Real Numbers

Theorems

1. Let p be a prime number.
   If p divides a^2, then p divides a,
   where a is a positive integer
2. $\sqrt{2}, \sqrt{3}$ are irrational numbers

Prime Factorization Method

To obtain HCF & LCM
For any two positive integers,
a and b
HCF (a, b) × LCM $(a, b) = a \times b$
For Example

$f(x) = 3x^2y$
and $g(x) = 6xy^2$
HCF $= 3xy$
and LCM $= 6x^2y^2$

Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur

Composite Number
$x = p_1 \times p_2 \times p_3 \ldots \times p_n$,
where $p_1, p_2, \ldots, p_n$
are prime numbers

Trace the Mind Map
- First Level
- Second Level
- Third Level

Polynomials

Degree of Polynomial

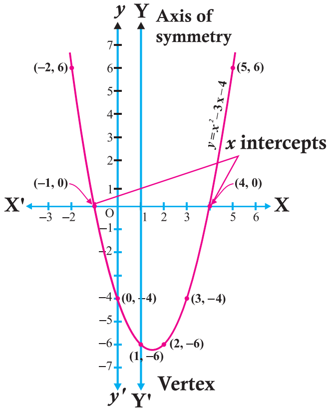
Highest power of x in Polynomial, $p(x)$

Types

Polynomial	Degree	General Form
Linear	1	$f(x) = ax + b$ $a \neq 0$
Quadratic	2	$f(x) = ax^2 + bx + c$ $a \neq 0$
Cubic	3	$f(x) = ax^3+bx^2+cx+d$ $a \neq 0$

Graphical Representation of Quadratic Polynomial

Parabola

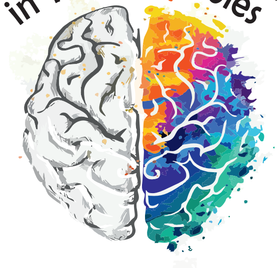


$y = x^2 - 3x - 4$

Trace the Mind Map

- First Level
- Second Level
- Third Level

Pair of Linear Equations in Two Variables



General Form

$a_1x + b_1y + c_1 = 0,$
$a_2x + b_2y + c_2 = 0$
where $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers.

Solution Graphically

Each solution (x, y), corresponds to a point on the line representing the equation and vice-versa

Graphical Representation

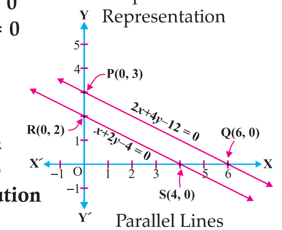
Pair of Lines : $x + 2y - 4 = 0$, $2x + 4y - 12 = 0$

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4}, \frac{c_1}{c_2} = \frac{-4}{-12}$

Compare the Ratios : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Algebraic Interpretation : No solution *i.e.,* Inconsistent

Graphical Representation



Parallel Lines

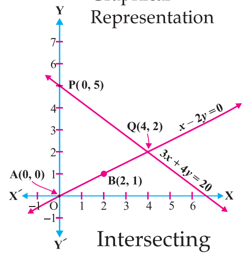
Pair of Lines : $x - 2y = 0$, $3x + 4y - 20 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{4}, \frac{c_1}{c_2} = \frac{0}{-20}$

Compare the Ratios: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Algebraic Interpretation : Exactly one solution *i.e.,* consistent (unique)

Graphical Representation



Intersecting Lines

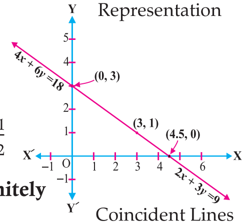
Pair of Lines : $2x + 3y - 9 = 0$, $4x + 6y - 18 = 0$

$\frac{a_1}{a_2} = \frac{2}{4}, \frac{b_1}{b_2} = \frac{3}{6}, \frac{c_1}{c_2} = \frac{-9}{-18}$

Compare the Ratios : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Algebraic Interpretation = Infinitely many solutions *i.e.,* consistent.

