SETH M.R. JAIPURIA SCHOOLS BANARAS, PARAO CAMPUS

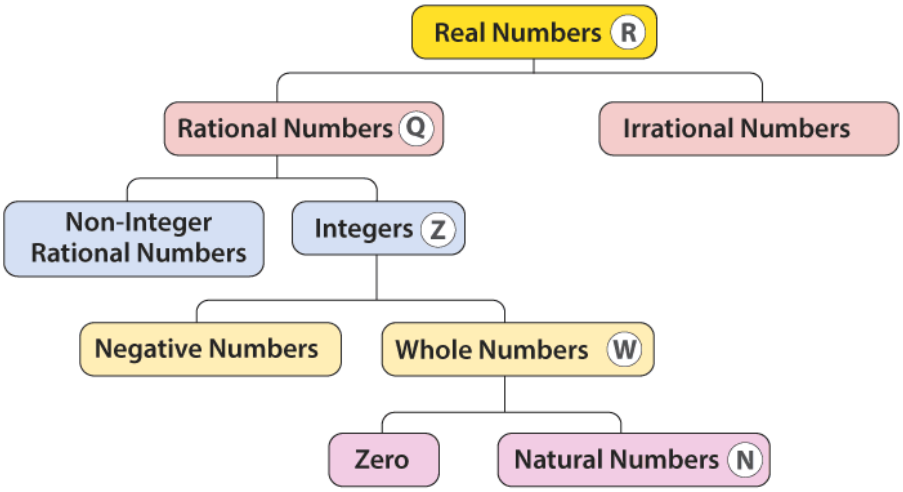
CLASS-10 (MATHS SAMPLE COPY)

CHAPTER - 1 (REAL NUMBERS)

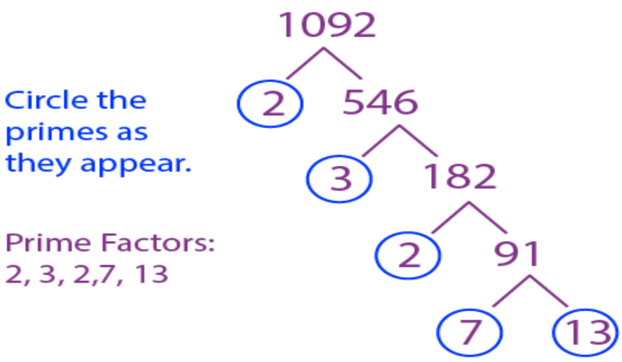
Real Numbers

Positive integers, negative integers, irrational numbers, and fractions are all examples of real numbers. In other words, we can say that any number is a real number, except for complex numbers. Examples of real numbers include -1, ½, 1.75, √2, and so on.
In general,

- Real numbers constitute the union of all rational and irrational numbers.
- Any real number can be plotted on the number line.



The Fundamental Theorem of Arithmetic



Prime Factorisation

- Prime Factorisation is the method of expressing a natural number as a product of prime numbers.
- Example: $36=2\times2\times3\times3$ is the prime factorisation of 36.

Fundamental Theorem of Arithmetic

- The Fundamental Theorem of Arithmetic states that the prime factorisation for a given number is unique if the arrangement of the prime factors is ignored.
- Example: $36=2\times2\times3\times3$ OR, $36=2\times3\times2\times3$
- Therefore, 36 is represented as a product of prime factors (Two 2s and two 3s) ignoring the arrangement of the factors.

Method of Finding LCM

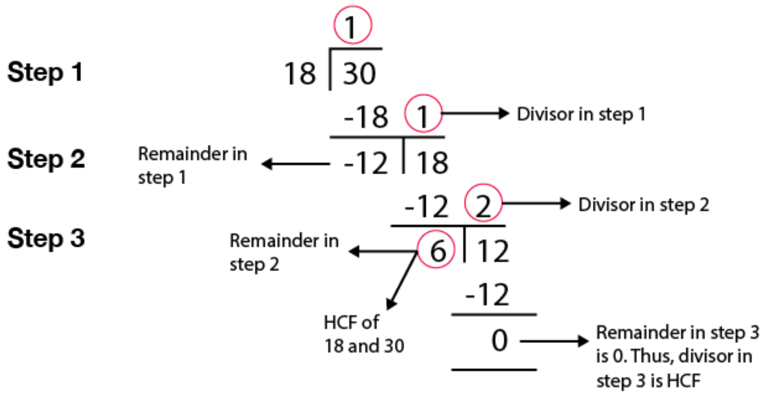
As we know, the smallest of the common multiples of two or more numbers is called the lowest common multiple (LCM).
Example: To find the Least Common Multiple (**L.C.M**) of 36 and 56,

1. $36=2\times2\times3\times3$
   $56=2\times2\times2\times7$
2. The common prime factors are 2×2
3. The uncommon prime factors are 3×3 for 36 and 2×7 for 56.
4. LCM of 36 and 56 = $2\times2\times3\times3\times2\times7$ which is 504

Method of Finding HCF

We know that the greatest number that divides each of the given numbers without leaving any remainder is the highest common factor (HCF) of two or more given numbers.
H.C.F can be found using two methods – Prime factorisation and Euclid's division algorithm.

- Prime Factorisation:
  - Given two numbers, we express both of them as products of their respective prime factors. Then, we select the prime factors that are common to both the numbers
  - Example – To find the H.C.F of 20 and 24
    $20=2\times2\times5$ and $24=2\times2\times2\times3$
  - The factor common to 20 and 24 is 2×2, which is 4, which in turn is the H.C.F of 20 and 24.
- Euclid's Division Algorithm:
  - It is the repeated use of Euclid's division lemma to find the H.C.F of two numbers.
  - Example: To find the HCF of 18 and 30



Finding the HCF of 18 and 30

  - The required HCF is **6**.

Product of Two Numbers = HCF X LCM of the Two Numbers

- For any **two** positive integers a and b,
  $a\times b=H.C.F\times L.C.M.$
- Example – For 36 and 56, the H.C.F is 4 and the L.C.M is 504
  $36\times56=2016$

4×504=2016
Thus, 36×56=4×504

- Let us consider another example:
  For 5 and 6, the H.C.F is 1 and the L.C.M is 30
  5 × 6 = 30
  1 × 30 =30
  Thus, 5 × 6 = 1 × 30
- The above relationship, however, doesn't hold true for 3 or more numbers.

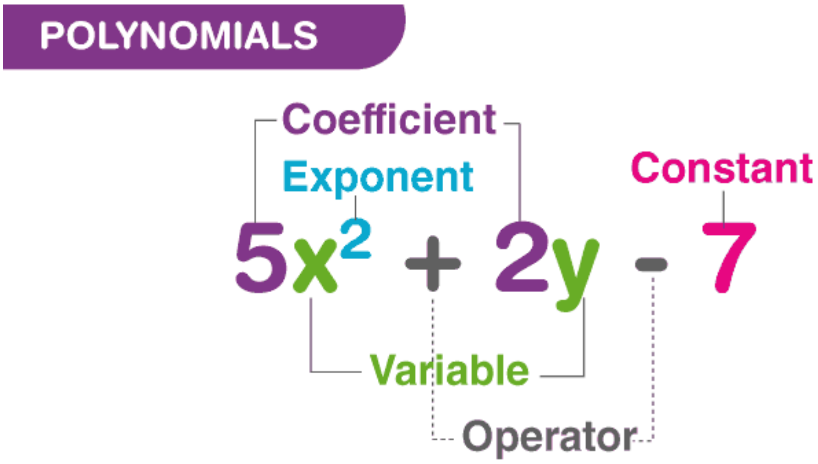
CH.1 REAL NUMBERS
NCERT C.W – EXE 1.1: 1, 5, 6, 7   EXE 1.2 – 1,3
NCERT H.W- EXE 1.1: 2, 3, 4   EXE 1.2 – 2
NCERT EXEMPLAR C.W- EXE 1.1: (4 to 10) EXE 1.2- (7 & 8)
NCERT EXEMPLAR H.W- EXE 1.3 – (10,11&12)
WORKBOOK C.W – CASE STUDY(Q.38 & 39)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.2 POLYNOMIALS

An algebraic expression can have **any number of terms**. The **coefficient** in each term can be **any real number**. There can be **any number of variables** in an algebraic expression. The **exponent** on the variables, however, must be **rational numbers.**

Polynomial

An algebraic expression can have exponents that are rational numbers. However, a polynomial is an algebraic expression in which the exponent on any variable is a whole number.



Let us consider a few more examples,
$5x^3+3x+1$ is an example of a polynomial. It is an algebraic expression as well.

$2x+3\sqrt{x}$ is an algebraic expression but not a polynomial. – since the exponent on x is 1/2, which is not a whole number..

Degree of a Polynomial

For a polynomial in one variable – the highest exponent on the variable in a polynomial is the degree of the polynomial.

Example: The degree of the polynomial x^2+2x+3 is 2, as the highest power of x in the given expression is x^2. Consider another example, the degree of the polynomial $x^8 + 2x^6 - 3x + 9$ is 8 since the greatest power in the given expression is 8.

Types Of Polynomials

Polynomials can be classified based on the following.
a) Number of terms
b) Degree of the polynomial.

Types of Polynomials Based on the Number of Terms

a) Monomial – A polynomial with just one term. Example: $2x$, $6x^2$, $9xy$

b) Binomial – A polynomial with two unlike terms. Example: $4x^2+x$, $5x+4$

a) Trinomial – A polynomial with three unlike terms. Example: x^2+3x+4

Types of Polynomials based on Degree

Linear Polynomial

A polynomial whose degree is one is called a linear polynomial.
For example, $2x+1$ is a linear polynomial.

Quadratic Polynomial

A polynomial of degree two is called a quadratic polynomial.
For example, $3x^2+8x+5$ is a quadratic polynomial.

Cubic Polynomial

A polynomial of degree three is called a ***cubic polynomial***.
For example, $2x^3+5x^2+9x+15$ is a cubic polynomial.

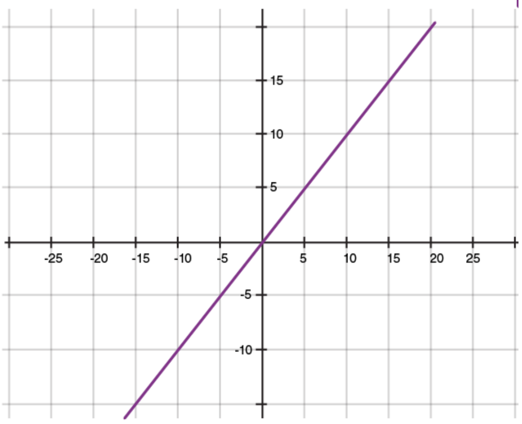
Graphical Representations

Let us learn here how to represent polynomial equations on the graph.

Representing Equations on a Graph

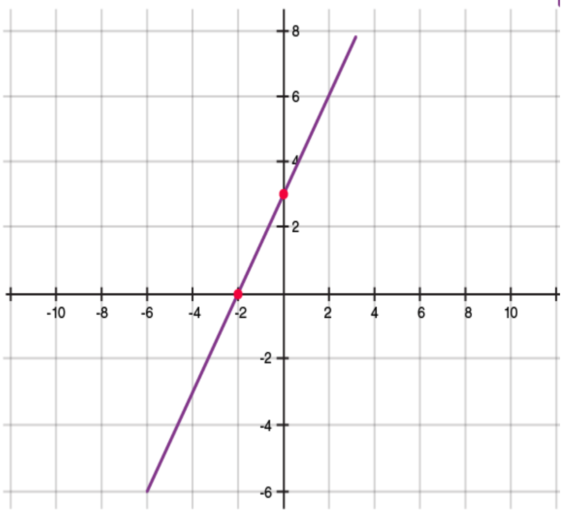
Any equation can be represented as a graph on the Cartesian plane, where each point on the graph represents the x and y coordinates of the point that satisfies the equation. An equation can be seen as a constraint placed on the x and y coordinates of a point, and any point that satisfies that constraint will lie on the curve.

For example, the equation y = x, on a graph, will be a straight line that joins all the points which have their x coordinate equal to their y coordinate. Example – (1,1), (2,2) and so on.



Geometrical Representation of a Linear Polynomial

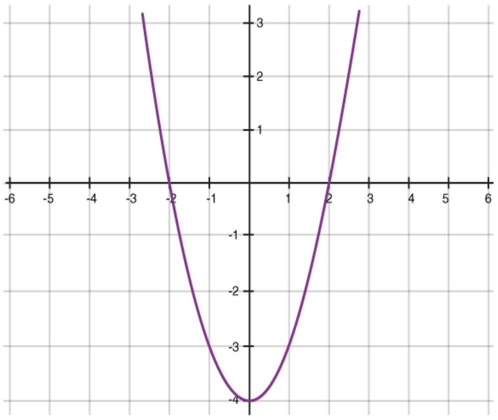
The graph of a linear polynomial is a straight line. It cuts the X-axis at exactly one point.



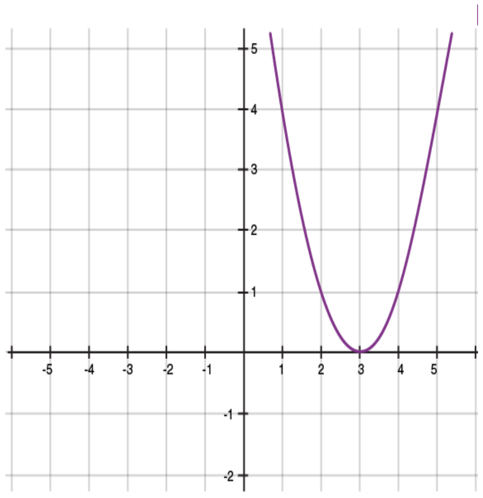
Linear graph

Geometrical Representation of a Quadratic Polynomial

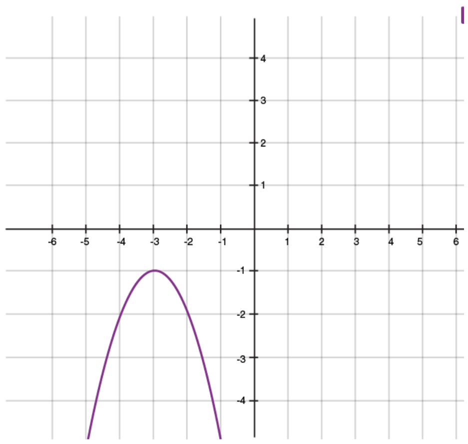
- The graph of a quadratic polynomial is a parabola
- It looks like a U, which either opens upwards or opens downwards depending on the value of ‘a’ in ax^2+bx+c
- If ‘a’ is positive, then parabola opens upwards and if ‘a’ is negative then it opens downwards
- It can cut the x-axis at 0, 1 or two points



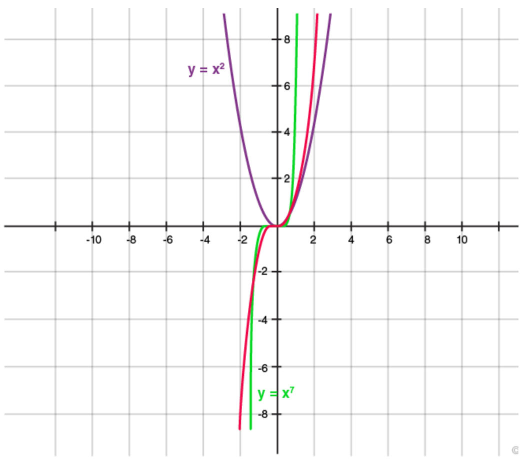
Graph of a polynomial which cuts the x-axis in two distinct points (a>0)



Graph of a Quadratic polynomial which touches the x-axis at one point (a>0)



Graph of a Quadratic polynomial that doesn’t touch the x-axis (a<0)



Graph of polynomials with different degrees.

Zeroes of a Polynomial

A zero of a polynomial **p(x)** is the value of x for which the value of p(x) is 0. If k is a zero of p(x), then **p(k)=0.**

For example, consider a polynomial $p(x)=x^2-3x+2$.

