MATHEMATICS
CLASS XII

1. Show that the relation R in the set R of real numbers, defined as $R = \{(a, b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

2. Check whether the relation R in R defined by $R = \{(a, b): a \leq b^3\}$ is reflexive, symmetric or transitive.

3. Give an example of a relation. Which is
(i) Symmetric but neither reflexive nor transitive.
(ii) Transitive but neither reflexive nor symmetric.
(iii) Reflexive and symmetric but not transitive.
(iv) Reflexive and transitive but not symmetric.
(v) Symmetric and transitive but not reflexive.

4. Let A = {1,2,3}. Then, the number of relations containing (1,2) and (1,3) which are reflexive and symmetric but not transitive is
a) 1 b) 2 c) 3 d) 4

5. Let A = {1,2,3}. Then the number of equivalence relations containing (1,2) is
a) 1 b) 2 c) 3 d) 4

6. Show that the relation R in the set Z of integers given by R = {(a, b): 2 divides a − b} is an equivalence relation.

7. The maximum number of equivalence relations on the set A = {1,2,3} are
a) 1 b) 2 c) 3 d) 5

8. Prove that: $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$ [CBSE Delhi 2011, 2014; (AI) 2009; (F) 2016]

9. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x, -\frac{1}{\sqrt{2}} \leq x \leq 1$ [CBSE (AI) 2011, 2014]

10. What is the principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$? [CBSE (AI) 2011]

11. Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

12. If A and B are square matrices of the same order 3 , such that $|A| = 2$ and $AB = 2I$, write the value of $|B|$.

[CBSE Delhi 2019]

13. Show that the matrix $B^T AB$ is symmetric or skew-symmetric according as A is symmetric or skew-symmetric. [HOTS]

14. If A and B are square matrices of the same order, then $(A + B)(A - B)$ is equal to
(a) $A^2 - B^2$ (b) $A^2 - BA - AB - B^2$
(c) $A^2 - B^2 + BA - AB$ (d) $A^2 - BA + B^2 + AB$

15. If A and B are matrices of same order, then $(AB' - BA')$ is a
(a) skew-symmetric matrix (b) null matrix
(c) symmetric matrix (d) unit matrix

16. Show that all the diagonal elements of a skew symmetric matrix are zero. [CBSE Delhi 2017]

17. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $A^2 = 4A - 3I$. Hence find A^{-1}.

[CBSE (F) 2015]

18. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$.

19. Find the value of $\sin^{-1}\left(\sin \frac{4\pi}{5}\right)$. [CBSE (AI) 2010]

20. Let A be a 3×3 square matrix such that $A(\text{adj } A) = 2I$, where I is the identity matrix. The value of $|\text{adj } A|$ is
(a) 4 (b) -4 (c) 0 (d) None of these

21. If A is square matrix of order 3×3 such that $|A| = 2$, then write the value of $|\text{adj (adj } A)|$.
(a) -16 (b) 16 (c) 0 (d) 2

22. If A is a square matrix of order 3 , such that $A(\text{adj } A) = 10I$, then $|\text{adj } A|$ is equal to
(a) 1 (b) 10 c) 100 (d) 101 [CBSE 2020 (65/5/1)]

23. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$.

[CBSE (AI) 2017]

24. Find the value of a and b such that the function $f(x)$ defined by:

$$f(x) = \begin{cases} 5; \text{ if } x \leq 2 \\ ax + b; \text{ if } 2 < x < 10 \\ 21; \text{ if } x \geq 10 \end{cases} \text{ is a continuous function}$$

[CBSE Delhi 2011]

25. If $y = x^{\sin x} + (\sin x)^{\cos x}$, find $\frac{dy}{dx}$. [CBSE Delhi 2009, (F) 2013]

OR

If $y = x^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$. [CBSE 2019 (65/4/1)]

26. If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$. [CBSE (AI) 2012; Delhi 2012]

27. If $y = \cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0 < x < \frac{\pi}{2}$ then find the value of $\frac{dy}{dx}$. [CBSE Delhi 2008; (F) 2013]

28. The function $f(x) = e^{|x|}$ is [NCERT Exemplar]

(a) continuous everywhere but not differentiable at $x = 0$
(b) continuous and differentiable everywhere
(c) not continuous at $x = 0$
(d) none of these

27. Let $f(x) = |\sin x|$. Then [NCERT Exemplar]

(a) f is everywhere differentiable
(b) f is everywhere continuos but not differentiable at $x = n\pi : n \in Z$
(c) f is everywhere continuous but not differentiable at $x = (2n + 1)\frac{\pi}{2}, n \in Z$

