## EXERCISE 4.5

*Using Cramer's rule, solve the following (1 to 7) systems of linear equations:*

1. (*i*) $3x + y = 5$
$x + 2y = 3$

(*ii*) $2x - y = 17$
$3x + 5y = 6.$

2. (*i*) $2x - 7y - 13 = 0$
$5x + 6y - 9 = 0$

(*ii*) $\frac{2}{x} + \frac{3}{y} = 2$
$\frac{5}{x} + \frac{8}{y} = \frac{31}{6}.$

**Hint.** (*ii*) Put $x = \frac{1}{u}$ and $y = \frac{1}{v}$.

3. $3x + ay = 4$
$2x + ay = 2,\ a \neq 0.$

4. (*i*) $5x - 7y + z = 11$
$6x - 8y - z = 15$
$3x + 2y - 6z = 7$

(*ii*) $x + y + z + 1 = 0$
$x + 2y + 3z + 4 = 0$
$x + 3y + 4z + 6 = 0.$

5. (*i*) $4x - 2y + 9z + 2 = 0$
$3x + 4y + z - 5 = 0$
$x - 3y + 2z - 8 = 0$

(*ii*) $x + y = 2$
$2x - z = 1$
$2y - 3z = 1.$

6. (*i*) $2x - 3y = 1$
$x + 3z = 11$
$x + 2y + z = 7$

(*ii*) $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$
$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$
$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2.$

7. $x + y + z = 1$
$ax + by + cz = k$
$a^2x + b^2y + c^2z = k^2$ where $a$, $b$, $c$ are all different.

8. The sum of three numbers is 20. If we multiply the first number by 2 and add the second number to the result and subtract the third number, we get 23. By adding second and third numbers to three times the first number, we get 46. Represent the above problem algebraically and use Cramer's rule to find the numbers from these equations.

9. Which of the following equations are consistent? If consistent, solve them.

(i) $3x + y = 5$
$6x + 2y = 11$

(ii) $x + 2y = 3$
$4x + 8y = 12$.

10. Which of the following equations are consistent? If consistent, solve them.

(i) $x - y + 3z = 6$
$x + 3y - 3z = -4$
$5x + 3y + 3z = 14$

(ii) $4x - 2y + 6z = 8$
$2x - y + 3z = 4$
$2x - y + 3z = 13$.

(iii) $2x + y - 2z = 4$
$x - 2y + z = -2$
$5x - 5y + z = -2$.

11. Which of the following systems has non-trivial solutions? If so, find these solutions.

(i) $5x + 5y + 2z = 0$
$2x + 5y + 4z = 0$
$4x + 5y + 2z = 0$

(ii) $2x - 3y - z = 0$
$x + 3y - 2z = 0$
$x - 3y = 0$.

12. If the system of equations $x - ky - z = 0$, $kx - y - z = 0$, $x + y - z = 0$ has a non-zero solution, then find the possible values of $k$.

13. Find the real value(s) of $a$ for which the system of equations
$x + ay = 0$, $y + az = 0$, $z + ax = 0$
has infinitely many solutions.

14. If the equations $x = cy + bz$, $y = az + cx$, $z = bx + ay$ are consistent, prove that $a^2 + b^2 + c^2 + 2abc = 1$.

## Answers

1. (i) $x = \frac{7}{5}, y = \frac{4}{5}$ (ii) $x = 7, y = -3$
2. (i) $x = 3, y = -1$ (ii) $x = 2, y = 3$
3. $x = 2, y = -\frac{2}{a}$
4. (i) $x = 1, y = -1, z = -1$ (ii) $x = 1, y = -1, z = -1$
5. (i) $x = 7, y = -3, z = -4$ (ii) $x = \frac{3}{4}, y = \frac{5}{4}, z = \frac{1}{2}$
6. (i) $x = 2, y = 1, z = 3$ (ii) $x = 2, y = 3, z = 5$
7. $x = \frac{(k-b)(c-k)}{(a-b)(c-a)}, y = \frac{(a-k)(c-k)}{(a-b)(b-c)}, z = \frac{(b-k)(k-a)}{(b-c)(a-b)}$
8. 13, 2, 5
9. (i) Inconsistent (ii) Consistent; $x = 3 - 2k, y = k$, $k$ is any real number
10. (i) Inconsistent (ii) Inconsistent
(iii) Consistent; $x = \frac{3}{5}(k+2), y = \frac{4}{5}(k+2), z = k$, $k$ is any real number
11. (i) Only trivial solution; $x = 0, y = 0, z = 0$
(ii) It has non-trivial solution; $x = 3k, y = k, z = 3k$ where $k$ is any real number
12. 1, −1
13. −1

The given system of equations is

$$x - y + 0z = 3$$
$$2x + 3y + 4z = 17$$
$$0x + y + 2z = 7$$

This system can be written as $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

or $\quad AX = C$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$.