When x=1, the value of p(x) will be equal to

$p(1)=12-3\times1+2$

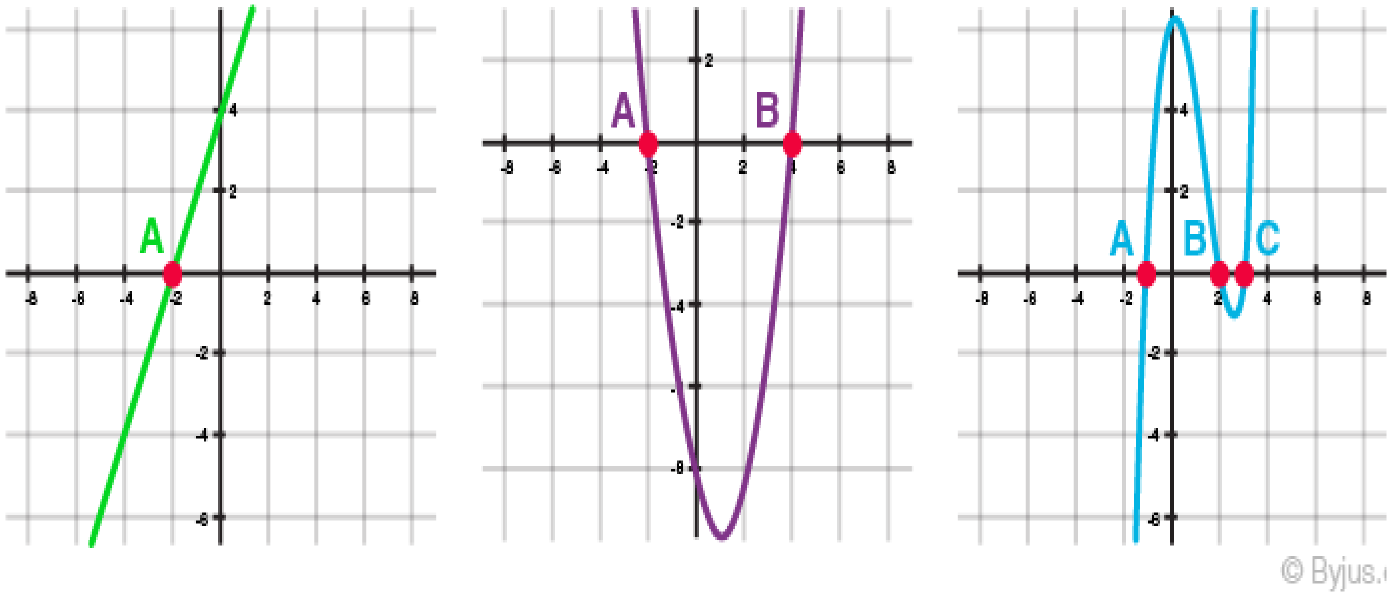
$=1-3+2$

$=0$

Since p(x)=0 at x=1, we say that 1 is a zero of the polynomial x^2-3x+2

Geometrical Meaning of Zeros of a Polynomial

Geometrically, zeros of a polynomial are the points where its graph cuts the x-axis.



(i) One zero (Linear Polynomial)   (ii) Two zeros (Quadratic Polynomial)   (iii) Three zeros (Cubic Polynomial)

Here A, B and C correspond to the zeros of the polynomial represented by the graphs.

Number of Zeros

In general, a polynomial of degree n has at most n zeros.

1. A linear polynomial has one zero,
2. A quadratic polynomial has at most two zeros.
3. A cubic polynomial has at most 3 zeros.

Factorisation of Polynomials

Quadratic polynomials can be factorized by splitting the middle term.

For example, consider the polynomial $2x^2-5x+3$

Splitting the middle term:

The middle term in the polynomial $2x^2-5x+3$ is -5x. This must be expressed as a sum of two terms such that the product of their coefficients is equal to the product of 2 and 3 (coefficient of x^2 and the constant term)

-5 can be expressed as $(-2)+(-3)$, as $-2\times-3=6=2\times3$

Thus, $2x^2-5x+3=2x^2-2x-3x+3$

Now, identify the common factors in individual groups

$2x^2-2x-3x+3=2x(x-1)-3(x-1)$

Taking $(x-1)$ as the common factor, this can be expressed as:

$2x(x-1)-3(x-1)=(x-1)(2x-3)$

Relationship between Zeroes and Coefficients of a Polynomial

For Quadratic Polynomial:
If α and β are the roots of a quadratic polynomial ax^2+bx+c, then,

$\alpha + \beta = -b/a$

Sum of zeroes = -coefficient of x /coefficient of x^2

$\alpha\beta = c/a$

Product of zeroes = constant term / coefficient of x^2

For Cubic Polynomial

If α,β and γ are the roots of a cubic polynomial ax^3+bx^2+cx+d, then

$\alpha+\beta+\gamma = -b/a$

$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$

$\alpha\beta\gamma = -d/a$

CH.2 POLYNOMIALS

NCERT C.W EX-2.1 (Q.No.1), EX-2.2(Q.1)
NCERT H.W EX-2.2 (Q.2)
NCERT EXEMPLAR C.W - EX-2.1(Q.5 TO Q.11) EX-2.3(Q.7 TO Q.10)
NCERT EXEMPLAR H.W- EX-2.4(Q.1)
WORKBOOK C.W- CASE STUDY(Q.38)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.3 PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Basics Revisited

Equation

An equation is a statement that two mathematical expressions having one or more variables are equal.

Linear Equation

Equations in which the powers of all the variables involved are one are called linear equations. The degree of a linear equation is always one.

General Form of a Linear Equation in Two Variables

The general form of a linear equation in two variables is $ax + by + c = 0$, where a and b cannot be zero simultaneously.

Representing Linear Equations for a Word Problem

To represent a word problem as a linear equation:

- Identify unknown quantities and denote them by variables.
- Represent the relationships between quantities in a mathematical form, replacing the unknowns with variables.

Example: The cost of 5 pens and 7 pencils is Rs.50 whereas the cost of 7 pens and 5 pencils is Rs. 65. Represent the given word problem in linear equations form.

Solution: Let us say the cost of 1 pen is Rs.x, and the cost of 1 pencil is Rs.y
Given,
The cost of 5 pens and 7 pencils is Rs.50.
So, we can write in the form of a linear equation;
$5x + 7y = 50$
Again given,
The cost of 7 pens and 5 pencils is Rs.65
So, we can write in the form of a linear equation;
$7x + 5y = 65$
Hence, $5x + 7y = 50$ and $7x + 5y = 65$ are the pairs of linear equations in two variables as per the given word problem.

Solution of a Linear Equation in 2 Variables

The solution of a linear equation in two variables is a pair of values (x, y), one for ‘x’ and the other for ‘y’, which makes the two sides of the equation equal.
Eg: If $2x + y = 4$, then (0,4) is one of its solutions as it satisfies the equation. A linear equation in two variables has infinitely many solutions.

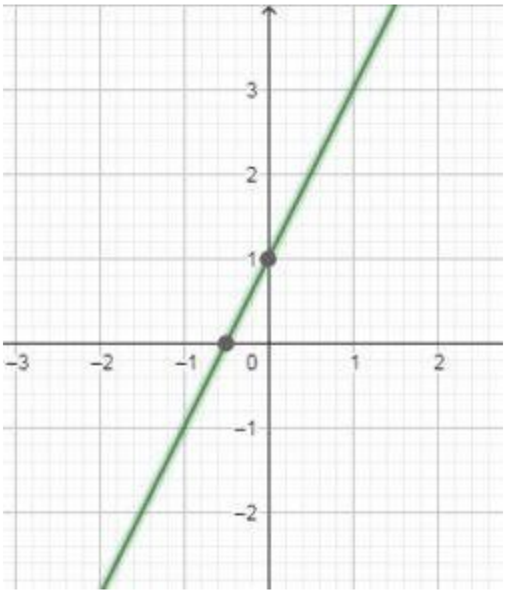
Geometrical Representation of a Linear Equation

Geometrically, a linear equation in two variables can be represented as a straight line.
$2x – y + 1 = 0$
$\Rightarrow y = 2x + 1$
Let us draw the graph for the given equation.

Step 1: Put the value of $x = 0$, such that $y = 1$.

Step 2: Put the value of $x = -1/2$ or 0.5, such that $y = 0$

Hence, the coordinates are (0, 1) and (-0.5, 0). We can find more points to plot the graph by putting different values for x. Now based on these coordinates, we can draw a straight line in the graph connecting the two points (0, 1) and (-0.5, 0) as shown in the figure below.



Graph of y = 2*x*+1

General Form of a Pair of Linear Equations in 2 Variables

A pair of linear equations in two variables can be represented as follows:

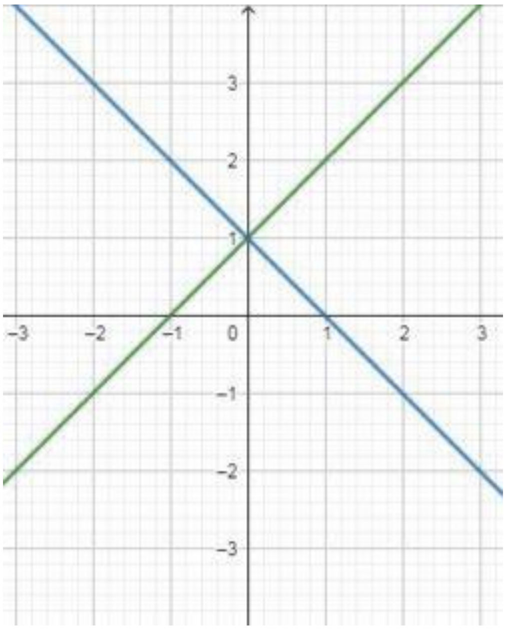
$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

The coefficients of x and y cannot be zero simultaneously for an equation.

Nature of 2 Straight Lines in a Plane

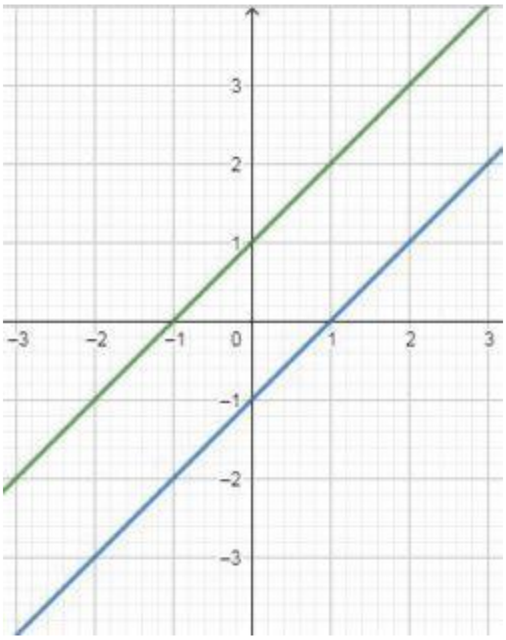
For a pair of straight lines on a plane, there are three possibilities.

i) They intersect at exactly one point

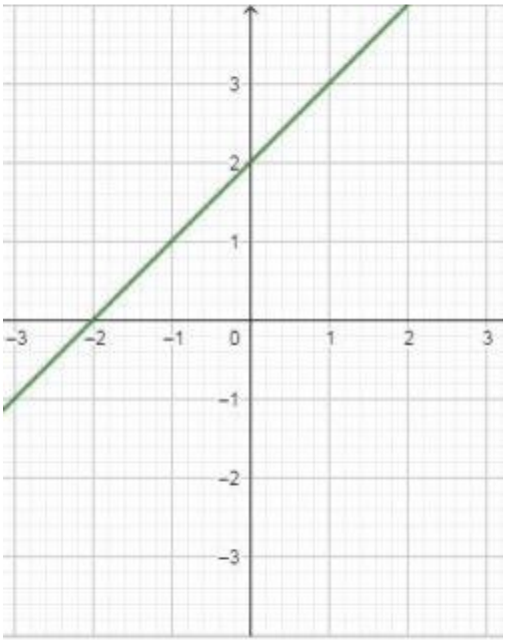


pair of linear equations which intersect at a single point.

ii) They are parallel



pair of linear equations which are parallel.
iii) They are coincident



pair of linear equations which are coincident.

Graphical Solution

Representing Pair of LE in 2 Variables Graphically

Graphically, a pair of linear equations in two variables can be represented by a pair of straight lines.

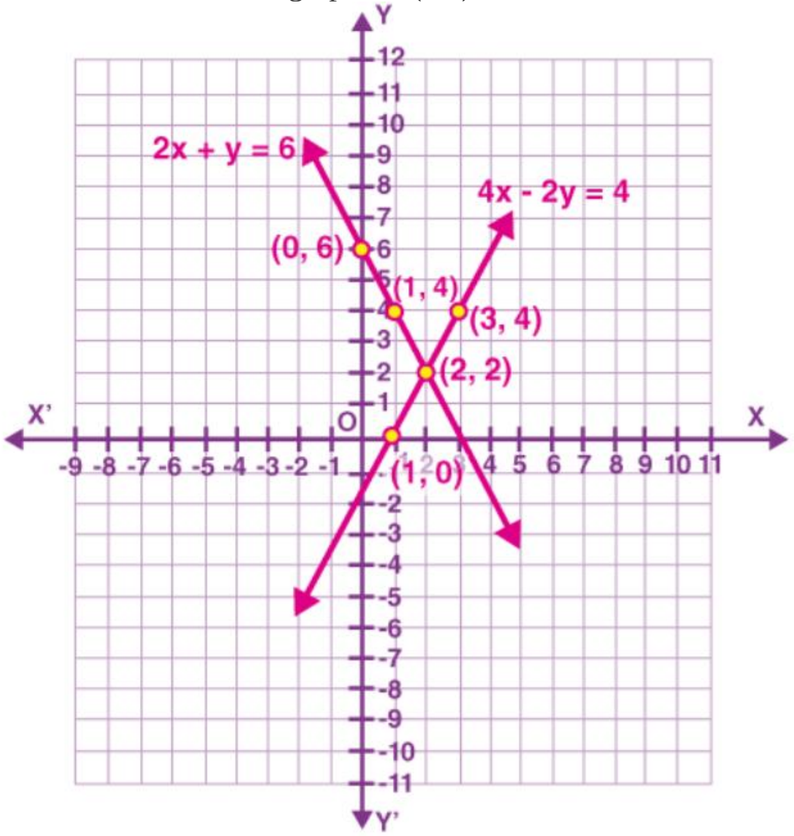
Graphical Method of Finding Solution of a Pair of Linear Equations

The Graphical Method of finding the solution to a pair of linear equations is as follows:

- Plot both the equations (two straight lines)
- Find the point of intersection of the lines.

The point of intersection is the solution.
For example, the graph of two linear equations $2x + y - 6 = 0$ and $4x - 2y - 4 = 0$ is shown below. The point of

intersection for the two graphs is (2,2).



Comparing the Ratios of Coefficients of a Linear Equation

i) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of equations are said to be **consistent**. Graphs of the two equations intersect at a unique point. The pair of linear equations have **exactly one solution**.

ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the equations are said to be **dependent**. One equation can be obtained by multiplying the other equation with a non-zero constant. In this case, graphs of both the equations coincide. Dependent equations are consistent. The pair linear equations have **infinitely many solutions.**

iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the equations are said to be **inconsistent**. The graphs of the equations are parallel to each other. The pair of linear equations have **no solution**.

CH.3 PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

NCERT C.W EX-3.1(Q.No.1,3,5&7), EX-3.2 Q.1&3 EX-3.3(Q.1)

NCERT H.W EX-3.1(Q.No.2,4,6) EX-3.2(Q.2) EX-3.3(Q.2)

NCERT EXEMPLAR C.W EX-3.1(Q.No.7 TO 13), EX-3.3(Q.No.1 TO 5)

NCERT EXEMPLAR H.W EX-3.3(Q.No.6 TO 13)

WORKBOOK C.W- CASE STUDY(Q.38)

WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.4 QUADRATIC EQUATIONS

Quadratic Equation

When we equate a quadratic polynomial to a constant, we get a quadratic equation.

Any equation of the form p(x) = k, where p(x) is a polynomial of degree 2, and c is a constant, is a quadratic equation.

The Standard Form of a Quadratic Equation

The standard form of a quadratic equation is $ax^2+bx+c=0$, where a,b and c are real numbers and $a\neq0$.
'a' is the coefficient of x^2. It is called the quadratic coefficient. 'b' is the coefficient of x. It is called the linear coefficient. 'c' is the constant term.

Solving Quadratic Equations by Factorisation

Roots of a Quadratic Equation

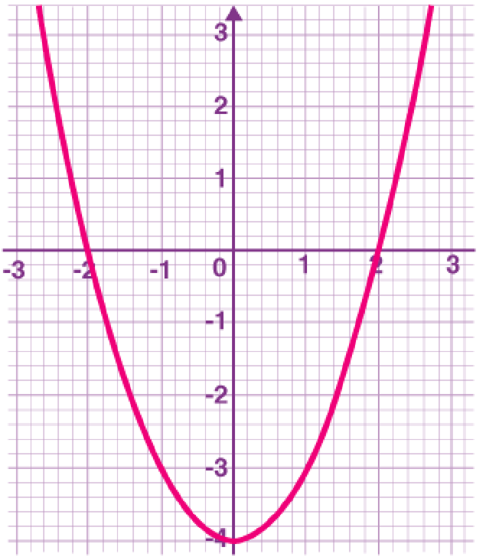
The values of x for which a quadratic equation is satisfied are called the roots of the quadratic equation.