(d) none of these

28. The number of points of discontinuity of f defined by $f(x) = |x| - |x+1|$ is [CBSE 2020(65/4/1)]

29. If $y = (\cos\ x)^{(\cos\ x)^{(\cos\ x)^{-\infty}}}$, then show that $\frac{dy}{dx} = \frac{y^2 \tan\ x}{y \log\ \cos\ x - 1}$. [NCERT Exemplar]

30. Show that the function $f(x) = 2x - |x|$ is continuous but not differentiable at $x = 0$. [CBSE (F) 2013]

31. Find the value of ' a ' for which the function f defined as

$$f(x) = \begin{cases} a\sin\ \frac{\pi}{2}(x+1), & x \leq 0 \\ \frac{\tan\ x - \sin\ x}{x^3}, & x > 0 \end{cases}$$ is continuous at $x = 0$. [CBSE Delhi 2011; (South) 2016]

32. Differentiate the following function with respect to x :

$$y = (\sin\ x)^x + \sin^{-1}\ \sqrt{x}$$ (*CBSEDelhi*2009,2013,2017)

33. Differentiate with respect to x :

$\sin^{-1}\ \left(\frac{2^{x+1} \cdot 3x}{1+(36)^x}\right)$ [CBSE (AI) 2013]

34. Differentiate $\tan^{-1}\ \left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1}\ x^2$. [CBSE (South) 2016, 2019 (65/4/1)]

35. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum?

36. Prove that the volume of the largest cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ of the volume of the sphere. [CBSE Delhi 2016; (AI) 2014; (F) 2013]

OR

Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$. [CBSE (F) 2012]

[Hint: replace a by R and you can get the result]

37. Show that the right-circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base. [CBSE (AI) 2011] [HOTS]

38. Find the maximum area of the isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of major axis. [CBSE Bhubaneshwar 2015, (AI) 2008]

39. Prove that the surface area of a solid cuboid, of square base and given volume, is minimum when it is a cube. [CBSE (AI) 2017; (F) 2009; CBSE 2005]

40. If the sum of hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$. [CBSE Delhi 2017; (AI) 2009, 2014; (Central) 2016]

41. A man 1.6 m tall walks at the rate of 0.3 m/sec away from a street light that is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening?

42. Using the method of integration, find the area of the triangle whose vertices are (1,0), (2,2) and (3,1). (All India 2019)

43. Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1, y = 3x + 1$ and x = 4. (All India 2019, 2011C; Delhi 2011)

44. Using integration, find the area of the region bounded by the curves $y = |x + 1| + 1, x = -3$, x = 3 and y = 0. (Delhi 2014C)

45. Using integration, find the area of the region bounded by the lines $2x + y = 4, 3x - 2y = 6$ and $x - 3y + 5 = 0$. (All India 2014C; Foreign 2011; Delhi 2009)

46. Find $\int \frac{\sin^6 x}{\cos^8 x} dx$. (All India 2014C)

47. Evaluate $\int \frac{dx}{\sin^2 x \cos^2 x}$. (Delhi 2014C; Foreign 2014)

48. Evaluate $\int \cos^{-1}(\sin x)\, dx$. (Delhi 2014C)

49. Find $\int \frac{dx}{\sin x + \sin 2x} dx$. (Delhi 2015)

50. Integrate w.r.t. x, $\frac{x^2 - 3x + 1}{\sqrt{1 - x^2}}$ (Delhi 2015)

51. Evaluate the following integral $\int_{-1}^{2} |x \sin \pi x| dx$

52. Evaluate $\int_{-1}^{\frac{3}{2}} |x \sin(\pi x)| dx$.

53. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1 + e^x} dx$ is equal to -

a) $\frac{\pi^2}{4} - 2$ b) $\frac{\pi^2}{4} + 2$ c) $\pi^2 - e^{\frac{\pi}{2}}$ d) $\pi^2 + e^{\frac{\pi}{2}}$

