JEE MAINS AND ADVANCED

1. For any natural number m, evaluate
$$\int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{1/m} dx, x > 0$$

2. Let A be a 3 × 3 matrix of non-negative real elements such that $A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Then the maximum value of det(A) is...................

3. Let $\alpha \in (0, \infty)$ and $A = \begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. If $\det\left(\text{adj}(2A - A^T)\right) \cdot \text{adj}(A - 2A^T) = 2^8$, then $(\det(A))^2$ is equal to

(a) 1   (b) 16   (c) 36   (d) 49

4. Let the sum of the maximum and the minimum values of the function $f(x) = \frac{2x^2 - 3x + 8}{2x^2 + 3x + 8}$ be $\frac{m}{n}$, where $\gcd(m, n) = 1$. Then $m + n$ is equal to

(a) 182   (b) 217   (c) 201   (d) 195

5. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is

(a) $2x^2 + 11x + 12 = 0$ (b) $x^2 + 8x + 12 = 0$

(c) $4x^2 + 14x + 12 = 0$ (d) $x^2 + 10x + 16 = 0$

6. If the domain of the function: $\sin^{-1}\left(\frac{3x-22}{2x-19}\right) + \log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha, \beta]$, then $3\alpha + 10\beta$ is equal to

(a) 95   (b) 97   (c) 98   (d) 100