If α is a root of the quadratic equation $ax^2+bx+c=0$, then $a\alpha^2+b\alpha+c=0$.

A quadratic equation can have two distinct real roots, two equal roots or real roots may not exist.

Graphically, the roots of a quadratic equation are the points where the graph of the quadratic polynomial cuts the x-axis.

Consider the graph of a quadratic equation $x^2-4=0$



Graph of a Quadratic Equation

In the above figure, -2 and 2 are the roots of the quadratic equation $x^2-4=0$
Note:

- If the graph of the quadratic polynomial cuts the x-axis at two distinct points, then it has real and distinct roots.
- If the graph of the quadratic polynomial touches the x-axis, then it has real and equal roots.

- If the graph of the quadratic polynomial does not cut or touch the x-axis then it does not have any real roots.

Solving Quadratic Equation Using Quadratic Formula

Quadratic Formula

Quadratic Formula is used to directly obtain the roots of a quadratic equation from the standard form of the equation.

For the quadratic equation $ax^2+bx+c=0$,

$x= [-b\pm \sqrt{(b^2-4ac)}]/2a$

By substituting the values of a,b and c, we can directly get the roots of the equation.

Example: If $x^2 – 5x + 6 = 0$ is the quadratic equation, find the roots.

Solution: Given, $x^2 – 5x + 6 = 0$ is the quadratic equation.

On comparing with the standard quadratic equation, we have;

$ax^2 + bx + c = 0$

$a = 1$, $b = -5$ and $c = 6$

Since,

$b^2 – 4ac = (-5)^2 – 4 \times 1 \times 6 = 25 – 24 = 1 > 0$
Hence, the roots are real.
Using quadratic formula,

$x = [-b \pm \sqrt{(b^2 – 4ac)}]/ 2a$

$= [-(-5) \pm \sqrt{1}]/ 2(1)$

$= [5 \pm 1]/ 2$

i.e. $x = (5 + 1)/2$ and $x = (5 – 1)/2$

$x = 6/2, x = 4/2$

$x = 3, 2$

Therefore, the roots of the quadratic equation are 3 and 2.

Discriminant

For a quadratic equation of the form $ax^2+bx+c=0$, the expression b^2-4ac is called the discriminant (denoted by D) of the quadratic equation.

The discriminant determines the nature of the roots of the quadratic equation based on the coefficients of the quadratic equation.

Nature of Roots

Based on the value of the discriminant, $D=b^2-4ac$, the roots of a quadratic equation, $ax^2 + bx + c = 0$, can be of three types.

Case 1: If **D>0**, the equation has two **distinct real roots**.

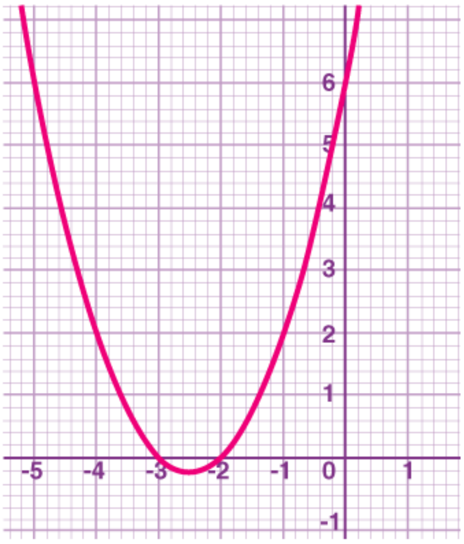
Case 2: If **D=0**, the equation has two **equal real roots**.

Case 3: If **D<0**, the equation has **no real roots**.

Graphical Representation of a Quadratic Equation

The graph of a quadratic polynomial is a parabola. The roots of a quadratic equation are the points where the parabola cuts the x-axis i.e. the points where the value of the quadratic polynomial is zero.

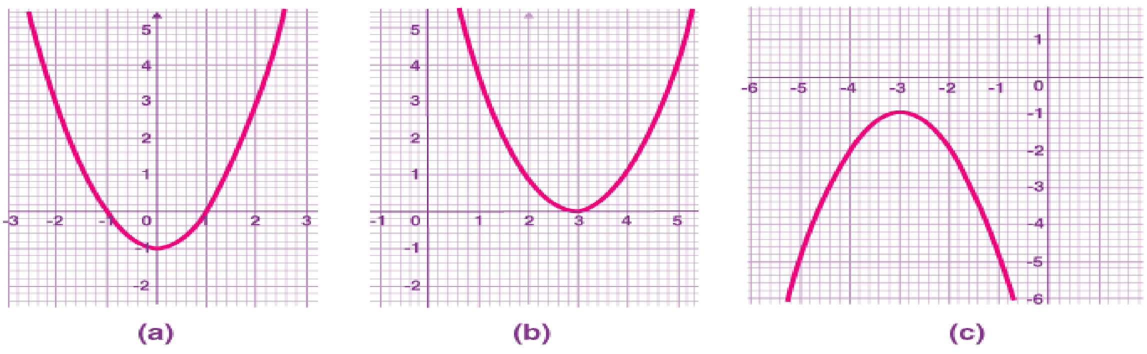
Now, the graph of $x^2+5x+6=0$ is:



In the above figure, -2 and -3 are the roots of the quadratic equation
$x^2+5x+6=0$.

For a quadratic polynomial ax^2+bx+c,

If a>0, the parabola opens upwards.
If a<0, the parabola opens downwards. If a = 0, the polynomial will become a first-degree polynomial and its graph is linear. The discriminant, $D=b^2-4ac$



Nature of graph for different values of D.

If **D>0**, the parabola cuts the x-axis at exactly two distinct points. The roots are distinct. This case is shown in the above figure in a, where the quadratic polynomial cuts the x-axis at **two distinct points.**

If **D=0**, the parabola just touches the x-axis at one point and the rest of the parabola lies above or below the x-axis. In this case, the roots are equal.
This case is shown in the above figure in b, where the quadratic polynomial touches the x-axis at **only one point**.

If **D<0**, the parabola lies entirely above or below the x-axis and there is no point of contact with the x-axis. In this case, there are no real roots.

This case is shown in the above figure in c, where the quadratic polynomial **neither cuts nor touches** the x-axis.

CH.4 QUADRATIC EQUATIONS
NCERT C.WEX-4.1(Q.No.1) EX-4.2(Q.No.1,2,3&4) EX-4.3(Q.1,3&5)
NCERT H.WEX-4.1(Q.No.2) EX-4.2 (Q.No.5&6) EX-4.3(Q.2&4)
NCERT EXEMPLAR C.W EX-4.1(Q.4 TO Q.10) EX-4.3 (Q.1) EX-4.4 (Q.1TO Q.4)
NCERT EXEMPLAR H.W EX-4.3 (Q.2) EX-4.4 (Q.5TO Q.8)
WORKBOOK C.W CASE STUDY(Q.38)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.5 ARITHMETIC PROGRESSION

Introduction to AP

Sequences, Series and Progressions

- A **sequence** is a finite or infinite list of numbers following a specific pattern. For example, 1, 2, 3, 4, 5,… is the sequence, an infinite sequence of natural numbers.
- A **series** is the sum of the elements in the corresponding sequence. For example, 1+2+3+4+5….is the series of natural numbers. Each number in a sequence or a series is called a term.
- A **progression** is a sequence in which the general term can be can be expressed using a mathematical formula.

For any finite sequence, it is generally represented as a_1, a_2, a_3, ……a_n, where 1, 2, 3, …, n represents the position of the term. As the series is represented as the sum of sequences, it is represented as $a_1 + a_2 + a_3 + …. + a_n$.

For any infinite sequence, it is generally represented as a_1, a_2, a_3, a_4, … and the infinite series is represented as $a_1 + a_2 + a_3 + ….$

Arithmetic Progression

An arithmetic progression (AP) is a progression in which the **difference** between two **consecutive** terms is constant.

In arithmetic progression, the first term is represented by the letter "a", the last term is represented by "l", the common difference between two terms is represented by "d", and the number of terms is represented by the letter "n".

Thus, the standard form of the arithmetic progression is given by the formula,

a, a + d, a + 2d, a + 3d, a + 4d, ….

Now, consider the infinite arithmetic progression 2, 5, 8, 11, 14….

Here, first term, a = 2

Common difference = 3

Here, the common difference is calculated as follows:

Second term – first term = 5 – 2 = 3

Third term – second term = 8 – 5 = 3

Fourth term – third term = 11 – 8 = 3

Fifth term – fourth term = 14 – 11 = 3

Since the difference between two consecutive terms is constant (i.e., 3), the given progression is an arithmetic progression.

Common Difference

The difference between two consecutive terms in an AP (*which is constant*) is the "**common difference**"**(d)** of an A.P. In the progression: *2, 5, 8, 11, 14* …the common difference is 3.
As it is the difference between any two consecutive terms, for any A.P, if the common difference is:

- **Positive**, the AP is **increasing**.
- **Zero**, the AP is **constant**.
- **Negative**, the A.P is **decreasing.**

The formula to find the common difference between the two terms is given as:

Common difference, d = (a_n – a_{n-1})

Where,

a_n represents the nth term of a sequence

a_{n-1} represents the previous term. i.e., $(n-1)^{th}$ term of a sequence.

Finite and Infinite AP

- A finite AP is an A.P in which the number of terms is finite. For example the A.P: *2, 5, 8……32, 35, 38*
- An **infinite** A.P is an A.P in which the **number of terms is infinite**. For example: *2, 5, 8, 11…..*

A finite A.P will have the last term, whereas an infinite A.P won't.

General Term of AP

In Arithmetic progression, a_n is called the general term, where n represents the position of the term in the given sequence.

The nth Term of an AP

The nth term of an A.P is given by $T_n = a+(n-1)d$, where **a** is the first term, **d** is a common difference and **n** is the number of terms.

Finding nth Term:

Determine the tenth term of the arithmetic progression 2, 7, 12, ….

Solution:

Given Arithmetic sequence: 2, 7, 12, …

Here, first term, a = 2

Common difference, d = 5

I.e., 7 – 2 = 5 and 12 – 7 = 5.

And now, we have to find the 10th term of AP.

Hence, n = 10

Thus, the formula to find the nth term of AP is $a_n = a + (n-1)d$

Now, substituting the values in the formula, we get

$a_{10} = 2 + (10 – 1)5$

$a_{10} = 2 + 9(5)$

$a_{10} = 2 + 45$

$a_{10} = 47$.

Therefore, 10^{th} term of the given arithmetic sequence 2, 7, 12, … is 47.

The General Form of an AP

The general form of an A.P is: ***(a, a + d, a+2d, a+3d……)*** where **a** is the first term and **d** is a common difference. Here, d=0, OR d > 0, OR d < 0

The Sum of Terms in an AP

The Formula for the Sum to n Terms of an AP

The sum to n terms of an A.P is given by:

$S_n= n/2(2a+(n−1)d)$

Where **a** is the first term, **d** is the common difference and **n** is the number of terms.

The sum of n terms of an A.P is also given by

$S_n= n/2(a+l)$

Where **a** is the first term, **l** is the last term of the A.P. and **n** is the number of terms.

Arithmetic Mean (A.M)

The Arithmetic Mean is the simple average of a given set of numbers. The arithmetic mean of a set of numbers is given by:

A.M= Sum of terms/Number of terms

The arithmetic mean is defined for any set of numbers. The numbers need not necessarily be in an A.P.

For example, of x, y and z are in Arithmetic progression, then y = (x + z)/2, and we can say that y is the arithmetic mean of x and z.

Basic Adding Patterns in an AP

The sum of two terms that are equidistant from either end of an AP is constant.
For example: in an A.P: *2,5,8,11,14,17…*
$T_1+T_6=2+17=19$
$T_2+T_5=5+14=19$ and so on….
Algebraically, this can be represented as

$T_r+T_{(n-r)+1}$=constant

The Sum of First n Natural Numbers

The **sum** of first **n** natural numbers is given by:

$S_n=n(n+1)/2$

This formula is derived by treating the sequence of natural numbers as an A.P where the first term (a) = 1 and the common difference (d) = 1.

Finding the Sum of First n Natural Numbers:

For example, if we want to find the sum of the first 10 natural numbers, we can find it as follows:

Here, n = 10.

Now, substitute the value in the formula,

$S_n=n(n+1)/2$

S_{10} = [10 (10+1)]/2

S_{10} = [10(11)]/2

S_{10} = 110/2

S_{10} = 55.

All the formulas related to Arithmetic Progression class 10 are tabulated below:

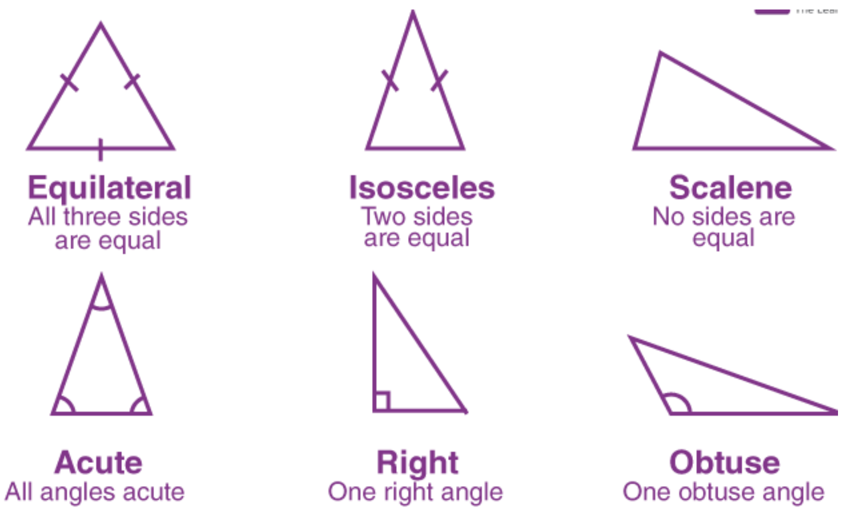
First term	a
Common difference	d
General form of AP	a, a + d, a + 2d, a + 3d,….
nth term	a_n = a + (n – 1)d
Sum of first n terms	S_n = (n/2) [2a + (n – 1)d]
Sum of all terms of AP	S = (n/2)(a + l) n = Number of terms l = Last term

CH.12 ARITHMETIC PROGRESSION
NCERT C.W EX-5.1(Q.No.1,3) EX-5.2(Q.NO. 2,5,7,9,12,14,16,18,20) EX-5.3(Q.NO. 1,3,4,8,11,15,16,18,19,20)
NCERT H.WEX- 5.1(Q.No.2,4) EX-5.2(Q.NO. 1,3,4,6,8,10,11,13,15,17,19) EX-5.3(Q.NO. 2,5,6,7,9,10,12,13,14,17)
NCERT EXEMPLAR C.W EX-5.1(Q.10 TO 18),EX-5.3(Q.21,26,33,34,35)
EX-5.4(Q.6 TO 10)
NCERT EXEMPLAR H.W EX-5.4(Q.1 TO 5)
WORKBOOK C.W -CASE STUDY(Q.38)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.6 TRIANGLES

What Is a Triangle?