Integrate the following –

54. $\frac{e^{2x} - 1}{e^{2x} + 1}$

55. $\frac{1}{1 + \cot x}$

56. $\frac{1}{1 - \tan x}$

57. $\frac{1}{\sin x \cos^3 x}$

58. $\frac{1}{\cos(x - a)\cos(x - b)}$

53. $\frac{x^3 + x + 1}{x^2 - 1}$

54. $\frac{1 - x^2}{x(1 - 2x)}$

55. $\frac{1}{x(x^n + 1)}$

56. $\frac{1}{(e^x - 1)}$

57. $\frac{xe^x}{(1 + x)^2}$

58. $e^x\left(\frac{1+\sin x}{1+\cos x}\right)$

59. $\frac{(x-3)e^x}{(x-1)^3}$

60. Evaluate $\int_{-1}^{2}|x^3-x|dx$

61. Evaluate $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$

62. Evaluate $\int_0^{\frac{\pi}{2}}\log\ \sin\ xdx$

63. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\sin^7\ xdx$

64. $\int_2^8|x-5|dx$

65. $\int_0^{\frac{\pi}{4}}\log(1+\tan\ x)dx$

66. Find $\int[\sqrt{\cot\ x}+\sqrt{\tan\ x}]dx$

67. Evaluate $\int_0^{\pi}\frac{xdx}{a^2\cos^2\ x+b^2\sin^2\ x}$

68. $\frac{1}{\sqrt{\sin^3\ x\sin(x+\alpha)}}$

69. $\tan^{-1}\ \sqrt{\frac{1-x}{1+x}}$

70. $\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$

71. $\int_1^4[|x-1|+|x-2|+|x-3|]dx$

72. Prove that: $\int_{-1}^{1}x^{17}\cos^4\ xdx=0$

73. Find $\int\frac{x^2-3x+1}{\sqrt{1-x^2}}$dx Delhi 2015

80. Evaluate $\int\frac{\sqrt{1-sinx}}{\sqrt{1+cosx}}e^{-\frac{x}{2}}$dx All India 2013 C

81.Find $\int\frac{sinx}{sin^3x+cos^3x}$ dx CBSE Sample Paper 2017

82. Evaluate $\int\frac{1}{sin^4x+sin^2xcos^2x+cos^4x}$dx All India 2014

83. The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

83. In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs 100 double itself in 10 years ($\log_e 2 = 0.6931$).

84. In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years ($e^{0.5} = 1.648$).

85. In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

86. $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0; y = \frac{\pi}{4}$ when $x = 1$

87. $2xy + y^2 - 2x^2\frac{dy}{dx} = 0; y = 2$ when $x = 1$

88. $\cos^2 x\frac{dy}{dx} + y = \tan x\left(0 \le x < \frac{\pi}{2}\right)$

89. $(1 + x^2)dy + 2xydx = \cot xdx(x \neq 0)$

90. $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}; y = 0$ when $x = 1$

91. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

92. If $\vec{a} = 2\hat{\imath} - \hat{\jmath} - 2\hat{k}$ and $\vec{b} = 7\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ then express $\vec{b}$ in the form of $\vec{b} = \vec{b}_1 + \vec{b}_2$ where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$.

94. Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

95. Find the shortest distance between the lines whose vector equations are $\vec{r} = (1 - t)\hat{\imath} + (t - 2)\hat{\jmath} + (3 - 2t)\hat{k}$ and $\vec{r} = (s + 1)\hat{\imath} + (2s - 1)\hat{\jmath} - (2s + 1)\hat{k}$

96. Find the vector equation of the line passing through the point $(1,2,-4)$ and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

97. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

98. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

99. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

100. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

100. Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is

a) $\frac{4}{5}$ b) $\frac{1}{2}$ c) $\frac{1}{5}$ d) $\frac{2}{5}$

101. A and B throw a die alternatively till one of them gets a ' 6 ' and wins the game. Find their respective probabilities of winning, if A starts first.

102. If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?

103. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$).

104. An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:

$$\text{P(A fails)} = 0.2$$
$$\text{P(B fails alone)} = 0.15$$
$$\text{P(A and B fail)} = 0.15$$

Evaluate the following probabilities
(i) P (A fails|B has failed)
(ii) P (A fails alone)

106. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

107. If m and n, respectively, are the order and the degree of the differential equation $\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)\right]^4 = 0$, then $m + n =$

a) 1 b) 2 c) 3 d) 4

108. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) or 'x' is/are

a) 3 b) $\sqrt{3}$ c) $-\sqrt{3}$ d) $\sqrt{3}, -\sqrt{3}$

109. Find the direction ratio and direction cosines of a line parallel to the whose equations are $6x - 12 = 3y + 9 = 2z - 2$.

110. Given that $\boldsymbol{A}$ is a square matrix of order 3 and $|\boldsymbol{A}| = -2$, then $|\boldsymbol{adj}(2\boldsymbol{A})|$ is equal to

a) -2^6 b) +4 c) -2^8 d) 2^8

111. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If the events of their solving the problem are independent then the probability that the problem will be solved, is

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

112. $\int \frac{x^2}{(x\cos x - \sin x)^2}\,dx$