [by 7.4 (5)]

$$= \sin^{-1}\frac{x+2}{3} + \mathrm{C}_2 \qquad \ldots (3)$$

Substituting (2) and (3) in (1), we obtain

$$\int \frac{x+3}{\sqrt{5-4x-x^2}} = -\sqrt{5-4x-x^2} + \sin^{-1}\frac{x+2}{3} + \mathrm{C}, \text{ where } \mathrm{C} = \mathrm{C}_2 - \frac{\mathrm{C}_1}{2}$$

EXERCISE 7.4

Integrate the functions in Exercises 1 to 23.

1. $\frac{3x^2}{x^6+1}$ **2.** $\frac{1}{\sqrt{1+4x^2}}$ **3.** $\frac{1}{\sqrt{(2-x)^2+1}}$

4. $\frac{1}{\sqrt{9-25x^2}}$ **5.** $\frac{3x}{1+2x^4}$ **6.** $\frac{x^2}{1-x^6}$

7. $\frac{x-1}{\sqrt{x^2-1}}$ **8.** $\frac{x^2}{\sqrt{x^6+a^6}}$ **9.** $\frac{\sec^2 x}{\sqrt{\tan^2 x+4}}$

10. $\dfrac{1}{\sqrt{x^2+2x+2}}$ **11.** $\dfrac{1}{9x^2+6x+5}$ **12.** $\dfrac{1}{\sqrt{7-6x-x^2}}$

13. $\dfrac{1}{\sqrt{(x-1)(x-2)}}$ **14.** $\dfrac{1}{\sqrt{8+3x-x^2}}$ **15.** $\dfrac{1}{\sqrt{(x-a)(x-b)}}$

16. $\dfrac{4x+1}{\sqrt{2x^2+x-3}}$ **17.** $\dfrac{x+2}{\sqrt{x^2-1}}$ **18.** $\dfrac{5x-2}{1+2x+3x^2}$

19. $\dfrac{6x+7}{\sqrt{(x-5)(x-4)}}$ **20.** $\dfrac{x+2}{\sqrt{4x-x^2}}$ **21.** $\dfrac{x+2}{\sqrt{x^2+2x+3}}$

22. $\dfrac{x+3}{x^2-2x-5}$ **23.** $\dfrac{5x+3}{\sqrt{x^2+4x+10}}$.

Choose the correct answer in Exercises 24 and 25.

24. $\int \dfrac{dx}{x^2+2x+2}$ equals

(A) $x \tan^{-1}(x+1) + C$ (B) $\tan^{-1}(x+1) + C$

(C) $(x+1)\tan^{-1}x + C$ (D) $\tan^{-1}x + C$

25. $\int \dfrac{dx}{\sqrt{9x-4x^2}}$ equals

(A) $\dfrac{1}{9}\sin^{-1}\left(\dfrac{9x-8}{8}\right)+C$ (B) $\dfrac{1}{2}\sin^{-1}\left(\dfrac{8x-9}{9}\right)+C$

(C) $\dfrac{1}{3}\sin^{-1}\left(\dfrac{9x-8}{8}\right)+C$ (D) $\dfrac{1}{2}\sin^{-1}\left(\dfrac{9x-8}{9}\right)+C$

7.5 Integration by Partial Fractions

Recall that a rational function is defined as the ratio of two polynomials in the form $\dfrac{P(x)}{Q(x)}$, where P (x) and Q(x) are polynomials in x and Q$(x) \neq 0$. If the degree of P(x) is less than the degree of Q(x), then the rational function is called proper, otherwise, it is called improper. The improper rational functions can be reduced to the proper rational