A triangle can be defined as a polygon which has three angles and three sides. The interior angles of a triangle sum up to 180 degrees, and the exterior angles sum up to 360 degrees. Depending upon the angle and its length, a triangle can be categorized into the following types-



1. Scalene Triangle – All three sides of the triangle are of different measure
2. Isosceles Triangle – Any two sides of the triangle are of equal length
3. Equilateral Triangle – All three sides of a triangle are equal and each angle measures 60 degrees
4. Acute angled Triangle – All the angles are smaller than 90 degrees
5. Right angle Triangle – Anyone of the three angles is equal to 90 degrees
6. Obtuse-angled Triangle – One of the angles is greater than 90 degrees

Similarity Criteria of Two Polygons Having the Same Number of Sides

Any two polygons which have the same number of sides are similar if the following two criteria are met-

1. Their corresponding angles are equal, and
2. Their corresponding sides are in the same ratio (or proportion)

Similarity Criteria of Triangles

Mathematical Expression for Similarity

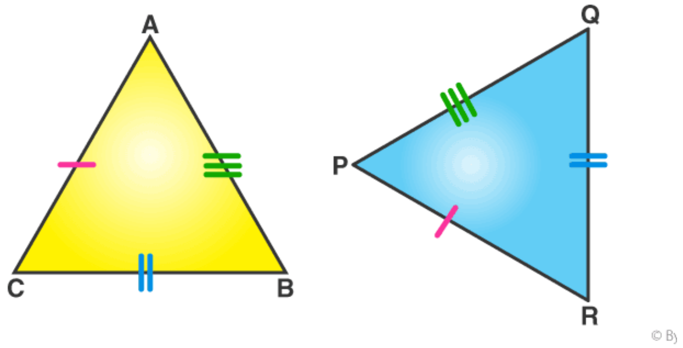
In Δ ABC and Δ DEF, if

(i) ∠ A = ∠ D, ∠ B = ∠ E, ∠ C = ∠ F and

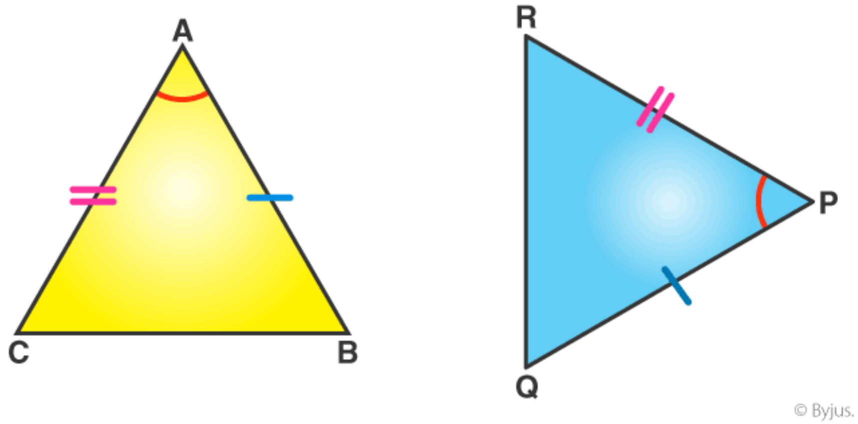
(ii)AB/DE = BC/ EF = CA/ FD, then the two triangles are similar.

To find whether the given two triangles are similar or not, it has four criteria. They are:

- **Side-Side-Side (SSS) Similarity Criterion –** When the corresponding sides of any two triangles are in the same ratio, then their corresponding angles will be equal, and the triangle will be considered similar triangles.

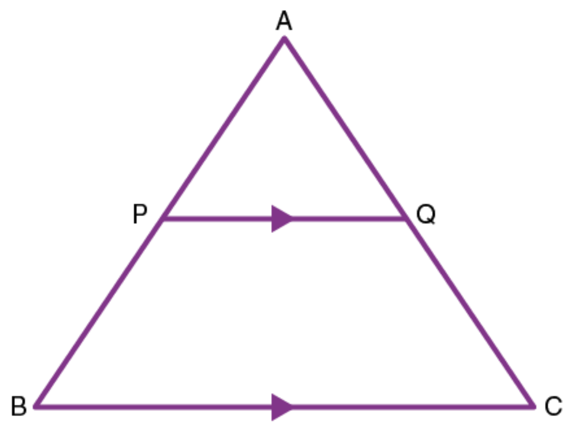


- **Angle Angle Angle (AAA) Similarity Criterion** – When the corresponding angles of any two triangles are equal, then their corresponding side will be in the same ratio, and the triangles are considered to be similar.
- **Angle-Angle (AA) Similarity Criterion –** When two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are considered similar.
- **Side-Angle-Side (SAS) Similarity Criterion –** When one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are said to be similar.



Basic Proportionality Theorem

The basic proportionality theorem states that “If a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.”



Now, let us understand the Basic proportionality theorem with the help of a diagram.

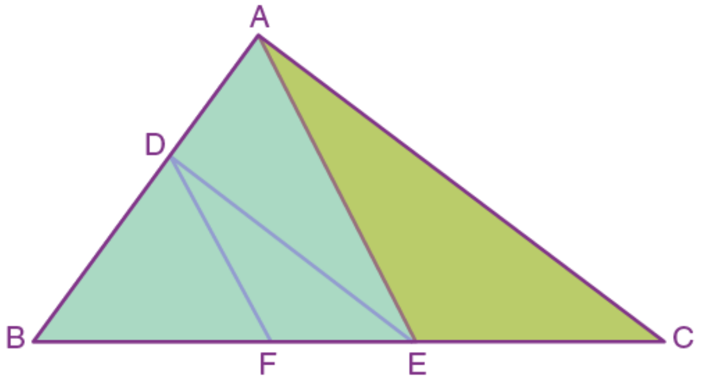
Consider the triangle ABC, as depicted in the diagram. We draw a line PQ parallel to the side BC of ABC and intersect the sides AB and AC in P and Q, respectively.

Thus, according to the Basic proportionality theorem,
AP/PB = AQ/QC

.

Example 1:

In the given figure, DE is parallel to AC, and DF is parallel to AE. Prove that BF/FE = BE/EC.



Solution:

Given that DE || AC and DF||AE.

To prove: BF/FE = BE/EC

In a triangle ABC, DE || AC.

We know that the line drawn parallel to one side of a triangle intersects the other two sides in distinct points, then it divides the other two sides in the same ratio.

Therefore, BE/EC = BD/DA …(1)

Now, consider the triangle AEB,

DF || AE.

Thus, we can say that

BF/FE = BD/DA …(2)

By comparing the equation (1) and (2), we can say

BE/EC = BF/FE

Hence, proved.

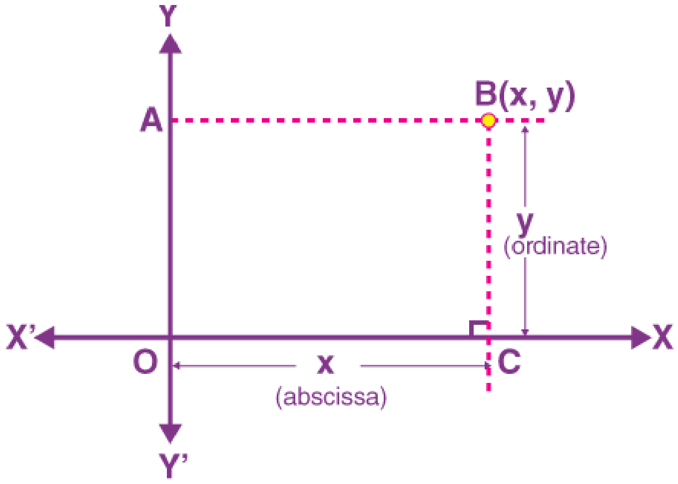
CH.6 TRIANGLES
NCERT C.W EX-6.1(Q.1& 2) EX-6.2(Q.1,3,5,7,8,9) EX-6.3(Q.1,3,7,8,10,12,14,16)
NCERT H.W EX-6.1(Q.3) EX-6.2(Q.2,4,6,10) EX-6.3(Q.2,4,5,6,9,11,13,15)
NCERT EXEMPLAR C.W EX-6.3(Q.2,3,5,7,14,15)
NCERT EXEMPLAR H.W EX-6.4(Q.4,7,8,16)

WORKBOOK C.W- CASE STUDY(Q.40)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.7 COORDINATE GEOMETRY

Points on a Cartesian Plane

A pair of numbers locate points on a plane called the **coordinates**. The distance of a point from the y-axis is known as **abscissa** or x-coordinate. The distance of a point from the x-axis is called **ordinate** or y-coordinate.

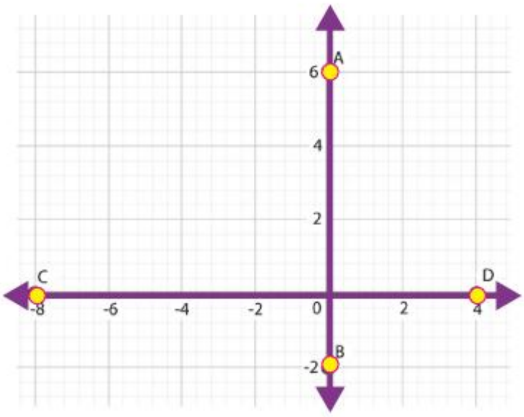


Example: Consider a point P(3, 2), where 3 is the abscissa, and 2 is the ordinate. 3 represents the distance of point P from the y-axis, and 2 represents the distance of point P from the x-axis.

Distance Formula

Distance between Two Points on the Same Coordinate Axes

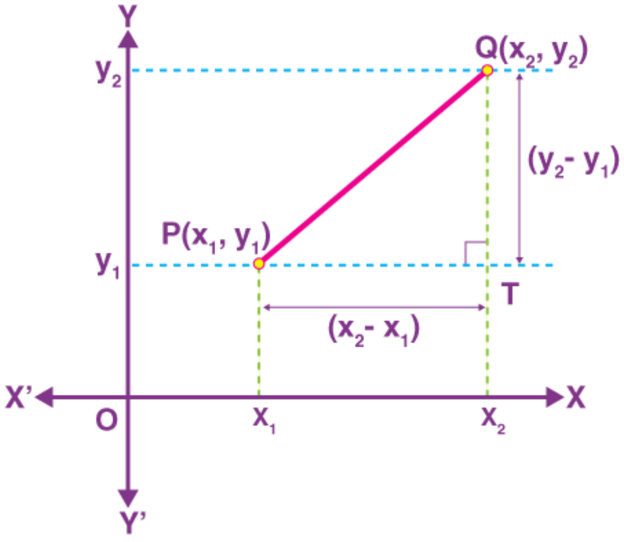
The distance between two points that are on the same axis (x-axis or y-axis) is given by the difference between their ordinates if they are on the y-axis, else by the difference between their abscissa if they are on the x-axis.



Distance AB = 6 – (-2) = 8 units

Distance CD = 4 – (-8) = 12 units

Distance between Two Points Using Pythagoras Theorem



Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points on the cartesian plane.

Draw lines parallel to the axes through P and Q to meet at T.

ΔPTQ is right-angled at T.

By **Pythagoras Theorem**,

$PQ^2 = PT^2 + QT^2$

$= (x_2 – x_1)^2 + (y_2 – y_1)^2$

$PQ = \sqrt{[x_2 – x_1)^2 + (y_2 – y_1)^2]}$

Distance Formula

Distance between any two points (x_1, y_1) and (x_2, y_2) is given by

$d = \sqrt{[x_2 – x_1)^2+(y_2 – y_1)^2]}$

Where d is the distance between the points (x_1,y_1) and (x_2,y_2).

Section Formula

If the point P(x, y) **divides** the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ **internally** in the **ratio m:n**, then, the coordinates of P are given by the **section formula** as:

$P(x, y)=(mx_2+nx_1 /m+n,my_2+ny_1 /m+n)$

.

Finding Ratio given the points

To find the ratio in which a given point P(x, y) divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$,

- Assume that the ratio is k : 1
- Substitute the ratio in the section formula for any of the coordinates to get the value of k.

x=kx2+x1/ k+1

When x1, x2 and x are known, k can be calculated. The same can be calculated from the y- coordinate also.

Example: Find the ratio when point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?

Solution: Let the ratio be m:n.

We can write the ratio as:

m/n:1 or k:1

Suppose (-4, 6) divide the line segment AB in k:1 ratio.

Now using the section formula, we have the following;

$(−4,6)=(3k−6k+1,−8k+10k+1)$

$-4 = (3k-6)/(k+1)$

$– 4k – 4 = 3k – 6$

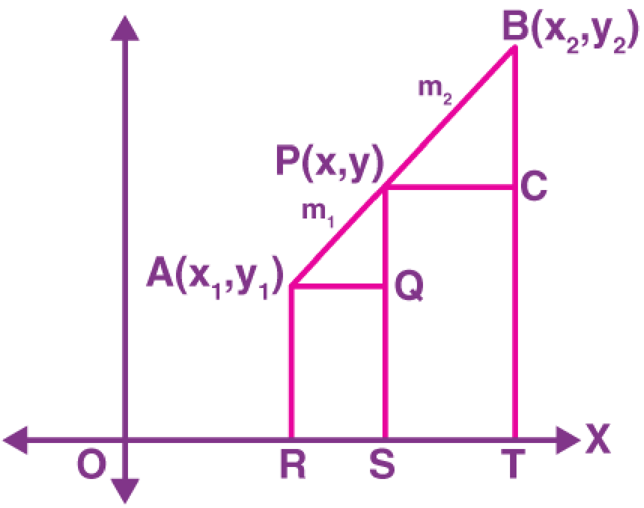
$7k =2$

$k : 1 = 2 : 7$

Thus, the required ratio is 2:7.

Midpoint

The **midpoint** of any line segment divides it in the ratio **1 : 1**.



The coordinates of the midpoint(P) of line segment joining A(x_1, y_1) and B(x_2, y_2) is given by

$p(x, y)=(x1+x2 /2,y1+y2 /2)$

Example: What is the midpoint of line segment PQ whose coordinates are P (-3, 3) and Q (1, 4), respectively.

Solution: Given, P (-3, 3) and Q (1, 4) are the points of line segment PQ.

Using midpoint formula, we have;

Midpoint of $PQ=(−3+12,−3+42)$

$= (-2/2, 1/2)$

$= (-1, 1/2)$

Points of Trisection

To find the points of trisection P and Q, which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ into three equal parts:

i) **AP : PB = 1 : 2**

ii) **AQ : QB = 2 : 1**

Example: Find the coordinates of the points of trisection of the line segment joining the points A(2, – 2) and B(– 7, 4).

Solution: Let P and Q divide the line segment AB into three parts.

So, P and Q are the points of trisection here.

Let P divides AB in 1:2, thus by section formula, the coordinates of P are (1, 0)

Let Q divides AB in 2:1 ratio, then by section formula, the coordinates are (-4,2)

Thus, the point of trisection for line segment AB are (1,0) and (-4,2).

Centroid of a Triangle

If $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ are the vertices of a ΔABC, then the coordinates of its centroid(P) are given by

$p(x, y)=(x_1+x_2+x_3 /3, y_1+y_2+y3 /3)$

Example: Find the coordinates of the centroid of a triangle whose vertices are given as (-1, -3), (2, 1) and (8, -4)

Solution: Given,

The coordinates of the vertices of a triangle are (-1, -3), (2, 1) and (8, -4)

The Centroid of a triangle is given by:

$G = ((x_1+x_2+x_3)/3 , (y_1+y_2+y_3)/3)$

$G = ((-1+2+8)/3 , (-3+1-4)/3)$

$G =(9/3 , -6/3)$

$G = (3, -2)$

Therefore, the centroid of a triangle, G = (3, -2)

Collinearity Condition

If three points A, B and C are collinear and B lies between A and C, then,

- AB + BC = AC. AB, BC, and AC can be calculated using the distance formula.
- The ratio in which B divides AC, calculated using the section formula for both the x and y coordinates separately, will be equal.
- The area of a triangle formed by three collinear points is zero.

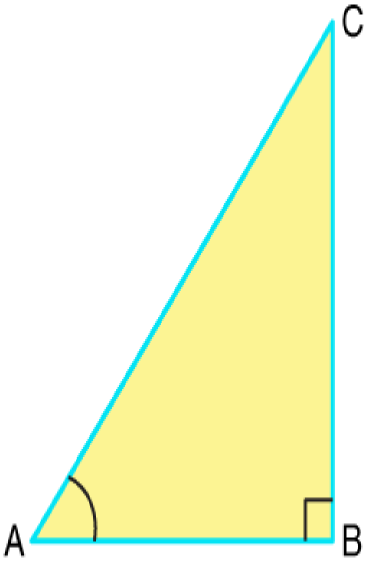
CH.7 COORDINATE GEOMETRY
NCERT C.W – EXE 7.1:(Q.NO.1,3,5,8,10) EXE 7.2 – :(Q.NO.1,3,4,6,8,10)
NCERT H.W- EXE 7.1:(Q.NO.2,4,6,7,9) EXE 7.2 – :(Q.NO.2,5,7,9)
NCERT EXEMPLAR EX 7.3 C.W- :(Q.NO.13,15,18,20)
NCERT EXEMPLAR H.W-EX-7.4(Q.1 & 4)
WORKBOOK C.W - CASE STUDY(Q.38,39)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.8 INTRODUCTION TO TRIGONOMETRY

Trigonometric Ratios

Opposite & Adjacent Sides in a Right-Angled Triangle

In the ΔABC right-angled at B, BC is the side opposite to ∠A, AC is the hypotenuse, and AB is the side adjacent to ∠A.