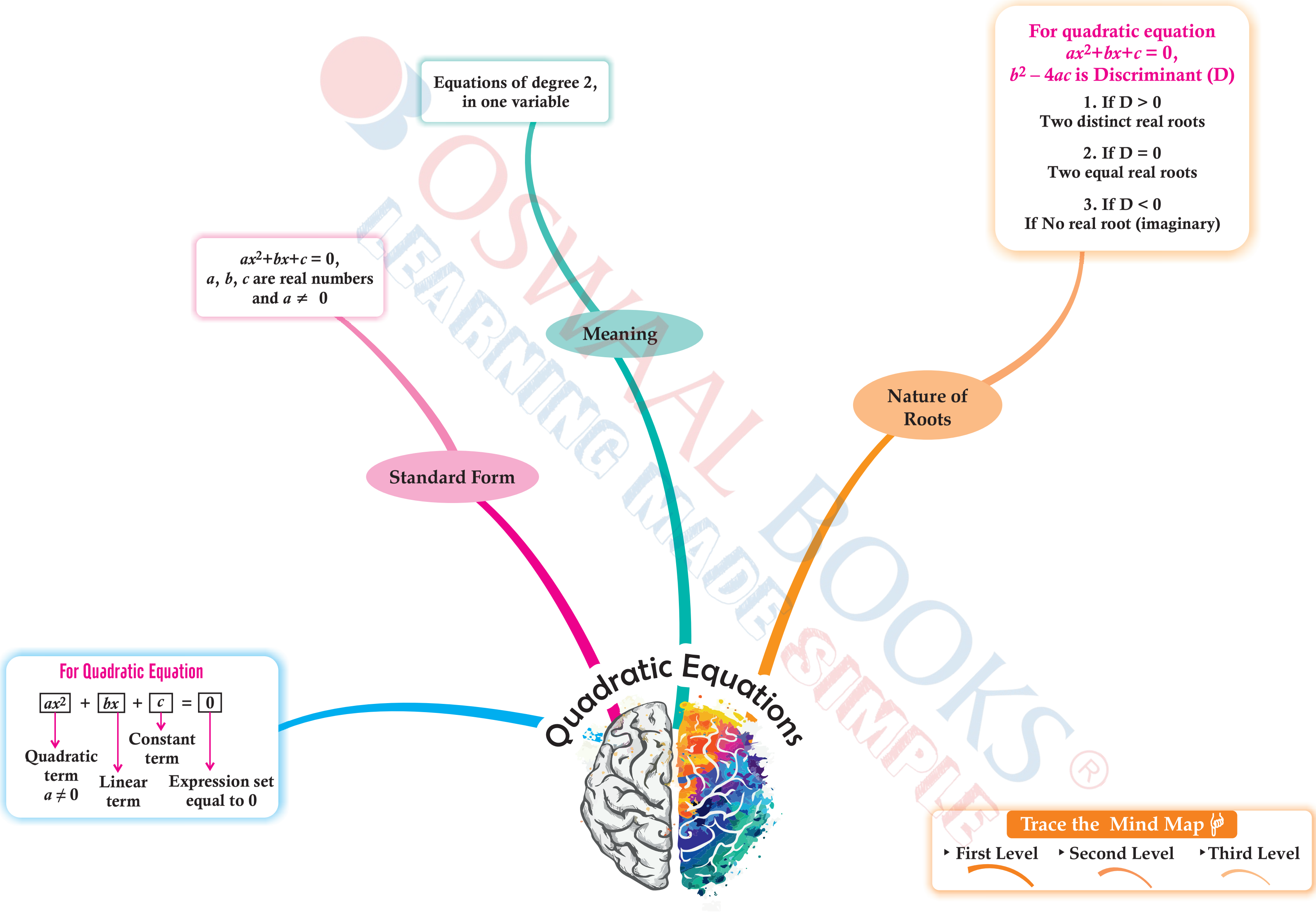
Graphical Representation



Coincident Lines

Trace the Mind Map

- First Level
- Second Level
- Third Level


Quadratic Equations
Meaning
Equations of degree 2, in one variable
Standard Form
$ax^2+bx+c=0$, a, b, c are real numbers and $a \neq 0$
Nature of Roots
For quadratic equation $ax^2+bx+c=0$, b^2-4ac is Discriminant (D)
1. If D > 0 Two distinct real roots
2. If D = 0 Two equal real roots
3. If D < 0 If No real root (imaginary)
For Quadratic Equation
ax^2 + bx + c = 0
Quadratic term $a \neq 0$
Linear term
Constant term
Expression set equal to 0
Trace the Mind Map
First Level
Second Level
Third Level

Arithmetic Progressions

Definition
Sequence of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

Arithmetic mean
If a, b, c, are in A.P.

