COMPETENCY BASED QUESTIONS

Objective Qs — 1 mark

1. If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then, $f(x)$ is:

(a) $x^3 + \frac{1}{x^3} - \frac{129}{8}$ (b) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(c) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (d) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

2. The value of $\int \sin^{-1}\left(\cos\left(\frac{\pi}{2} - 3x\right)\right)dx$ is:

(a) $\frac{x^2}{2} + C$ (b) $\frac{3x^2}{2} + C$

(c) $\frac{\sin 3x^2}{2} + C$ (d) none of these

3. The value of $\int \frac{(x^3+8)(x-1)}{x^2-2x+4}dx$ is:

(a) $\frac{x^3}{3} + \frac{x^2}{2} + 2x + C$

(b) $\frac{x^3}{3} + \frac{x^2}{2} - 2x + C$

(c) $\frac{x^3}{3} + x^2 - 2x + C$

(d) none of these

4. The value of $\int \frac{2x}{(2x+1)^2}dx$ is:

(a) $\frac{1}{2}\left[\log|2x+1| + \frac{1}{(2x+1)}\right] + C$

(b) $\frac{1}{2}\left[\log|2x+1| - \frac{1}{(2x+1)}\right] + C$

(c) $\left[\log|2x+1| + \frac{1}{(2x+1)}\right] + C$

(d) none of these

5. $\int \frac{1}{\sqrt{1+\cos 2x}}dx =$

(a) $\frac{1}{\sqrt{2}}\log|\sec x + \tan x| + C$

(b) $\frac{1}{\sqrt{2}}\log|\sec x - \tan x| + C$

(c) $\log|\sec x + \tan x| + C$

(d) none of these

6. $\int \frac{(x+1)e^x}{\cos^2(xe^x)}dx =$

(a) $-\cot(xe^x) + C$ (b) $\tan(xe^x) + C$

(c) $\tan(e^x) + C$ (d) $\cot(e^x) + C$

7. $\int \frac{dx}{\sqrt{16-6x-x^2}}$ is equal to:

(a) $\sin^{-1}(x+3) + C$

(b) $\frac{1}{5}\sin^{-1}\left(\frac{x+3}{5}\right) + C$

(c) $\sin^{-1}\left(\frac{x+3}{5}\right) + C$

(d) $\sin^{-1}\left(\frac{x-3}{5}\right) + C$

8. $\int \sqrt{1-25x^2}\,dx$ is equal to:

(a) $\frac{x}{2}\sqrt{1-25x^2}-\frac{1}{10}\sin^{-1}5x+C$

(b) $\frac{x}{2}\sqrt{1-25x^2}+\frac{1}{10}\sin^{-1}5x+C$

(c) $\frac{x}{2}\sqrt{25-x^2}+\frac{1}{10}\sin^{-1}5x+C$

(d) $\frac{x}{2}\sqrt{25-x^2}-\frac{1}{10}\sin^{-1}5x+C$

9. If $\int\frac{dx}{(x+2)(x^2+1)}=a\log|1+x^2|+b\tan^{-1}x+\frac{1}{5}\log|x+2|+C$, then:

(a) $a=\frac{-1}{10}, b=\frac{-2}{5}$ (b) $a=\frac{1}{10}, b=\frac{-2}{5}$

(c) $a=\frac{-1}{10}, b=\frac{2}{5}$ (d) $a=\frac{1}{10}, b=\frac{2}{5}$

10. $\int e^x\left(\frac{x\log x+1}{x}\right)dx$ is equal to:

(a) $\log(e^x\log x)+C$ (b) $\frac{e^x}{x}+C$

(c) $x\log x+e^x+C$ (d) $e^x\log x+C$

11. $\int_0^{2/3}\frac{dx}{4+9x^2}$ equals:

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$

(c) $\frac{\pi}{24}$ (d) $\frac{\pi}{4}$

12. The value of $\int_{e^2}^{e^3}\frac{dx}{x\log x}$ is:

(a) $\log\frac{2}{3}$ (b) $\log\frac{3}{2}$

(c) log 3 (d) log 2

13. $\int_0^{\pi/2}\sin^2 x\,dx$ is equal to:

(a) $\frac{\pi}{3}$ (b) 0

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

14. $\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ equals:

(a) 2 (b) $\frac{3}{4}$

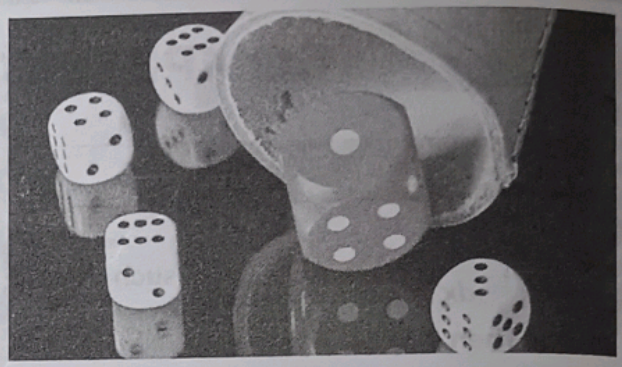
(c) 0 (d) −2

15. $\int_0^{\pi/2}\sqrt{1-\sin 2x}\,dx$ equals:

(a) $2\sqrt{2}$ (b) $2(\sqrt{2}+1)$

(c) 2 (d) $2(\sqrt{2}-1)$

16. Samrat, Kanchan and few friends were playing a game with die. They discussed ways to calculate the probability of rolling a particular digit. Samrat came up with the idea of calculating the probability from the probability distribution function, which he assumed to be $e^{|x|}$.



$\int_{-1}^{1}e^{|x|}\,dx=$

(a) $2e-3$ (b) $2e+2$

(c) $2e-4$ (d) $2e-2$

17. $\int[e^x\log(\sec x+\tan x)+e^x\sec x]\,dx$ equal to:

(a) $e^x\log\tan x+C$

(b) $e^x\log\sec x+C$

(c) $e^x\log(\sec x+\tan x)+C$

(d) $e^x\log(\text{sex}\,x-\tan x)+C$

In the question number 18 to 25, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option as:

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

18. Assertion (A): $\int x^2dx=\frac{x^3}{3}+C$

Reason (R): $\int e^{x^2}dx=e^{x^3/3}+C$

19. Assertion (A): $\int x^x(1+\log x)\,dx=x^x+C$

Reason (R): $\frac{d}{dx}(x^x)=x^x(1+\log x)$

20. Assertion (A): $\int e^x[\sin x + \cos x]\,dx = e^x \cos(x) + C$

Reason (R): $\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C$

21. Assertion (A): $\int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx = \frac{\pi}{4}$

Reason (R): $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

22. Assertion (A): $\int_{-3}^{3} (x^3 + 5)dx = 30$

Reason (R): $f(x) = x^3 + 5$ is an odd function.

23. Assertion (A): $\frac{d}{dx}\left[\int_0^{x^2} \frac{dt}{t^2+4}\right] = \frac{2x}{x^4+4}$

Reason (R): $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

24. Assertion (A): $\int_{-1}^{1} (x^3 + \sin x + 2)\,dx = 0$

Reason (R): $\int_{-a}^{a} f(x)dx$

$$= \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is an even function } i.e., f(-x) = f(x) \\ 0, & \text{if } f(x) \text{ is an odd function } i.e., f(-x) = -f(x) \end{cases}$$

25. Assertion (A): The value of the integral $\int_{-\pi/4}^{\pi/4} \cos\theta \log\left[\frac{5 + 2\sin\theta}{5 - 2\sin\theta}\right] d\theta$ is zero

Reason (R): $\int_{-a}^{a} f(x)dx = 0$, if $f(-x) = -f(x)$.

Case Based Qs — 4 & 5 marks

26. *A function f(x) is a continuous function defined on [0, a], then* $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Based on the given information, answer the following questions.

(A) If $f(x) = \frac{\sin x - \cos x}{1 + \sin x \cos x}$, then what is the value of $f\left(\frac{\pi}{2} - x\right)$? Hence, evaluate $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

(B) Evaluate $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$.

(C) Find the value of $\int_0^{\pi/2} \cos^2 x\,dx$.

Very Short & Short Qs — 1 - 3 marks

27. Find the integration of $\frac{\text{cosec}^2 x}{\sec^2 x}$.

28. Evaluate: $\int \frac{x^2}{\sqrt{x^6 + b^6}} dx$

29. Evaluate: $\int 3^{3^{3^x}} 3^{3^x} 3^x dx$

30. Evaluate: $\int \frac{x^2+3}{x^2-1} dx$

31. Find $\int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x} dx$

32. Evaluate: $\int_0^3 x\sqrt{x+6}\,dx$

33. By using of properties of definite integrals, evaluate: $\int_0^2 (2-x)^m x\,dx$

34. Show that $\int_0^a f(x)\,h(x)\,dx = 3\int_0^a f(x)\,dx$, if f and h are defined as $f(x) = f(a-x)$ and $h(x) + h(a-x) = 6$.