Trigonometric Ratios

For the right ΔABC, right-angled at ∠B, the trigonometric ratios of the ∠A are as follows:

- sin A=opposite side/hypotenuse=BC/AC
- cos A=adjacent side/hypotenuse=AB/AC
- tan A=opposite side/adjacent side=BC/AB
- cosec A=hypotenuse/opposite side=AC/BC

- sec A=hypotenuse/adjacent side=AC/AB
- cot A=adjacent side/opposite side=AB/BC

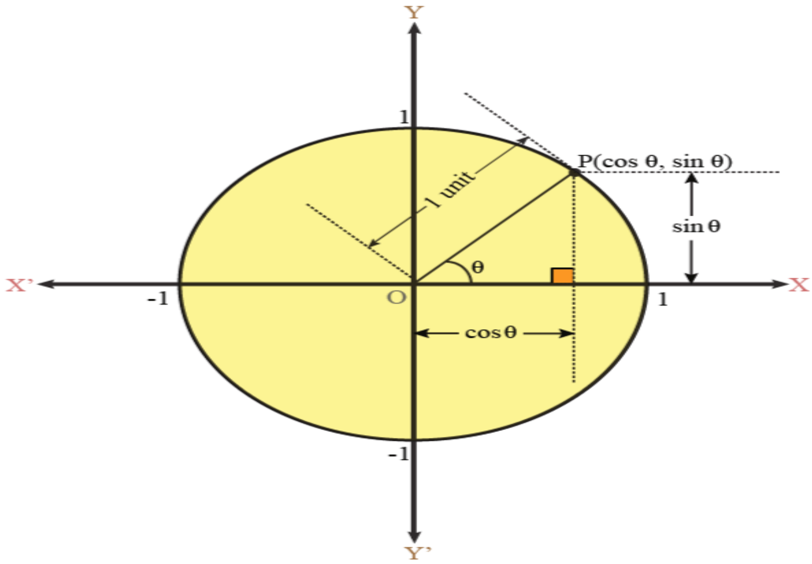
Relation between Trigonometric Ratios

- cosec θ =1/sin θ
- sec θ = 1/cos θ
- tan θ = sin θ/cos θ
- cot θ = cos θ/sin θ=1/tan θ

Visualization of Trigonometric Ratios Using a Unit Circle

Draw a circle of the unit radius with the origin as the centre. Consider a line segment OP joining a point P on the circle to the centre, which makes an angle θ with the x-axis. Draw a perpendicular from P to the x-axis to cut it at Q.

- sin θ=PQ/OP=PQ/1=PQ
- cos θ=OQ/OP=OQ/1=OQ
- tan θ=PQ/OQ=sin θ/cos θ
- cosec θ=OP/PQ=1/PQ
- sec θ=OP/OQ=1/OQ
- cot θ=OQ/PQ=cos θ/sin θ



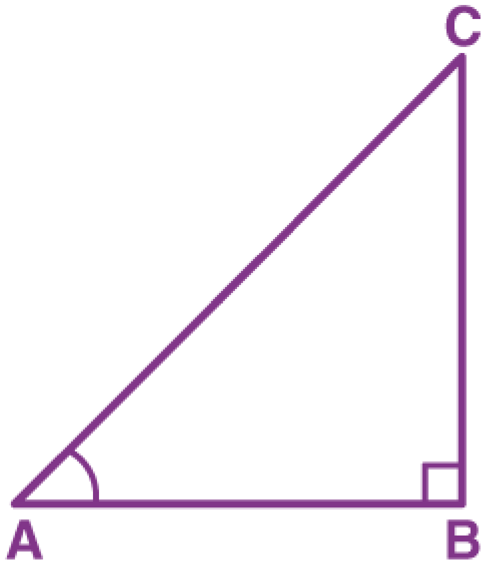
Visualisation of Trigonometric Ratios Using a Unit Circle

Trigonometric Ratios of Specific Angles

The specific angles that are defined for trigonometric ratios are 0°, 30°, 45°, 60° and 90°.

Trigonometric Ratios of 45°

If one of the angles of a right-angled triangle is 45°, then another angle will also be equal to 45°.



Let us say ABC is a right-angled triangle at B, such that;

∠ A = ∠ C = 45°

Thus, BC = AB = a (say)

Using Pythagoras theorem, we have;

$AC^2 = AB^2 + BC^2$

$= a^2 + a^2$

$= 2a^2$

AC = a√2

Now, from the trigonometric ratios, we have;

- sin 45° = (Opp. side to angle 45°)/Hypotenuse = BC/AC = a/a√2 = 1/√2
- cos 45° = (Adj. side to angle 45°)/Hypotenuse = AB/AC = a/a√2 = 1/√2
- tan 45° = BC/AB = a/a = 1

Similarly,

- cosec 45° = 1/sin 45° = √2
- sec 45° = 1/cos 45° = √2
- cot 45° = 1/tan 45° = 1

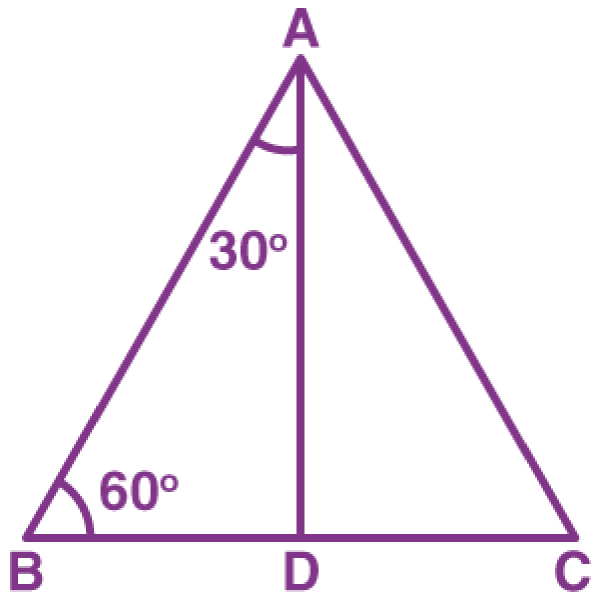
Trigonometric Ratios of 30° and 60°

Here, we will consider an equilateral triangle ABC, such that;

AB = BC = AC = 2a

∠A = ∠B = ∠C = 60°

Now, draw a perpendicular AD from vertex A that meets BC at D



According to the congruency of the triangle, we can say;

Δ ABD ≅ Δ ACD

Hence,

BD = DC

∠ BAD = ∠ CAD (By CPCT)

Now, in triangle ABD, ∠ BAD = 30° and ∠ ABD = 60°

Using Pythagoras theorem,

$AD^2 = AB^2 – BD^2$

$= (2a)^2 – (a)^2$

$= 3a^2$

AD = a√3

So, the trigonometric ratios for a 30-degree angle will be;

sin 30° = BD/AB = a/2a = 1/2

cos 30° = AD/AB = a√3/2a = √3/2

tan 30° = BD/AD = a/a√3 = 1/√3

Also,

cosec 30° = 1/sin 30 = 2

sec 30° = 1/cos 30 = 2/√3

cot 30° = 1/tan 30 = √3

Similarly, we can derive the values of trigonometric ratios for 60°.

- sin 60° = √3/2
- cos 60° = 1/2
- tan 60° = √3
- cosec 60° = 2/√3

- sec 60° = 2
- cot 60° = 1/√3

Trigonometric Ratios of 0° and 90°

If ABC is a right-angled triangle at B, if ∠A is reduced, then side AC will come near to side AB. So, if ∠ A is nearing 0 degree, then AC becomes almost equal to AB, and BC get almost equal to 0.

Hence, Sin A = BC /AC = 0

and cos A = AB/AC = 1

tan A = sin A/cos A = 0/1 = 0

Also,

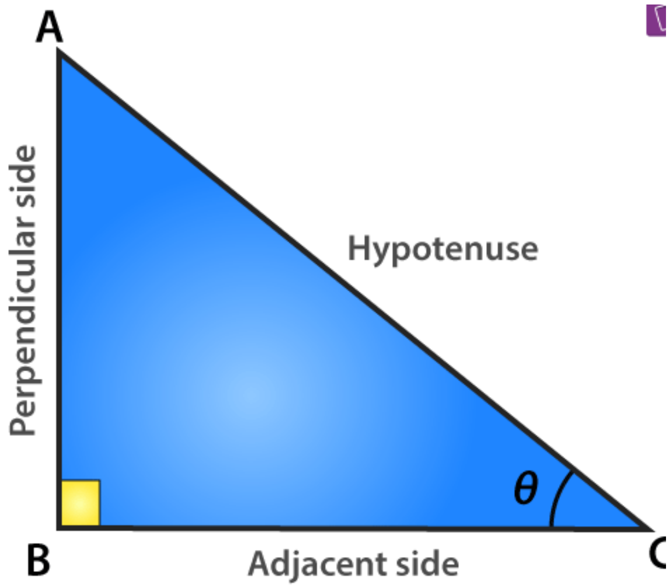
cosec A = 1/sin A = 1/0 = not defined

sec A = 1/cos A = 1/1 = 1

cot A = 1/tan A = 1/0 = not defined

In the same way, we can find the values of trigonometric ratios for a 90-degree angle. Here, angle C is reduced to 0, and the side AB will be nearing side BC such that angle A is almost 90 degrees and AB is almost 0.

Variation of Trigonometric Ratios from 0 to 90 Degrees



Standard Values of Trigonometric Ratios

∠A	0°	30°	45°	60°	90°
sin A	0	1/2	1/√2	√3/2	1
cos A	1	√3/2	1/√2	1/2	0
tan A	0	1/√3	1	√3	not defined
cosec A	not defined	2	√2	2/√3	1

sec A	1	2/√3	√2	2	not defined
cot A	not defined	√3	1	1/√3	0

Trigonometric Identities

The three most important trigonometric identities are:

- $\sin^2\theta+\cos^2\theta=1$
- $1+\cot^2\theta=\text{coesc}^2\theta$
- $1+\tan^2\theta=\sec^2\theta$

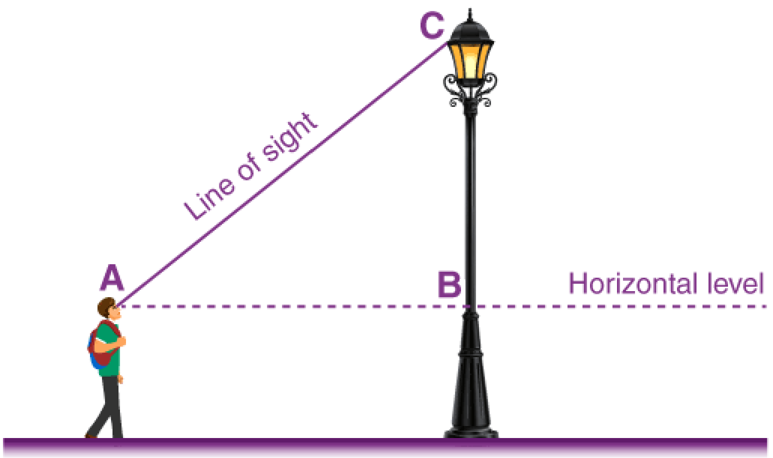
CH.8 INTRODUCTION TO TRIGONOMETRY
NCERT C.W – EXE 8.1: Q.1,4,6,8,10 EXE 8.2 – Q.1, 3 EXE 8.3 Q.1,4
NCERT H.W- EXE 8.1: Q.2,3,5,7,9&11 EXE 8.2 – Q.2, 4 EXE 8.3 Q.2,3
NCERT EXEMPLAR C.W- EXE 8.4 Q.4,5,9,10,11,12
NCERT EXEMPLAR H.W- EXE 8.4 Q.1,2
WORKBOOK C.W - CASE STUDY(Q.38)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.9 SOME APPLICATIONS OF TRIGONOMETRY

Horizontal Level and Line of Sight

Line of sight is the line drawn from the eye of the observer to the point on the object viewed by the observer.

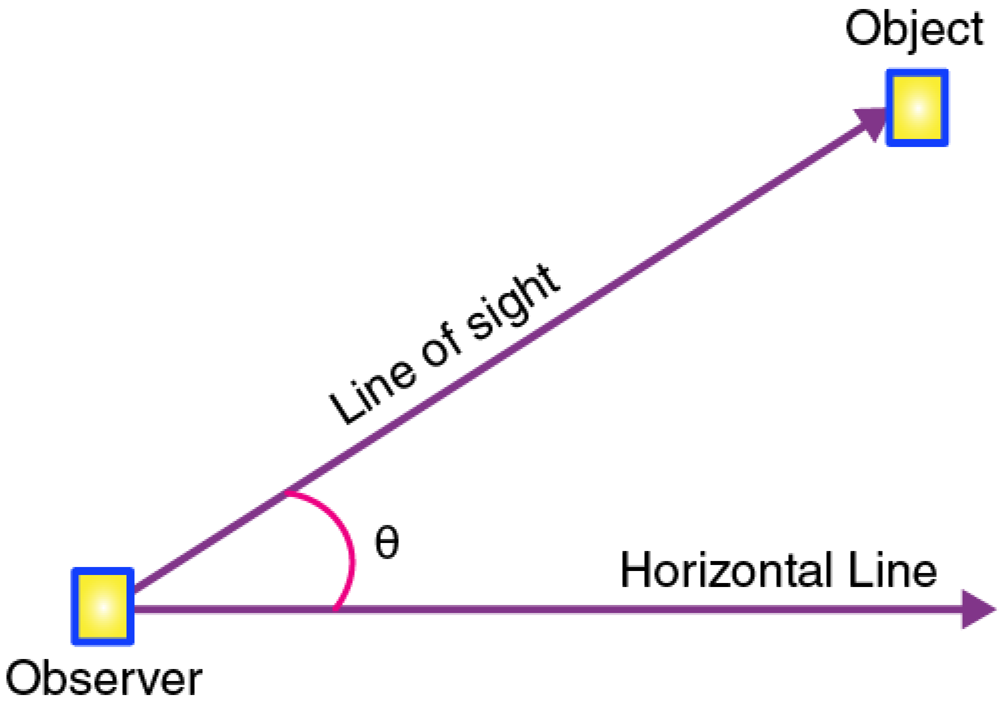
The **horizontal level** is the horizontal line through the eye of the observer.



Line of sight and horizontal level

Angle of Elevation

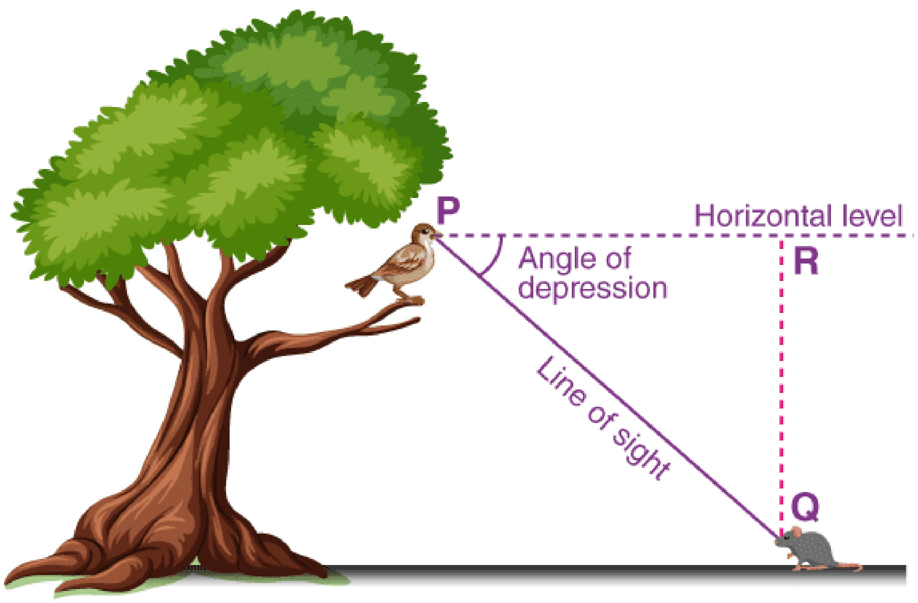
The **angle of elevation** is relevant for objects **above** the horizontal level. It is the **angle** formed by the **line of sight** with the **horizontal level**. In the below-mentioned diagram, “θ” denotes the angle of elevation.



Angle of elevation

Angle of Depression

The **angle of depression** is relevant for objects **below** the horizontal level. It is the **angle** formed by the **line of sight** with the **horizontal level**.



Angle of depression

Calculating Heights and Distances

To calculate heights and distances, we can make use of trigonometric ratios.