Then $b = \frac{a+c}{2}$

here,
$b \rightarrow$ arithmetic mean

General form
$a, a+d, a+2d, a+3d, ...a+(n-1)d$

Common Difference
- Fixed number in arithmetic progression which provides the to and fro terms by adding/subtracting from the present number.
- Can be positive or negative.
- It is denoted by d

n^{th} term

From the end

$a_n = l - (n-1)d$
here, $l \rightarrow$ last term
$d \rightarrow$ common difference
$a_n \rightarrow n^{th}$ term

From the beginning

$a_n = a + (n-1)d$
here, $a \rightarrow$ first term
$d \rightarrow$ common difference
$a_n \rightarrow n^{th}$ term

Sum (S)

When first term and common difference are given :

$S_n = \frac{n}{2}(2a + (n-1)d)$

here, $a \rightarrow$ first term
$d \rightarrow$ common difference
$n \rightarrow$ total no. of terms

When first & last terms are given :

$S_n = \frac{n}{2}(a+a_n)$ or $S_n = \frac{n}{2}(a+l)$

here, $a \rightarrow$ first term
$n \rightarrow$ total no. of terms
$a_n \rightarrow n^{th}$ term
$l \rightarrow$ last term

Example

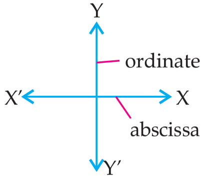
Sum of first n positive integers
Let $S_n = 1 + 2 + 3 + ... n$
here, $a = 1$, last term, $l = n$

$S_n = \frac{n(a+l)}{2} = \frac{n(1+n)}{2}$

Trace the Mind Map
- First Level
- Second Level
- Third Level

Co-ordinate Geometry

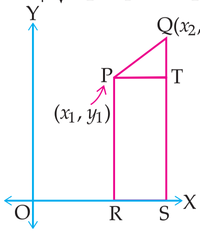
Co-ordinate axes



Horizontal = X–axis (Abscissa)
Vertical = Y–axis (Ordinate)

Distance formula

$$PQ = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$



Example

Are the following points vertices of a square : (1, 7), (4, 2), (–1, –1), (–4, 4)?
A = (1, 7); B = (4, 2); C = (–1, –1); D = (–4, 4)

$AB = \left|\sqrt{(1-4)^2 + (7-2)^2}\right| = \sqrt{34}$ Unit

$BC = \left|\sqrt{(4+1)^2 + (2+1)^2}\right| = \sqrt{34}$ Unit

$CD = \left|\sqrt{(-1+4)^2 + (-1-4)^2}\right| = \sqrt{34}$ Unit

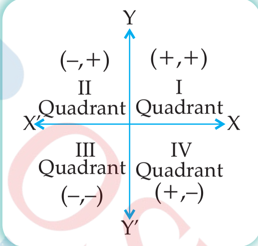
$DA = \left|\sqrt{(1+4)^2 + (7-4)^2}\right| = \sqrt{34}$ Unit

$AC = \left|\sqrt{(1+1)^2 + (7+1)^2}\right| = \sqrt{68}$ Unit

$BD = \left|\sqrt{(4+4)^2 + (2-4)^2}\right| = \sqrt{68}$ Unit

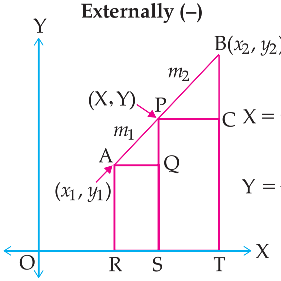
Since, AB = BC = CD = DA and AC = BD.
All four sides and diagonals are equal
Hence, ABCD is a square