As $A^{-1}$ exists, the given system has a unique solution $X = A^{-1} C$

$$\Rightarrow X = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\Rightarrow x = 2, y = -1, z = 4.$

Hence, the solution of the given system of equations is

$$x = 2, y = -1, z = 4.$$

## EXERCISE 4.6

1. Use matrix method to solve the following system of equations:

(i) $5x + 2y = 4$
$7x + 3y = 5$

(ii) $4x - 3y = 3$
$3x - 5y = 7.$

2. Using matrix method, determine whether the following system of equations is consistent or inconsistent:

(i) $x + 2y = 2$
$2x + 3y = 3$

(ii) $x + 2y = 9$
$2x + 4y = 7$

(iii) $3x - y + 2z = 3$
$2x + y + 3z = 5$
$x - 2y - z = 1$

(iv) $5x - y + 4z = 5$
$2x + 3y + 5z = 2$
$5x - 2y + 6z = -1$

3. Solve the following system of equations by using matrix method:

$x + y = 1$
$7x + 7y = 7.$

4. Using matrix method, solve the following system of equations:

(i) $x - 2y + 3z = 6$
$x + 4y + z = 12$
$x - 3y + 2z = 1$

(ii) $x + y + z = 6$
$x - y + z = 2$
$2x + y - z = 1$

(iii) $x + y + z = 9$
$2x + 5y + 7z = 52$
$2x + y - z = 0$

(iv) $x - 2y = 10$
$2x - y - z = 8$
$-2y + z = 7$

(v) $3x + y + z = 1$
$2x + 2z = 0$
$5x + y + 2z = 2$

(vi) $2x - 3y + 5z = 11$
$3x + 2y - 4z = -5$
$x + y - 2z = -3$

5. Solve the following system of linear equations using matrix method:

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=9,\ \frac{2}{x}+\frac{5}{y}+\frac{7}{z}=52,\ \frac{2}{x}+\frac{1}{y}-\frac{1}{z}=0.$

6. State the condition under which the following equations have a unique solution:

$x+2y-2z+5=0$

$-x+3y+4=0$

$-2y+z-4=0.$

Using Martin's rule, find the unique solution of the above system of linear equations.

7. The sum of three numbers is 6. If we multiply third number by 3 and add second to it we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

8. If $A=\begin{bmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{bmatrix}$, find $A^{-1}$. Using $A^{-1}$, solve the system of linear equations:

$x+2y-3z=-4,\ 2x+3y+2z=2,\ 3x-3y-4z=11.$

9. If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find $A^{-1}$. Using $A^{-1}$, solve the following system of equations:

$2x-3y+5z=11,\ 3x+2y-4z=-5,\ x+y-2z=-3.$

10. If $A=\begin{bmatrix}3 & 5\\ -2 & 3\end{bmatrix}$, find $A^{-1}$ and use it to solve the system of equations:

$3x-2y=7,\ 5x+3y=1.$

11. If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$, find $A^{-1}$ and use it to solve the system of equations:

$x+2y+z=4,\ -x+y+z=0,\ x-3y+z=2.$

12. Given two matrices A and B where $A=\begin{bmatrix}1 & -2 & 3\\ 1 & 4 & 1\\ 1 & -3 & 2\end{bmatrix}$ and $B=\begin{bmatrix}11 & -5 & -14\\ -1 & -1 & 2\\ -7 & 1 & 6\end{bmatrix}$, find AB and use this result to solve the following system of equations:

$x-2y+3z=6,\ x+4y+z=12,\ x-3y+2z=1.$

13. The equilibrium conditions for three competitive markets are described as given below, where $p_1$, $p_2$ and $p_3$ are the equilibrium price for each market respectively:

$p_1+2p_2+3p_3=85,\ 3p_1+2p_2+2p_3=105,\ 2p_1+3p_2+2p_3=110$

Using matrix method, find the values of respective equilibrium prices.

## Answers

1. (i) $x=2, y=-3$ (ii) $-\frac{6}{11}, -\frac{19}{11}$
2. (i) consistent (ii) inconsistent (iii) inconsistent (iv) consistent
3. $x=1-k, y=k$, where $k$ is any number
4. (i) $x=1, y=2, z=3$ (ii) $x=1, y=2, z=3$ (iii) $x=1, y=3, z=5$ (iv) $x=0, y=-5, z=-3$ (v) $x=1, y=-1, z=-1$ (vi) $x=1, y=2, z=3$

5. $x = 1, y = \frac{1}{3}, z = \frac{1}{5}$

6. Matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ must be inversible; $x = 1, y = -1, z = 2$.

7. The first, second and third numbers are 1, 2 and 3 respectively.

8. $\frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}; x = 3, y = -2, z = 1$

9. $\begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}; x = 1, y = 2, z = 3$

10. $\frac{1}{19}\begin{bmatrix} 3 & -5 \\ 2 & 3 \end{bmatrix}; x = \frac{23}{19}, y = -\frac{32}{19}$

11. $\frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}; x = \frac{9}{5}, y = \frac{2}{5}, z = \frac{7}{5}$

12. $AB = -8I; x = 1, y = 2, z = 3$

13. $p_1 = 15, p_2 = 20, p_3 = 10$