Go through the below trigonometric ratio table for reference:

Trigonometry Ratios Table								
Angles (In Degrees)	0°	30°	45°	60°	90°	180°	270°	360°

Angles (In Radians)	0°	π/6	π/4	π/3	π/2	π	3π/2	2π
sin	0	1/2	1/√2	√3/2	1	0	-1	0
cos	1	√3/2	1/√2	1/2	0	-1	0	1
tan	0	1/√3	1	√3	∞	0	∞	0
cot	∞	√3	1	1/√3	0	∞	0	∞
cosec	∞	2	√2	2/√3	1	∞	-1	∞
sec	1	2/√3	√2	2	∞	-1	∞	1

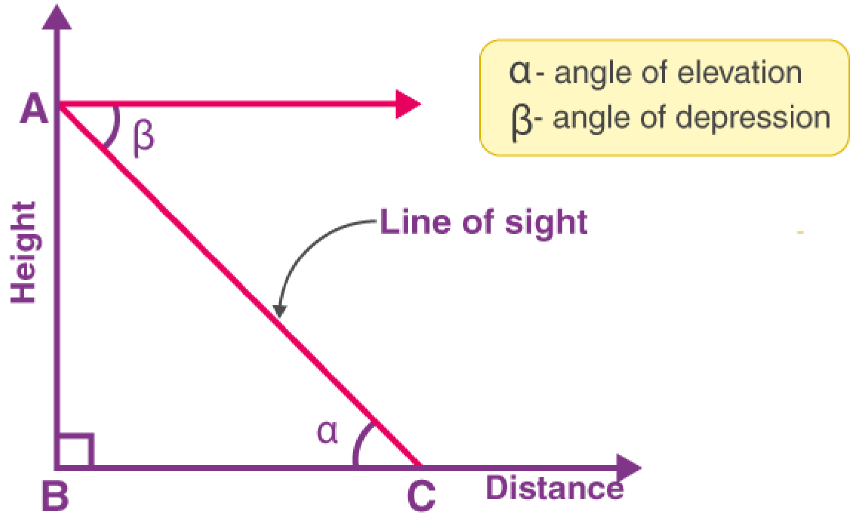
Step 1: Draw a **line diagram** corresponding to the problem.

Step 2: Mark all known heights, distances and angles and denote unknown lengths by variables.

Step 3: Use the values of various **trigonometric ratios** of the angles to obtain the unknown lengths from the known lengths.

Heights and Distance Summary

The below diagram gives the complete summary of "Heights and Distances".



From the given diagram, if "C" is the point of observation,

- AC is the line of sight
- BC is the distance between the observer and the object.
- AB is the height of the object
- α is the angle of elevation
- β is the angle of depression.

Solved Examples

Example 1: A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

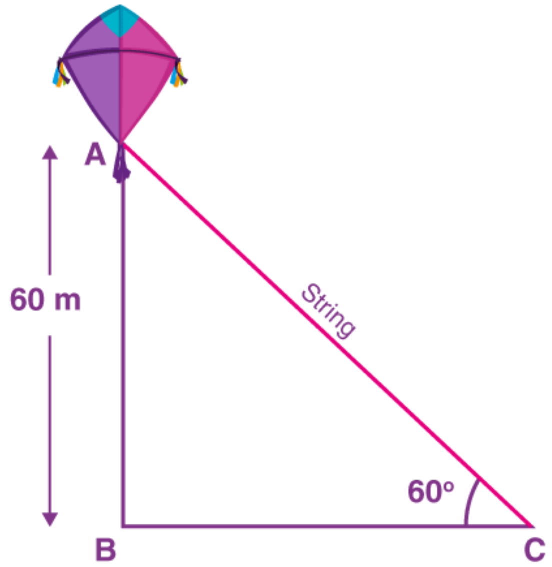
Solution:

Let A be the position of a kite at a height of 60 m above the ground.

Thus, AB = 60 m

Also, AC is the length of the string.

Angle of inclination = ∠ACB = 60



In right triangle ABC,

sin 60° = AB/AC

$\sqrt{3}/2 = 60/AC$

$AC = (60 \times 2)\sqrt{}/3$

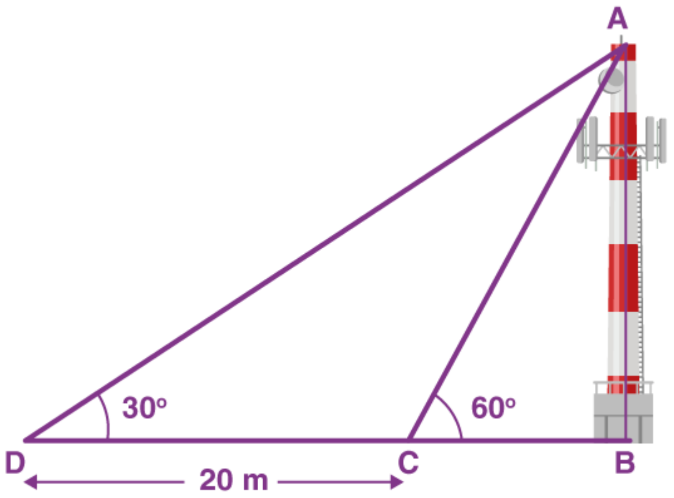
$= (120 \times \sqrt{3})/(\sqrt{3} \times \sqrt{3})$

$= (120\sqrt{3})/3$

$= 40\sqrt{3}$

Therefore, the length of the string is $40\sqrt{3}$ m.

Example 2: A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° as shown in the figure. Find the height of the tower and the width of the canal.



Solution:

Given,

AB is the height of the tower.

DC = 20 m (given)

In right ΔABD,

tan 30° = AB/BD

$1/\sqrt{3} = AB/(20 + BC)$

$AB = (20 + BC)/\sqrt{3}$….(i)

In right ΔABC,

tan 60° = AB/BC

$\sqrt{3} = AB/BC$

$AB = \sqrt{3}\ BC$….(ii)

From (i) and (ii),

$\sqrt{3}\ BC = (20 + BC)/\sqrt{3}$

3 BC = 20 + BC

2 BC = 20

BC = 10

Substituting the value of BC in equation (ii),

$AB = (20 + 10)/\sqrt{3} = 30/\sqrt{3} = 10\sqrt{3}$

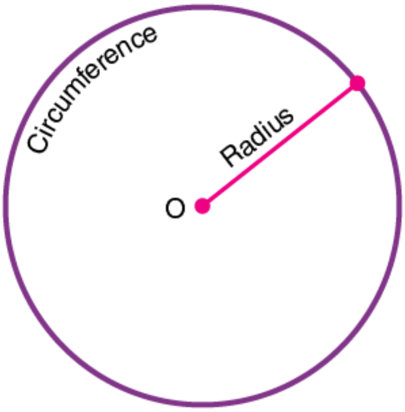
Therefore, the height of the tower is $10\sqrt{3}$ m, and the width of the canal is 10 m.

CH.9 SOME APPLICATIONS OF TRIGONOMETRY
NCERT C.W – EXE 9.1: Q.1,5,7,11,13,14&15   EXE 9.1 – Q.2,3,4,6,8,9,10,12
NCERT H.W- EXE 8.1: 4, 6, 7   EXE 8.2 – 2, 5
NCERT EXEMPLAR C.W- EXE 8.4 Q.8,15
NCERT EXEMPLAR H.W- EXE 8.4 Q.3,7,13
WORKBOOK C.W - CASE STUDY(Q.38)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.10 CIRCLES

Introduction to Circles

As we know that a circle is a closed two-dimensional geometrical figure, such that all points on the surface of a circle are equidistant from the point called the "centre". The distance from the centre to any point on the surface of a circle is called "Radius".



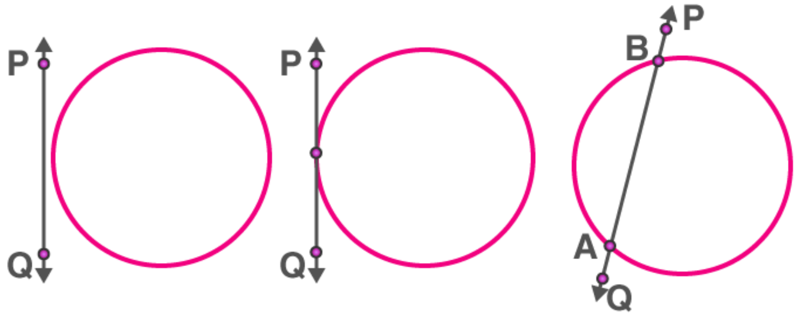
Circle and Line in a Plane

For a circle and a line on a plane, there can be **three** possibilities.

i) they can be **non-intersecting**

ii) they can have **a single common point:** in this case, the line touches the circle.

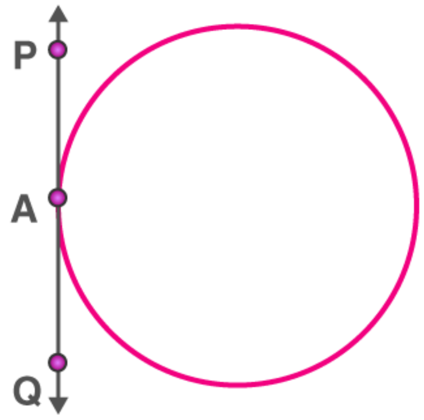
ii) they can have **two common points:** in this case, the line cuts the circle.



(i) Non-intersecting (ii) Touching (iii) Intersecting

Tangent

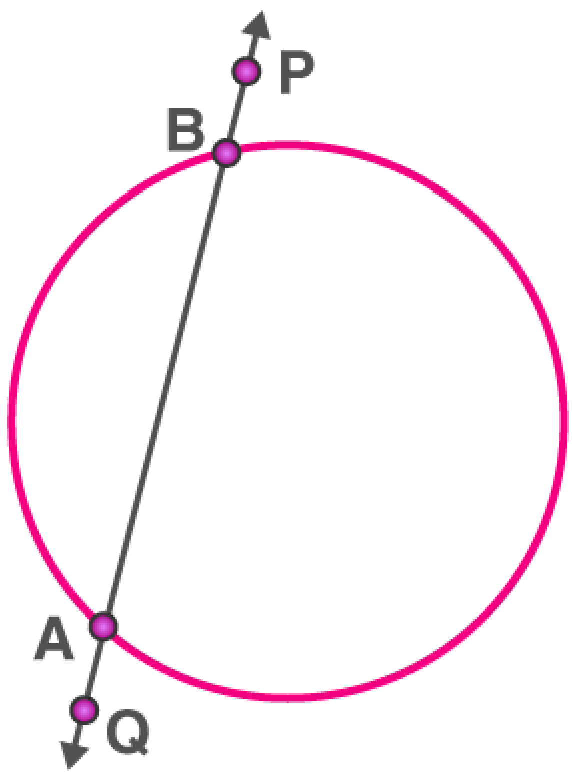
A **tangent to a circle** is a line that touches the circle at exactly one point. For every point on the circle, there is a unique tangent passing through it.



Tangent

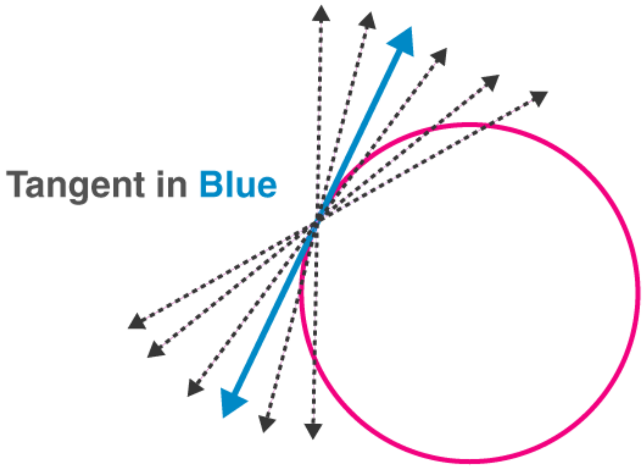
Secant

A **secant to a circle** is a line that has two points in common with the circle. It cuts the circle at two points, forming a chord of the circle.



Secant

Tangent as a Special Case of Secant

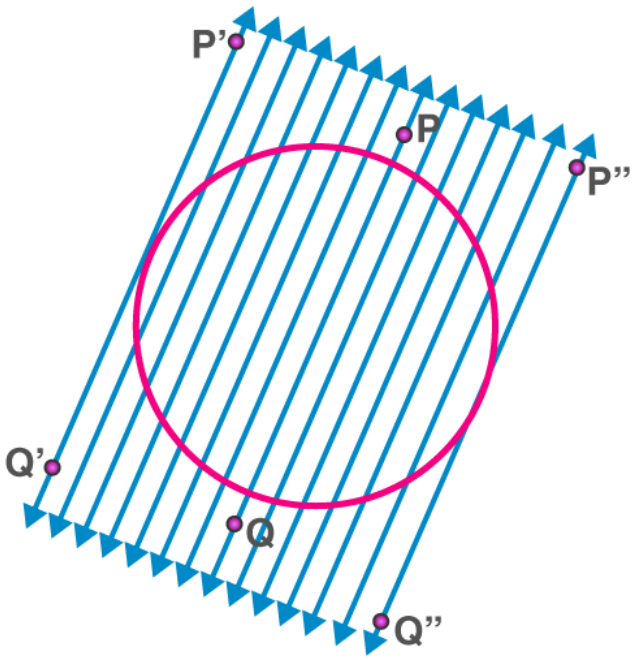


Tangent as a special case of Secant

The tangent to a circle can be seen as a special case of the secant when the two endpoints of its corresponding chord coincide.

Two Parallel Tangents at most for a Given Secant

For every given **secant** of a circle, there are **exactly two tangents which are parallel** to it and touches the circle at two **diametrically opposite points.**



Parallel tangents

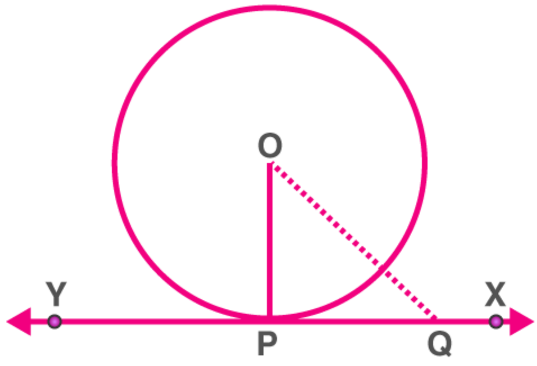
From the given diagram, we can observe the following points:

- PQ is the secant of a circle.
- P'Q' & P"Q" are two tangents which are parallel to PQ.

Theorems

Tangent Perpendicular to the Radius at the Point of Contact

Theorem: The theorem states that "the **tangent** to the circle at any point is the **perpendicular to the radius** of the circle that passes through the point of contact".



Tangent and radius

Here, O is the centre and OP⊥XY.

Theorem Proof:

Assume a circle with centre “O” and XY being the tangent to the circle at point “P”. Now, we have to prove that OP is perpendicular to the tangent XY.

Now, consider a point Q on the tangent line XY, other than P. Join the points OQ as shown in the figure.

Here, point Q should lie outside the circle. Because if the point Q lies inside the circle, XY will not be a tangent to the circle. It means that XY will become a secant of a circle.

So, OQ should be greater than the radius of the circle OP.

It means that

OQ > OP

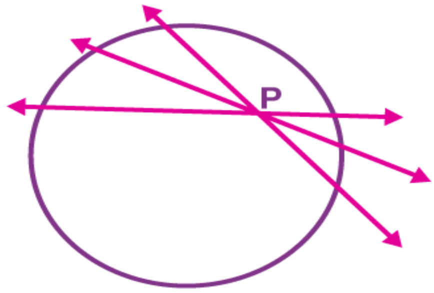
As, this condition is obeyed for all points on line XY except P, OP should be the shortest of all distances from the centre of the circle “O” to the points of line XY.

Thus, we can conclude that OP is perpendicular to XY.

Hence, the theorem is proved.

The Number of Tangents Drawn from a Given Point

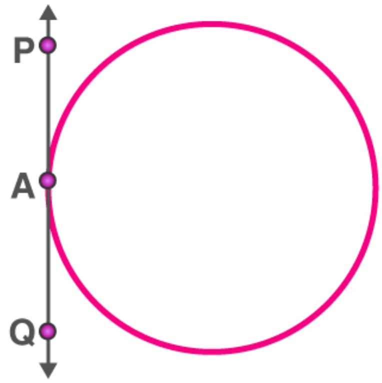
i) If the point is in an **interior region of the circle**, any line through that point will be a secant. So, **no tangent** can be drawn to a circle which passes through a point that lies inside it.