Quadrants



Section formula

Internally (+)
Externally (–)



$$X = \frac{m_1x_2 \pm m_2x_1}{m_1 \pm m_2}$$

$$Y = \frac{m_1y_2 \pm m_2y_1}{m_1 \pm m_2}$$

Example

Find point of trisection of line segment AB, A(2, –2) and B(–7, 4)

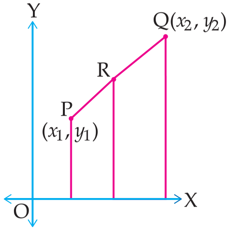
A (2, –2) — P — Q — B (–7, 4)

Co-ordinates of $P = \left(\frac{1(-7) + 2(2)}{1+2}, \frac{1(4) + 2(-2)}{1+2}\right)$
$= (-1, 0)$

Co-ordinates of Q

$Q = \left(\frac{2(-7) + 1(2)}{2+1}, \frac{2(4) + 1(-2)}{2+1}\right)$
$= (-4, 2)$

Mid-point of a Line Segment



$$R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Trace the Mind Map
First Level · Second Level · Third Level

Triangles

Theorems

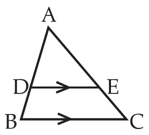
Summary

In ΔABC, let $DE \parallel BC$. Then,

(i) $\frac{AD}{DB} = \frac{AE}{EC}$

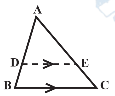
(ii) $\frac{AB}{DB} = \frac{AC}{EC}$

(iii) $\frac{AD}{AB} = \frac{AE}{AC}$



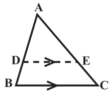
1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

If, $DE \parallel BC$ then $\frac{AD}{DB} = \frac{AE}{EC}$

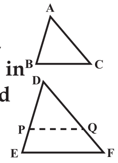


2. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

If $\frac{AD}{DB} = \frac{AE}{EC}$ then, $DE \parallel BC$



3. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.(AAA criterion)



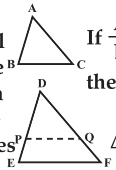
If $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

then, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

$\Delta ABC \sim \Delta DEF$

(By AAA Criterian)

4. If in two triangles, sides of one triangle are proportional to (*i.e.,* in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.(SSS criterion)



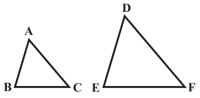
If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

then, $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

$\Delta ABC \sim \Delta DEF$

(By SSS Criterian)

5. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.(SAS criterion)



If $\frac{AB}{DE} = \frac{AC}{DF}$ & $\angle A = \angle D$

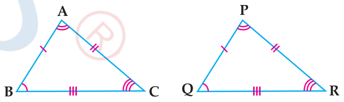
then, $\Delta ABC \sim \Delta DEF$

(By SAS Criterian)

Similarity

(i) Corresponding angles are equal

(ii) Corresponding sides are in the same ratio



$\Delta ABC \sim \Delta PQR$

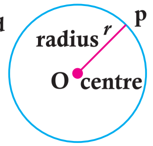
Trace the Mind Map

- First Level
- Second Level
- Third Level

Circles

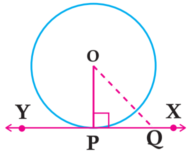
Definition

In a fixed plane, The locus of a point equidistant from a fixed point is called 'circle'. The fixed point is called the centre & separation of points is the radius of the circle.



Theorem

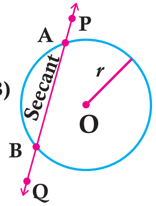
The tangent at any point of a circle is perpendicular to the radius through the point of contact.



$\angle OPQ = 90°$

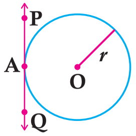
Secant

Two common points (A and B) between line PQ and circle.



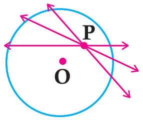
Tangent and tangent point

Only one common point (A) between circle and line PQ. Here, PQ is tangent at point A lying on the circle.

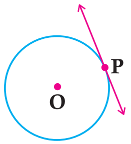


Facts

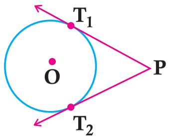
1. There is no tangent to a circle passing through a point lying inside the circle.



