CHAPTER 11 THREE DIMENSIONAL GEOMETRY

SUBJECT: MATHEMATICS **MAX. MARKS : 40**

CLASS : XII **DURATION : 1½ hrs**

General Instructions:

(i). **All** questions are compulsory.

(ii). This question paper contains 2**0** questions divided into five Sections A, B, C, D and E.

(iii). **Section A** comprises of **10 MCQs** of **1 mark** each. **Section B** comprises of 4 questions of **2 marks** each. **Section C** comprises of 3 questions of **3 marks** each. **Section D** comprises of 1 question of **5 marks** each and **Section E** comprises of 2 Case Study Based Questions of **4 marks** each.

(iv). There is no overall choice.

(v). Use of Calculators is not permitted

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. Two-line $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy plane has coordinates

(a) (2, 4, 7) (b) (-2, 4, 7) (c) (2, -4, -7) (d) (2, -4, 7)

2. Direction ratios of the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$ are

(a) 2, 6, 3 (b) –2, 6, 3 (c) 2, – 6, 3 (d) none of these

3. The vector equation of the line joining the points (3, -2, -5) and (3, -2, 6) is:

(a) $(4\hat{i}-4\hat{j}+5\hat{k})+\lambda(12\hat{k})$ (b) $(4\hat{i}-4\hat{j}+5\hat{k})+\lambda(12\hat{k})$

(c) $(6\hat{i}-2\hat{j}+2\hat{k})+\lambda(5\hat{k})$ (d) $(9\hat{i}-9\hat{j}-2\hat{k})+\lambda(2\hat{k})$

4. A point that lies on the line $\frac{x-1}{-2}=\frac{y+3}{4}=\frac{1-z}{7}$ is:

(a) (1, -3, 1) (b) (-2, 4, 7) (c) (-1, 3, 1) (d) (2, -4, -7)

5. The direction ratios of the line 6x – 2 = 3y + 1 = 2z – 2 are:

(a) 6, 3, 2 (b) 1, 1, 2 (c) 1, 2, 3 (d) 1, 3, 2

6. The straight line $\frac{x-3}{3}=\frac{y-2}{1}=\frac{z-1}{0}$ is:

(a) parallel to x-axis (b) parallel to y-axis

(c) parallel to z-axis (d) perpendicular to z-axis

7. If the equation of a line AB is $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-5}{4}$, find the direction ratios of a line parallel to AB.

(a) 1, 2, 4 (b) 1, 2, –4 (c) 1, –2, –4 (d) 1, –2, 4

8. If a line makes angles α, β, γ with the positive direction of co-ordinates axes, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

(a) 1 (b) 2 (c) 3 (d) 4

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A) :** The angle between the straight lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$ is 90°.
Reason (R) : Skew lines are lines in different planes which are parallel and intersecting.

10. **Assertion:** If the cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$, then its vector form is $\vec{r}=5\hat{i}-4\hat{j}+6\hat{k}+\lambda(3\hat{i}+7\hat{j}+2\hat{k})$
Reason: The cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x-3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is $\frac{x+3}{-2}=\frac{y-4}{4}=\frac{z+8}{-5}$.

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the z-axis.

12. Show that the line through the points (1, –1, 2), (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

13. Find the angle between the straight lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{1-x}{-1}=\frac{y+2}{2}=\frac{3-z}{3}$.

14. Find the coordinates of the point where the line $\frac{x+3}{3}=\frac{y-1}{-1}=\frac{z-5}{-5}$ cuts the XY plane.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P(1, 3, 3).

16. Find the vector equation of the line through the point (1, 2, –4) and perpendicular to the two lines $\vec{r}=(8\hat{i}-19\hat{j}+10\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$ and $\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$

17. Find the shortest distance between the lines
$\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$

SECTION – D

Questions 18 carry 5 marks.

18. Find the coordinates of the foot of perpendicular drawn from the point A(–1, 8, 4) to the line joining the points B(0, –1, 3) and C(2, –3, –1). Hence find the image of the point A in the line BC.

SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. Case-Study 1: Read the following passage and answer the questions given below.
The equation of motion of a missile are x = 3t, y = - 4t, z = t, where the time 't' is given in seconds, and the distance is measured in kilometers.

(a) Write the path of the missile.

(b) Find the distance of the rocket from the starting point (0, 0, 0) in 5 seconds.

(c) If the position of the rocket at a certain instant of the time is (5, -8, 10). Find the height of the rocket from the ground. (Ground considered as xy-plane)

OR

(c) Find the value of k for which the lines $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{k}$ and $\frac{x-2}{-2}=\frac{y-3}{-1}=\frac{z-5}{7}$; are perpendicular?

20. Case-Study 2: Read the following passage and answer the questions given below.

Two non-parallel and non-intersecting straight lines are called skew lines. For skew lines, the line segment of the shortest distance will be perpendicular to both the lines. If the lines are $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\vec{b_2}$.

Then, shortest distance is given as $d=\left|\frac{(\vec{b_1}\times\vec{b_2}).(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$

Here, $\vec{a_1},\vec{a_2}$ are position vectors of point through which the lines are passing and $\vec{b_1},\vec{b_2}$ are the vectors in the direction of a line.

(a) If a line has the direction ratios -18, 12, -4 then what are its direction cosines? (1)

(b) Write the condition for which the given two lines $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\mu\vec{b_2}$ are not coplanar in vector form. (1)

(c) Write the distance of a point P(a, b, c) from the x-axis (1)

(d) If the cartesian form of a line is $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$ then write the vector equation of line. (1)