No tangent can be drawn to a circle from a point inside it

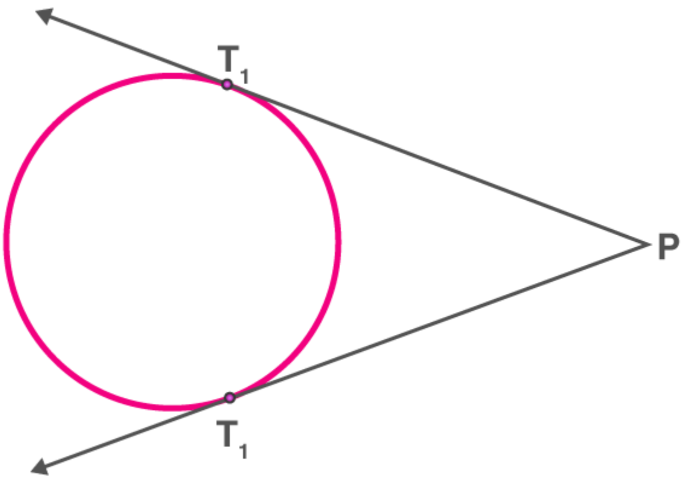
AB is a secant drawn through the point S

ii) When a point of tangency lies on the circle, there is **exactly one tangent** to a circle that passes through it.



A tangent passing through a point lying on the circle

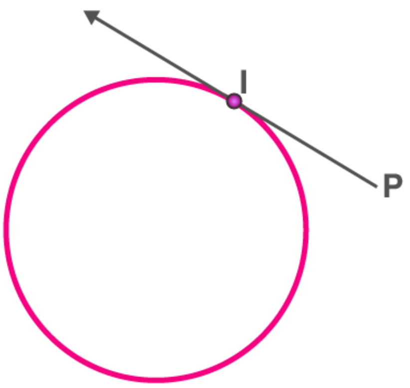
iii) When the point lies outside of the circle, there are **accurately two tangents** to a circle through it



Tangents to a circle from an external point

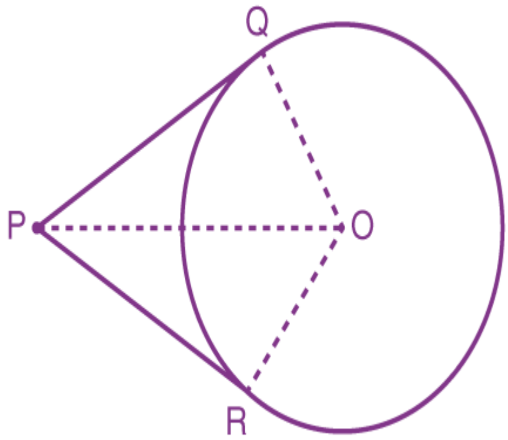
Length of a Tangent

The length of the tangent from the point (Say P) to the circle is defined as the segment of the tangent from the external point **P** to the point of tangency **I** with the circle. In this case, PI is the tangent length.



Lengths of Tangents Drawn from an External Point

Theorem: "The lengths of tangents drawn from an external point to a circle are equal."



Tangents to a circle from an external point

Assume that we are provided with a circle whose centre is "O" and P is the point that lies outside the circle.

The two tangents formed here are PQ ad PR on the circle from the point P as shown in the figure.

Now, we have to prove PQ = PR.

To prove this, now we have to join OP, OQ, and OR.

According to the theorem "The tangent to the circle at any point is perpendicular to the radius of the circle that passes through the point of contact", the angles formed between the tangents and radii are right angles.

So, two right angles are formed, such as ∠OQP and ∠ORP.

Thus, from the radii of the same circle, we can write OQ = OR.

Common side: OP = OP.

Hence, using the RHS congruence rule,

Δ OQP $\cong$ Δ ORP

Thus, it gives PQ = PR [Using CPCT].

Hence the theorem is proved.

Thus, the two important theorems in Class 10 Maths Chapter 10 Circles are:
Theorem 10.1: The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Theorem 10.2: The lengths of tangents drawn from an external point to a circle are equal.
Interesting facts about Circles and its properties are listed below:

- In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.
- The tangents drawn at the ends of a diameter of a circle are parallel.
- The perpendicular at the point of contact to the tangent to a circle passes through the centre.
- The angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.
- The parallelogram circumscribing a circle is a rhombus.
- The opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

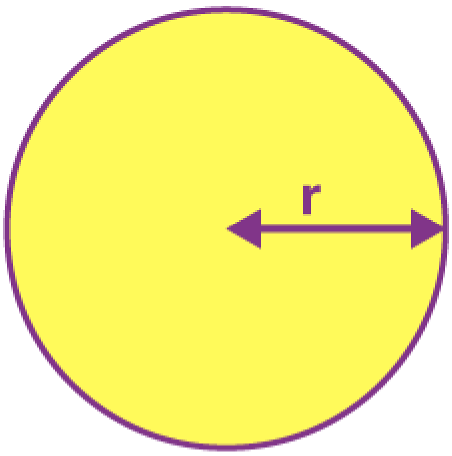
CH.10 CIRCLES
NCERT C.W – EXE 10.1: Q.1,3   EXE 10.2 – Q.1,3.7,8,9,11,12,13
NCERT H.W- EXE 10.1 – Q.2,4   EXE 10.2 – Q.2,4,5,6,10
NCERT EXEMPLAR C.W- EXE-9.4 Q.4,7,11,12
NCERT EXEMPLAR H.W- EXE-9.4 Q.1,2,3
WORKBOOK C.W - CASE STUDY(Q.39)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.11 AREAS RELATED TO CIRCLES

Introduction to Areas Related to Circles

In the chapter "Areas Related to Circles" for Class 10, we will learn to find the areas of circles, areas of segments and sectors of circles, circumference, length of the arc of a sector, etc. A circle is a two-dimensional figure. It is a curved shape that has all its points at an equal distance from the center. Let us learn now how to find areas related to the circle in this article.

Area of a Circle



The area of a circle is πr^2, where π=22/7 or ≈ 3.14 (can be used interchangeably for problem-solving purposes) and r is the radius of the circle.

π is the ratio of the circumference of a circle to its diameter.

Example: Find the area of a circle with radius = 7cm.

Solution: Given, radius of circle = 7cm

By the formula we know;

Area of circle = πr^2

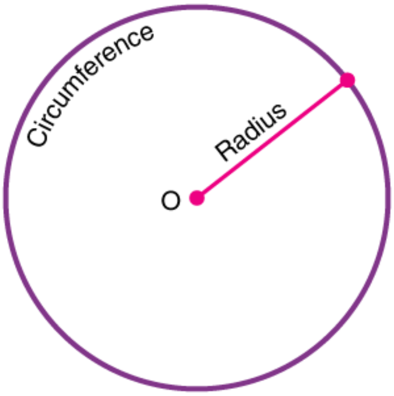
= $\pi(7)^2$

= $(22/7)\ (7)^2$

= 154 sq.cm.

Circumference of a Circle

The circumference of a circle is the distance covered by going around its boundary once.

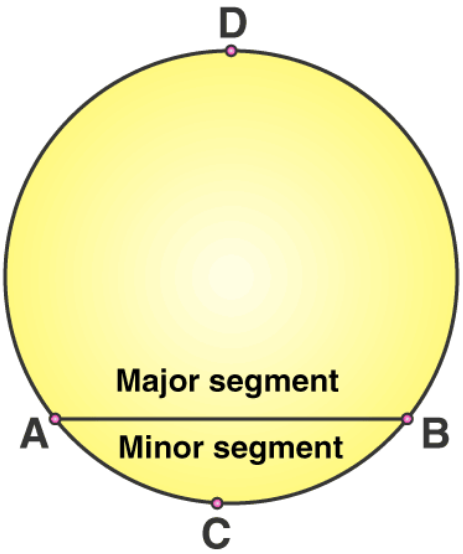


The perimeter of a circle has a special name: Circumference, which is π times the diameter which is given by the formula;
Circumference of a circle = $2\pi r$.

Example: The circumference of a circle whose radius is 21cm is given by;
$C = 2\pi r$
= 2 (22/7) (21)
= 132 cm

Segment of a Circle

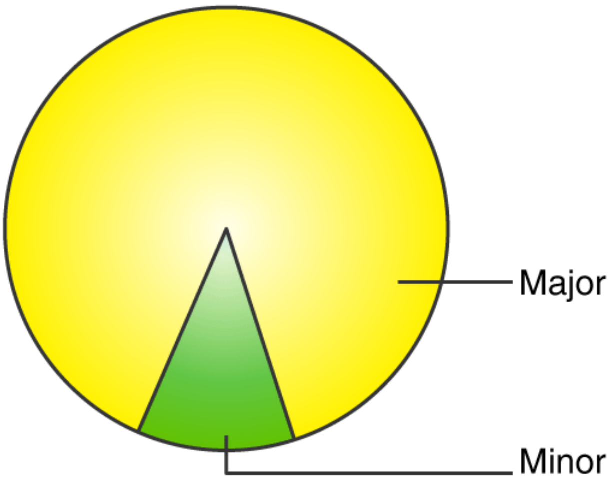
A circular segment is a region of a circle that is "cut off" from the rest of the circle by a secant or a chord.



Sector of a Circle

A circle sector/ sector of a circle is defined as the region of a circle enclosed by an arc and two radii. The smaller area is called the minor sector, and the larger area is called the major sector.

SECTOR OF A CIRCLE



Angle of a Sector

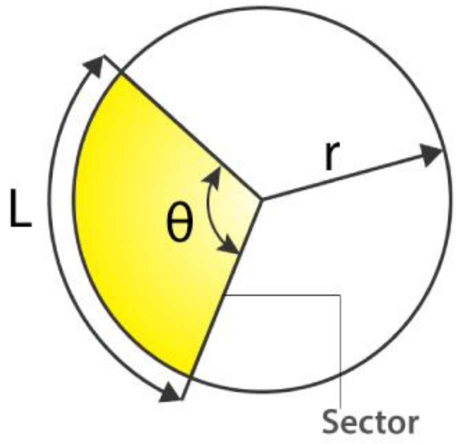
The angle of a sector is the angle that is enclosed between the two radii of the sector.

Area of a Sector of a Circle

The area of a sector is given by

$(\theta/360°)\times\pi r^2$

where $\angle\theta$ is the angle of this sector(minor sector in the following case) and r is its radius



Area of a sector

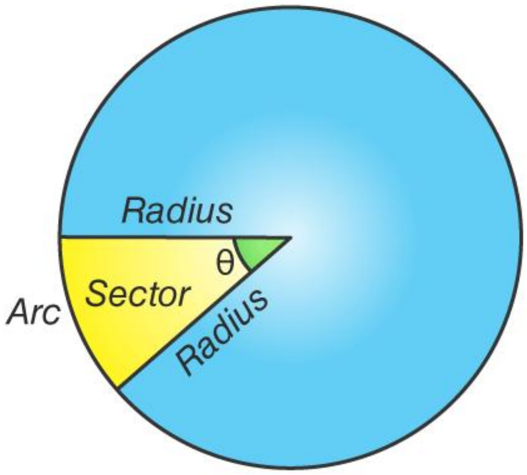
Example: Suppose the sector of a circle is 45° and radius is 4 cm, then the area of the sector will be:

Area = $(\theta/360°) \times \pi r^2$

= (45°/360°) × (22/7) × 4 × 4

= 44/7 sq. cm

Length of an Arc of a Sector



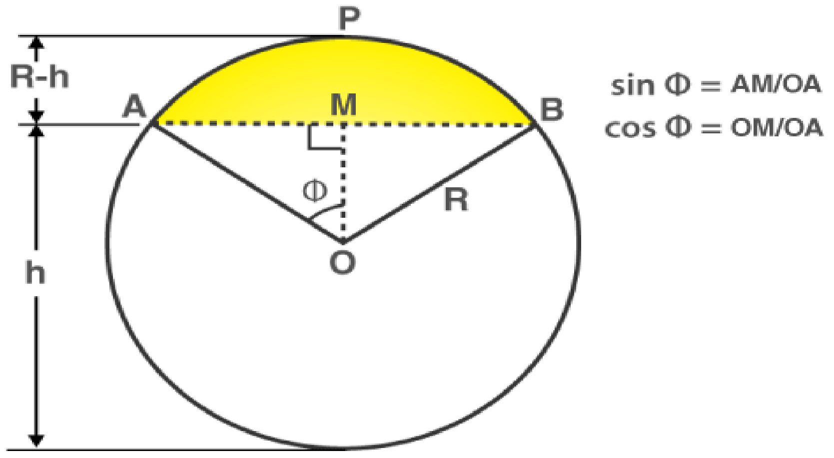
The length of the arc of a sector can be found by using the expression for the circumference of a circle and the angle of the sector, using the following formula:

$L= (\theta/360°)\times 2\pi r$
Where θ is the angle of sector and r is the radius of the circle.

Area of a Triangle

The area of a triangle is,
Area=(1/2)×base×height
If the triangle is an equilateral then,
Area=$(\sqrt{3}/4)\times a^2$ where "a" is the side length of the triangle.

Area of a Segment of a Circle



Area of segment APB (highlighted in yellow)
= (Area of sector OAPB) – (Area of triangle AOB)

$=[(\emptyset/360°)\times\pi r^2] - [(1/2)\times AB\times OM]$
[To find the area of triangle AOB, use trigonometric ratios to find OM (height) and AB (base)]

Also, the area of segment APB can be calculated directly if the angle of the sector is known using the following formula.

$=[(\theta/360°)\times\pi r^2] - [r^2\times\sin\theta/2 \times \cos\theta/2]$
Where θ is the angle of the sector and r is the radius of the circle.

Formulas List

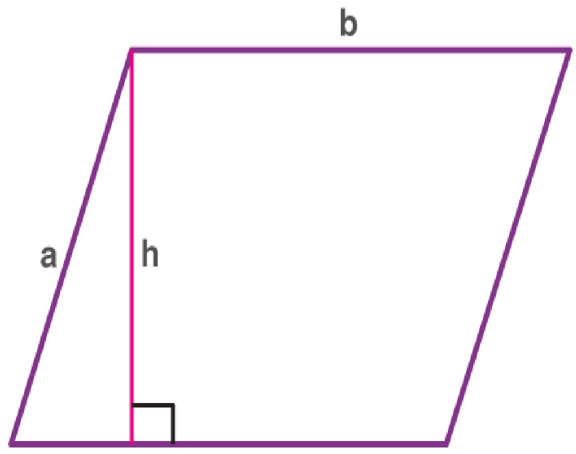
All these formulas are tabulated as given below for quick revision.

Parameters of Circles	**Formulas**
Area of the sector of angle θ	$(\theta/360°) \times \pi r^2$
Length of an arc of a sector of angle θ	$(\theta/360°) \times 2\pi r$
Area of major sector	$\pi r^2 – (\theta/360°) \times \pi r^2$
Area of a segment of a circle	Area of the corresponding sector – Area of the corresponding triangle
Area of the major segment	πr^2 – Area of segment (minor segment)

Visualizations

Areas of Different Plane Figures

– Area of a square (side l) $=l^2$
– Area of a rectangle $=l\times b$, where l and b are the length and breadth of the rectangle
– Area of a parallelogram $=b\times h$, where “b” is the base and “h” is the perpendicular height.



parallelogram

Area of a trapezium $=[(a+b)\times h]/2$,

where

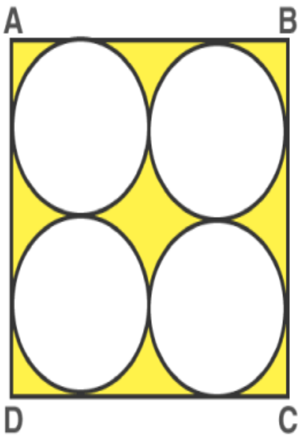
a & b are the length of the parallel sides

h is the trapezium height

Area of a rhombus $=pq/2$, where p & q are the diagonals.

Areas of Combination of Plane figures

For example: Find the area of the shaded part in the following figure: Given the ABCD is a square of side 28 cm and has four equal circles enclosed within.



Area of the shaded region

Looking at the figure we can visualize that the required shaded area = A(square ABCD) − 4 ×A(Circle).

Also, the diameter of each circle is 14 cm.

$=(l^2)-4\times(\pi r^2)$

$=(28^2)-[4\times(\pi\times49)]$

$=784-[4\times22/7\times49]$

$=784-616$

$=168cm^2$

CH.11 AREAS RELATED TO CIRCLES
NCERT C.W – EXE 11.1: Q.1,5,7,9,11,12,14
NCERT H.W- EXE 11.1: Q.2,3,4,6,8,10,13
NCERT EXEMPLAR C.W- EXE-11.4 Q.6,10,15,17
NCERT EXEMPLAR H.W- EXE-11.3 Q.9,11,12,13,15
WORKBOOK C.W - CASE STUDY(Q.38)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

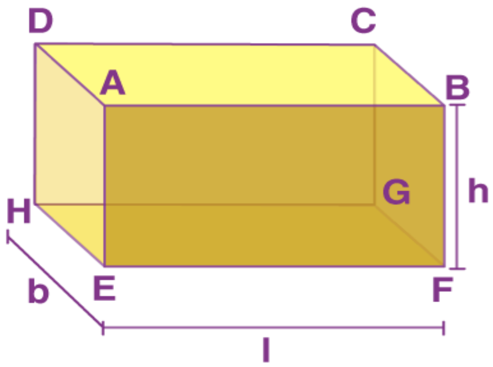
CH.12 SURFACE AREAS & VOLUMES

Surface Area and Volume of Cuboid

A cuboid is a region covered by its six rectangular faces. The surface area of a cuboid is equal to the sum of the areas of its six rectangular faces.