2. There is one and only one tangent to a circle passing through a point lying on the circle.



3. There are exactly two tangents to a circle through a point lying outside the circle.



Trace the Mind Map

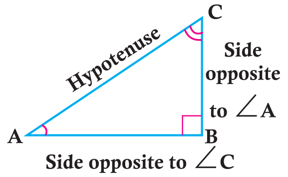
- First Level
- Second Level
- Third Level

Introduction to Trigonometry

Trigonometry

Study of relationships between the sides & angles of a triangle

Trigonometric Ratios



Sine of $\angle A = \frac{BC}{AC}$

Cosine of $\angle A = \frac{AB}{AC}$

Tangent of $\angle A = \frac{BC}{AB}$

Cotangent of $\angle A = \frac{AB}{BC}$

Secant of $\angle A = \frac{AC}{AB}$

Cosecant of $\angle A = \frac{AC}{BC}$

Note: How to learn the relation

"Some people have" $\sin\theta = \frac{P}{H}$

"Curly Brown Hair" $\cos\theta = \frac{B}{H}$

"through proper Brushing" $\tan\theta = \frac{P}{B}$

Values of trigonometric ratios between 0° to 90°

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	(∞) Not defined
cot A	(∞) Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	(∞) Not defined
cosec A	(∞) Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Relationships between the Ratios

Express cos A and tan A in terms of sin A

Solution : Since, $\cos^2 A + \sin^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$ *i.e.* $\cos A = \sqrt{1-\sin^2 A}$

$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}}$

Trigonometric Identities

$0 \le A \le 90°$ $\cos^2 A + \sin^2 A = 1$

$0 \le A < 90°$ $1 + \tan^2 A = \sec^2 A$

$0 < A \le 90°$ $\cot^2 A + 1 = \text{cosec}^2 A$

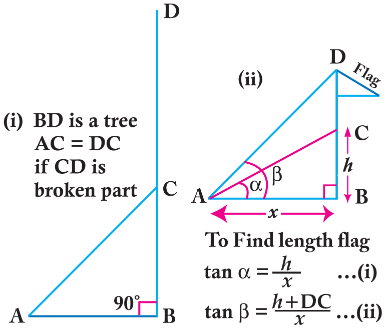
Trace the Mind Map

- First Level
- Second Level
- Third Level

Some Application of Trigonometry

Measuring Distance

Height / Length of an object



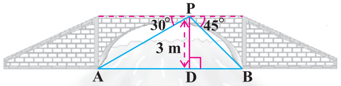
To Find length flag

$\tan \alpha = \frac{h}{x}$ ...(i)

$\tan \beta = \frac{h + DC}{x}$ ...(ii)

and solve it

To determine width AB



From figure, AB = AD + DB

In right ΔAPD, ∠A = 30°, ∠D = 90°

$\tan 30° = \frac{PD}{AD}$ i.e., $AD = 3\sqrt{3}$ m

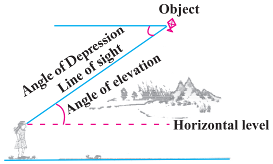
In right ΔBPD, ∠B = 45°, ∠D = 90°

$\tan 45° = \frac{PD}{BD}$ i.e., BD = 3

$\therefore AB = (3\sqrt{3} + 3)\text{ m} = 3(\sqrt{3} + 1)\text{ m}$

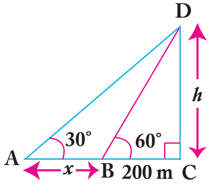
Difference of Angles

Angle of Elevation is equal to Angle of Depression



Measuring Height

Distance between two objects



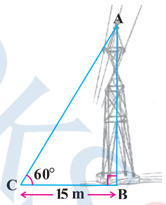
Find x and h

$\tan 60° = \frac{h}{200}$ ...(i)