35. Evaluate: $\int \frac{2x^4 + 7x^3 + 6x^2}{x^2 + 2x} dx$

36. Evaluate: $\int_0^{\sqrt{3}/2} \sin^{-1} x\,dx$

37. Evaluate: $\int_1^3 \frac{1}{x^2(x+1)} dx$

38. Evaluate the integral:

$$\int_0^{\frac{\pi}{2}} \frac{\sin\theta d\theta}{(25+\cos\theta)(26+\cos\theta)}$$

Show your steps.

39. Evaluate: $\int_0^{\pi} \log(1+\cos x)\,dx$, when

$$\int_0^{\pi/2} \log\sin x\,dx = -\frac{\pi}{2}\log 2$$

40. Evaluate: $\int_0^{\pi} \frac{e^0}{13e^0+12\cos x}dx$

41. Evaluate: $\int_{\pi/4}^{\pi/2} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$

Long Qs — 4 & 5 marks

42. Solve the following integrals.

(A) $\int \frac{x^3}{(x+3)^4}dx$

(B) $\int \frac{x^6+8}{x^2+2}dx$

43. Evaluate the following integrals:

(A) $\int \cos 2x\cos 4x\cos 6x\,dx$

(B) $\int \sin^4 x\cos^4 x\,dx$

44. Integrate the given function and show your steps.

$$\int x^7 \sin(2x^4)\,dx$$

45. Evaluate: $\int_0^{\pi/2} \frac{dx}{(a^2\cos^2 x+b^2\sin^2 x)^2}$

46. Evaluate the integral and show your steps.

$$\int \frac{1}{8-x^3}dx$$

47. $\int_1^4 \{|x-1|+|x-2|+|x-3|\}dx$

48. If $\int_{-\pi/4}^{\pi/4} e^{-x}|\cos 2x|\,dx = \alpha(e^{-\frac{\pi}{4}}+e^{\frac{\pi}{4}})$

then find the value of α.

NCERT, EXEMPLAR & DIKSHA QUESTIONS

Objective Qs — 1 mark

49. Integrate: $\frac{2^x}{3^x}dx$

(a) $x\log 2 - \log 3 + C$

(b) $\frac{\left(\frac{2}{3}\right)^x}{\log\left(\frac{2}{3}\right)}+C$

(c) $\frac{\left(\frac{3}{2}\right)^x}{\log\left(\frac{3}{2}\right)}+C$

(d) none of these [DIKSHA]

50. $\int \frac{x^3}{x+1}dx$ is:

(a) $x+\frac{x^2}{2}+\frac{x^3}{3}-\log|1-x|+C$

(b) $x+\frac{x^2}{2}-\frac{x^3}{3}-\log|1-x|+C$

(c) $x-\frac{x^2}{2}-\frac{x^3}{3}-\log|1+x|+C$

(d) $x-\frac{x^2}{2}+\frac{x^3}{3}-\log|1+x|+C$

51. $\int \frac{dx}{\sin(x-a)\sin(x-b)}$ is:

(a) $\sin(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+C$

(b) $\operatorname{cosec}(b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+C$

(c) $\operatorname{cosec}(b-a)\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+C$

(d) $\sin(b-a)\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+C$

52. $\int \frac{x}{(x-1)(x-2)}dx$ is:

(a) $\log\left|\frac{(x-1)^2}{x-2}\right|+C$

(b) $\log\left|\frac{(x-2)^2}{x-1}\right|+C$

(c) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+C$

(d) $\log|(x-1)(x-2)|+C$

53. $\int \frac{1}{x(x^2+1)}\,dx$ is equal to:

(a) $\log|x| - \frac{1}{2}\log(x^2+1) + C$

(b) $\log|x| + \frac{1}{2}\log(x^2+1) + C$

(c) $-\log|x| + \frac{1}{2}\log(x^2+1) + C$

(d) $\frac{1}{2}\log|x| + \frac{1}{2}\log(x^2+1) + C$

54. $\int \frac{x+\sin x}{1+\cos x}\,dx$ is equal to:

(a) $\log|1+\cos x| + C$ (b) $\log|1+\sin x| + C$

(c) $x - \tan\frac{x}{2} + C$ (d) $x\tan\frac{x}{2} + C$

55. $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to:

(a) $\frac{e^x}{1+x^2} + C$ (b) $\frac{-e^x}{1+x^2} + C$

(c) $\frac{e^x}{(1+x^2)^2} + C$ (d) $\frac{-e^x}{(1+x^2)^2} + C$

Very Short & Short Qs — 1 - 3 marks

56. Verify that: $\int \frac{2x-1}{2x+3}\,dx = x - \log|(2x+3)^2| + C$

57. Evaluate: $\int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}\,dx$

58. Evaluate: $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}}\,dx$

59. Evaluate: $\int \frac{dx}{x\sqrt{x^4-1}}$

60. Evaluate: $\int \sqrt{5-2x+x^2}\,dx$

61. Evaluate: $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x}\,dx$

62. Evaluate: $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}}\,dx$

63. Evaluate: $\int \sqrt{2ax - x^2}\,dx$

Long Qs — 4 & 5 marks

64. Evaluate: $\int \frac{x^2}{x^4 - x^2 - 12}\,dx$

65. Evaluate: $\int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x)\,dx$

PREVIOUS YEAR QUESTIONS

Objective Qs — 1 mark

66. Which of these is equal to $\int e^{(x\log 5)} e^x\,dx$, where C is the constant of integration?

(a) $\frac{(5e)^x}{\log 5e} + C$ (b) $\log 5^x + x + C$

(c) $5^x e^x + C$ (d) $(5e)^x \log x + C$

[CBSE Practice Set-1 2023]

67. For any integer n, the value of

$\int_{-\pi}^{\pi} e^{\cos^2 x} \sin^3(2n+1)x\,dx$ is:

(a) –1 (b) 0

(c) 1 (d) 2 **[CBSE SQP 2023]**

68. Anti-derivative of $\frac{\tan x - 1}{\tan x + 1}$ with respect to x is:

(a) $\sec^2\left(\frac{\pi}{4} - x\right) + c$

(b) $-\sec^2\left(\frac{\pi}{4} - x\right) + c$

(c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$

(d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + c$ **[CBSE 2023]**

69. The value of $\int_{-\pi/2}^{\pi/2} x^3 \sin^4 x \, dx$ is:

(a) 0 (b) $\frac{\pi}{2}$

(c) π (d) $\frac{\pi^2}{4}$

[CBSE Practice Set-2 2023]

70. The value of $\int_0^{\pi} \tan^2\left(\frac{\theta}{3}\right) d\theta$ is:

(a) $\pi+\sqrt{3}$ (b) $3\sqrt{3}-\pi$

(c) $\sqrt{3}-\pi$ (d) $\pi-\sqrt{3}$

[CBSE 2024]

Very Short & Short Qs 1 - 3 marks

71. Find: $\int \frac{2x}{(x^2+1)(x^2-4)} dx$. [CBSE 2024]

72. Evaluate: $\int_{-1}^{1} \log_e\left(\frac{2-x}{2+x}\right) dx$.

[CBSE SQP 2023]

73. Solve the integral:

$$I = \int \frac{3x+5}{x^2+4x+7} dx$$

Show your work.

[CBSE Practice Set-1 2023]

74. Using the properties of definite integrals, prove the following:

$\int_0^{\pi} h(\sin x)\, dx = 2\int_0^{\frac{\pi}{2}} h(\sin x)\, dx$, where $h(\sin x)$ is a function of $\sin x$. State the property used.

[CBSE Practice Set-1 2023]

75. Find: $\int \frac{2x^2+3}{x^2(x^2+9)} dx$; $x \neq 0$. [CBSE SQP 2023]

76. Find: $\int \sqrt{\frac{x}{1-x^3}} dx$; $x \in (0, 1)$.

[CBSE SQP 2023]

77. Evaluate: $\int_0^{\frac{\pi}{4}} \log(1+\tan x) dx$.

[CBSE SQP 2023]

78. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(e^x+e^{-x})(e^x-e^{-x})} dx$.

[CBSE 2023]

79. Evaluate $\int_{-1}^{1} |x^4 - x| dx$. [CBSE 2023]

80. Find $\int e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$. [CBSE 2023]

81. Find $\int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$

[CBSE Practice Set-2 2023]

82. Find $\int \frac{\sin\phi}{\sqrt{\sin^2\phi + 2\cos\phi + 3}} d\phi$

[CBSE Practice Set-2 2023]

83. Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$.

[CBSE Practice Set-2 2023]

84. Evaluate $\int_0^{\pi} \log(1+\cos x) dx$

[CBSE Practice Set-2 2023]

85. Find: $\int \sec^3\theta d\theta$ [CBSE 2024]

86. Evaluate: $\int_0^{\frac{\pi}{4}} \frac{x\, dx}{1+\cos 2x + \sin 2x}$

[CBSE 2024]

87. Find: $\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1+x^2}}\right] dx$

[CBSE 2024]

88. Evaluate: $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

[CBSE SQP 2022]