$\tan 30° = \frac{h}{x + 200}$ ...(ii)

and solve both equations

To determine height of object AB



In ΔABC, ∠B = 90°, ∠C = 60°

Here, $\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{15}$

i.e., $AB = 15\sqrt{3}$ m

Trace the Mind Map
- First Level
- Second Level
- Third Level

Areas Related to Circles

Sector of a circle

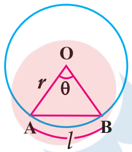
Formulae

Area of Sector

Area $= \frac{\theta}{360^\circ} \times$ area of circle

Area $= \frac{\theta}{360^\circ} \times \pi r^2$

Or, Area $= \frac{1}{2} \times l \times r$



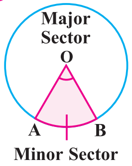
Length of arc

$l = \frac{\theta}{360^\circ} \times$ circumference

Or, $l = \frac{\theta}{360^\circ} \times 2\pi r$

Meaning

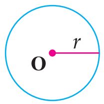
Portion of the circular region enclosed by two radius and the corresponding arc



Circle

Formulae

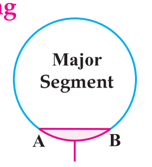
- Circumference $= \pi \times$ diameter $= 2\pi r$
- Area $= \pi r^2$



Segment

Meaning

Portion of the circular region enclosed between a chord and the corresponding arc

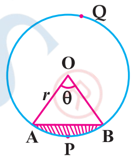


Formula

Area = Area of the corresponding sector – Area of the corresponding triangle

$= \frac{\theta}{360^\circ} \times \pi r^2$ – area of ΔOAB

$= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin\theta$



Trace the Mind Map

- First Level
- Second Level
- Third Level

Surface Areas and Volumes

Surface Area

Sum of surface areas of the faces of solid

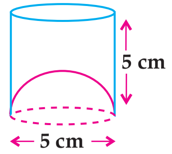
Volume

Quantity of 3-D space enclosed by a hollow/closed solid

Combination of Solids

Example

Given: Inner diameter of the Cylindrical glass = 5 cm
Height = 5 cm



Find: Actual capacity of cylindrical glass.

Solution : Apparent capacity of the glass $= \pi r^2 h$

$= 3.14 \times 2.5 \times 2.5 \times 5$

$= 98.125 \text{ cm}^3$

Volume of hemisphere $= \frac{2}{3}\pi r^3$, if $r = 2.5$ cm

$= \frac{2}{3} \times 3.14 \times (2.5)^3 \text{ cm}^3 = 32.71 \text{ cm}^3$

Actual capacity = Apparent capacity – Volume of hemisphere

$= 98.125 - 32.71$

$= 65.42 \text{ cm}^3$

Trace the Mind Map

- First Level
- Second Level
- Third Level

Statistics

Definition

A collection, analysis, interpretation of quantitative data

Cumulative Frequency

Frequency obtained by adding the frequencies of all the classes preceding the class

Class Mark

$$\left[\frac{\text{Upper class limit} + \text{Lower class limit}}{2}\right]$$

Median (For Ungrouped Data)

If n is odd,
Median $= \left(\frac{n+1}{2}\right)^{th}$ term
where, n = Sum of frequencies

If n is even,
Median $= \dfrac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$
where, n = Sum of frequencies

Grouped Data

Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

where, l = lower limit of modal class
f_1 = frequency of modal class
f_0 = frequency of class preceding to modal class
f_2 = frequency of class succeeding to modal class
h = width of class interval

Median $= l + \left(\dfrac{\frac{n}{2} - c.f.}{f}\right) \times h$

where, l = lower Limit of median class
n = Sum of frequencies
h = with of class interval
f = frequency of median class
$c.f.$ = Cumulative frequency of the class preceding to median class

Mean

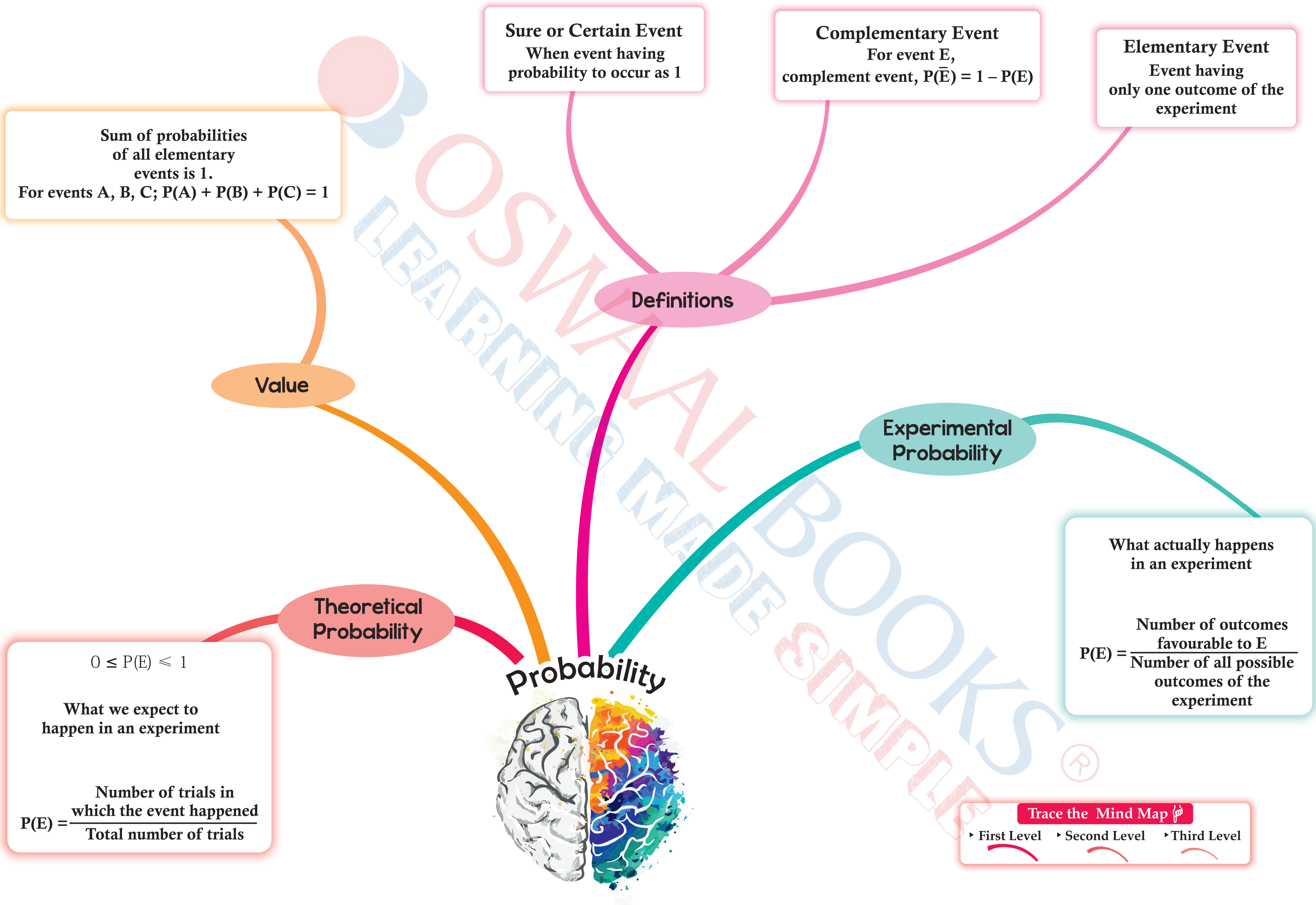
Direct Method $\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

where, x_i = class marks.
f_i = frequencies

Assumed Mean Method, $\bar{x} = a + \dfrac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$

Here, $d_i = x_i - a$
f_i = frequencies
x_i = class marks

Trace the Mind Map
- First Level
- Second Level
- Third Level


Probability
Definitions
Sure or Certain Event
When event having probability to occur as 1
Complementary Event
For event E, complement event, $P(\bar{E}) = 1 - P(E)$
Elementary Event
Event having only one outcome of the experiment
Value
Sum of probabilities of all elementary events is 1.
For events A, B, C; $P(A) + P(B) + P(C) = 1$
Experimental Probability
What actually happens in an experiment
$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$
Theoretical Probability
$0 \leq P(E) \leq 1$
What we expect to happen in an experiment
$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$
Trace the Mind Map
First Level
Second Level
Third Level