Surface Area of the Cuboid

Consider a cuboid whose dimensions are l × b × h, respectively.



Cuboid with length *l*, breadth b and height h

The total surface area of the cuboid (TSA) = Sum of the areas of all its six faces

TSA (cuboid) = $2(l \times b) + 2(b \times h) + 2(l \times h) = 2(lb + bh + lh)$

Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.

The lateral surface area of the cuboid = Area of face AEHD + Area of face BFGC + Area of face ABFE + Area of face DHGC

LSA (cuboid) = $2(b \times h) + 2(l \times h) = 2h(l + b)$

Length of diagonal of a cuboid =$\sqrt{(l^2 + b^2 + h^2)}$

Volume of a Cuboid

The volume of a cuboid is the space occupied within its six rectangular faces.

Volume of a cuboid = (base area) × height = (lb)h = lbh

Surface Area and Volume of Cube

A cube is a three-dimensional solid that has six square faces, twelve edges and eight vertices.

Surface Area of Cube

As we know, one of the important properties of a cube is length = breadth = height.

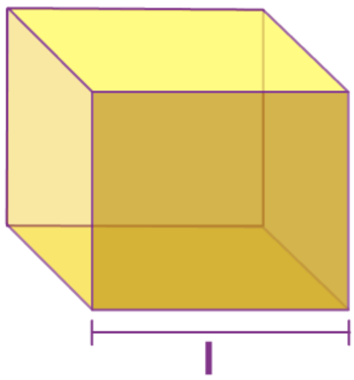
If we assume that the length of the cube is "l", and hence we get

l = breadth = height

So, obviously, here we get,

Breadth = l

Height = l



Cube with length *l*

The total surface area of the cube (TSA) = Sum of the areas of all its six faces.

In case of all faces has an equal area, TSA of Cube = 6 × area of Square = $6l^2$ square units.

Similarly, the Lateral surface area of cube = $2(l \times l + l \times l) = 4l^2$
Note: Diagonal of a cube $=\sqrt{3}l$

Volume of a Cube

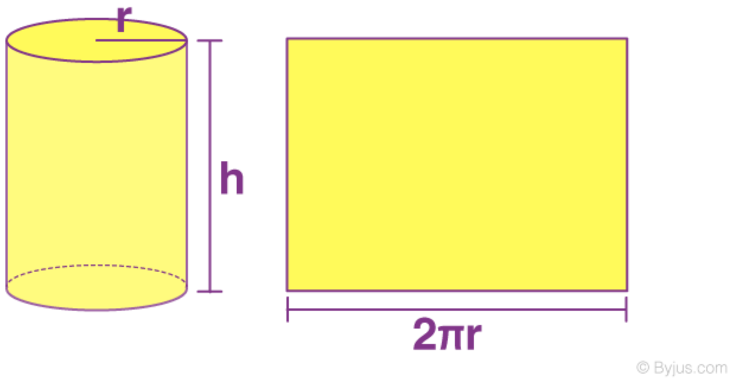
Volume of a cube = base area × height
Since all dimensions of a cube are identical, volume = l^3
Where *l* is the length of the edge of the cube.

Surface Area and Volume of Cylinder

A cylinder is a solid shape that has two circular bases connected with each other through a lateral surface. Thus, there are three faces, two circular and one lateral, of a cylinder. Based on these dimensions, we can find the surface area and volume of a cylinder.

Surface Area of Cylinder

Take a cylinder of base radius *r* and height *h* units. The curved surface of this cylinder, if opened along the diameter ($d = 2r$) of the circular base can be transformed into a rectangle of length $2\pi r$ and height *h* units. Thus,



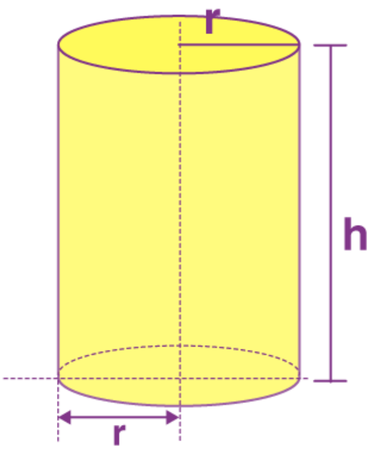
Transformation of a Cylinder into a rectangle.

CSA of a cylinder of base radius *r* and height h = $2\pi \times r \times h$
TSA of a cylinder of base radius *r* and height h = $2\pi \times r \times h$ + area of two circular bases

$=2\pi \times r \times h + 2\pi r^2$
$=2\pi r(h + r)$

Volume of a Cylinder

Volume of a cylinder = Base area × height = $(\pi r^2) \times h = \pi r^2 h$



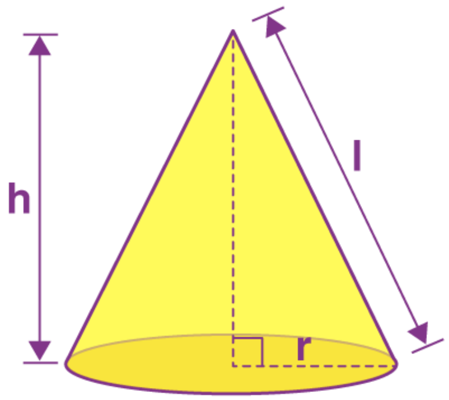
Cylinder with height h and base radius r

Surface Area and Volume of Right Circular Cone

A cone is a 3d shape that has one circular base and narrows smoothly from the base to a point called the vertex.

Surface Area of Cone

Consider a right circular cone with slant length *l*, radius *r* and height *h*.



Cone with base radius r and height h

CSA of right circular cone = πrl
TSA = CSA + area of base = $\pi rl + \pi r^2 = \pi r(l + r)$

Volume of a Right Circular Cone

The volume of a Right circular cone is 1/3 times that of a cylinder of the same height and base.
In other words, 3 cones make a cylinder of the same height and base.
The volume of a Right circular cone $=(1/3)\pi r^2 h$
Where '*r*' is the radius of the base and '*h*' is the height of the cone.

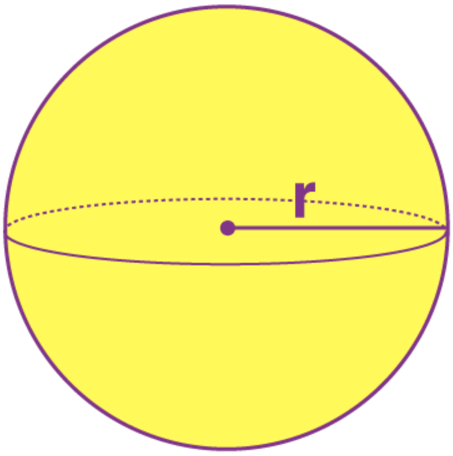
Surface Area and Volume of Sphere

A sphere is a solid that is round in shape, and the points on its surface are equidistant from the center.

Surface Area of Sphere

For a sphere of radius *r*

Curved Surface Area (CSA) = Total Surface Area (TSA) = $4\pi r^2$



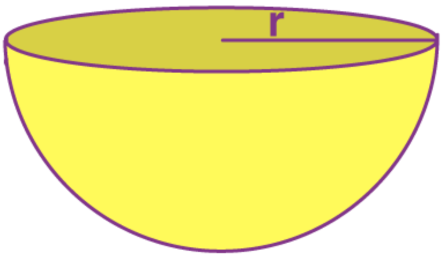
Sphere with radius r

Volume of Sphere

The volume of a sphere of radius $r = (4/3)\pi r^3$

Surface Area and Volume of Hemisphere

A hemisphere is a shape that is half of the sphere and has one flat surface. The other side of the hemisphere is shaped as a circular bowl. See the figure below.



Hemisphere of radius r

Surface Area of Hemisphere

We know that the CSA of a sphere $= 4\pi r^2$.

A hemisphere is half of a sphere.
∴ CSA of a hemisphere of radius $r = 2\pi r^2$

Total Surface Area = curved surface area + area of the base circle
⇒TSA = $3\pi r^2$

Volume of Hemisphere

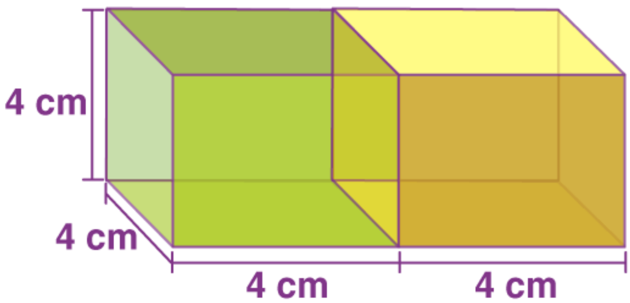
The volume (V) of a hemisphere will be half of that of a sphere.
∴ The volume of the hemisphere of radius $r = (2/3)\pi r^3$

Surface Area and Volume of Combination of Solids

The combination of solids explains the shapes formed when two different solids are combined together. Thus, the surface area and volume for such shapes will vary from the other basic solids.

Surface Area of Combined Figures

Areas of complex figures can be broken down and analysed as simpler known shapes. By finding the areas of these known shapes, we can find out the required area of the unknown figure.
Example: 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.
Length of each cube = $64^{(1/3)}$ = 4cm
Since these cubes are joined adjacently, they form a cuboid whose length l = 8 cm. But height and breadth will remain the same = 4 cm.



Combination of 2 equal cubes

∴ The new surface area, TSA = 2(lb + bh + lh)

TSA = 2 (8 x 4 + 4 x 4 + 8 x 4)

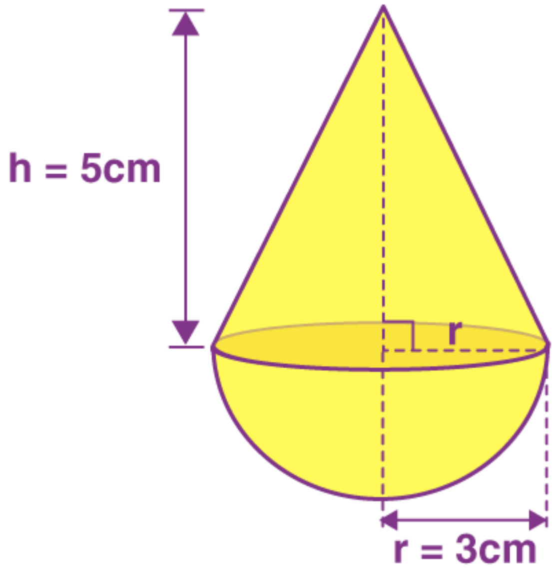
= 2(32 + 16 + 32)

= 2 (80)

TSA = 160 cm^2

Volume of Combined Solids

The volume of complex objects can be simplified by visualising them as a combination of shapes of known solids.
Example: A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 3 cm and the height of the cone is equal to 5 cm.
This can be visualised as follows :



Volume of combined solids

V(solid) = V(Cone) + V(hemisphere)

V(solid) = $(1/3)\pi r^2 h + (2/3)\pi r^3$

V(solid) = $(1/3)\pi(9)(5) + (2/3)\pi(27)$

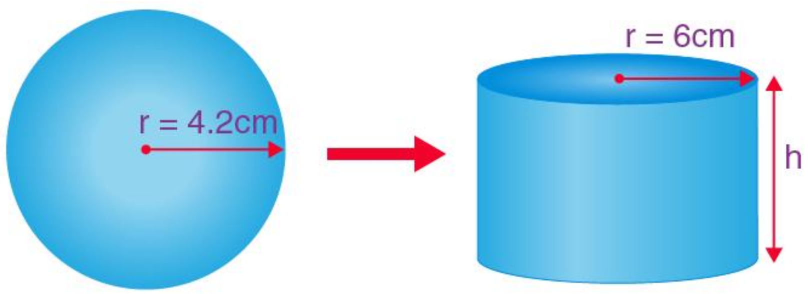
V(solid) = 33π cm^3

Conversion of Solid from One Shape to Another

When a solid is converted into another solid of a different shape (by melting or casting), the volume remains constant.

Example:

A metallic sphere with a radius of 4.2 cm is melted and recast in the shape of a cylinder with a radius of 6 cm. Determine the cylinder's height.



Solution:

Given that, the sphere is melted and recast into a cylinder.

So, we can write,

Sphere's volume = Cylinder's volume.

As we know, the volume of a sphere = $(4/3)\pi r^3$ cubic units.

And the volume of a cylinder = $\pi r^2 h$ cubic units.

So, we can write:

$(4/3)\pi r^3 = \pi r^2 h$ …(1)

Given that, the radius of the metallic sphere = 4.2 cm.

Radius of the cylinder = 6 cm.

Now, substitute the values in (1), we get

$(4/3)\pi(4.2)^3 = \pi(6)^2 h$

$(4/3)(4.2)^3 = 36h$

Hence, $h = [(4/3)(4.2)^3]/36$

$h = [4 \times 4.2 \times 4.2 \times 4.2]/[3 \times 36]$

By simplifying the above expression, we get

h = 2.744 cm

Hence, the height of the cylinder is 2.744 cm.

Surface Areas and Volumes Formulas – Summary

The following table presents a brief summary of different solid shapes with their surface area and volume formulas:

Shape	**Parameters**	**Surface Area (Square units)**	**Volume (Cubic units)**
Cuboid	Length = l Breadth = b Height = h	TSA = 2(lb + bh + lh) LSA = 2h(l + b)	$V = l \times b \times h$
Cube	Length = Breadth = Height = l	TSA = $6l^2$ LSA = $4l^2$	$V = l^3$
Cylinder	Radius = r Height = h	CSA = $2\pi \times r \times h$ TSA = $2\pi r(h + r)$	$V = \pi r^2 h$
Cone	Radius = r Height = h Slant Height = l	CSA = πrl TSA = $\pi r(l + r)$	$V = (1/3)\pi r^2 h$
Sphere	Radius = r	CSA = TSA = $4\pi r^2$	$V = (4/3)\pi r^3$
Hemisphere	Radius = r	CSA = $2\pi r^2$ TSA = $3\pi r^2$	$V = (2/3)\pi r^3$
Frustum	Radius of top circular part = r_1 Radius of bottom circular part = r_2	CSA = $\pi(r_1+r_2)l$ TSA = $\pi(r_1 + r_2)l + \pi(r_1^2 + r_2^2)$	$V = (1/3)\pi h(r_1^2 + r_2^2 + r_1 r_2)$

	Height = h Slant height = l		

CH.12 SURFACE AREAS & VOLUMES
NCERT C.W – EXE 12.1:Q.1,3,4,6,7&9 EXE 12.2 – Q.1,3,4,7,8
NCERT H.W- EXE 12.1: Q.2,5,8 EXE 12.2 –Q.2,5,6
NCERT EXEMPLAR C.W- EXE-13.4 Q.1,3,7,8
NCERT EXEMPLAR H.W- EXE-13.4 Q.4,5
WORKBOOK C.W - CASE STUDY(Q.38)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.13 STATISTICS

Introduction to Statistics

Ungrouped Data

Ungrouped data is data in its original or raw form. The observations are not classified into groups.

For example, the ages of everyone present in a classroom of kindergarten kids with the teacher are as follows:

3, 3, 4, 3, 5, 4, 3, 3, 4, 3, 3, 3, 3, 4, 3, 27.

This data shows that there is one grown-up person present in this class, and that is the teacher. Ungrouped data is easy to work with when the data set is small.

Grouped Data

In grouped data, observations are organized in groups.

For example, a class of students got different marks in a school exam. The data is tabulated as follows:

Mark interval	0-20	21-40	41-60	61-80	81-100
No. of Students	13	9	36	32	10

This shows how many students got the particular mark range. Grouped data is easier to work with when a large amount of data is present.

Frequency

Frequency is the number of times a particular observation occurs in data.

For example, if four students have scored marks between 90 and 100, then the marks scored between 90 and 100 have a frequency of 4.

Class Interval

Data can be grouped into class intervals such that all observations in that range belong to that class.

Class width = upper class limit – lower class limit

Example: Consider a class interval 31 – 40.

Here, the lower class limit is 31, and the upper class limit is 40.

Hence, the size/width of the class interval 31 – 40 is calculated as follows:

Class interval size = Upper class limit – Lower class limit

Class interval size = 40 – 31 = 9

Therefore, the size of the class interval is 9.

Mean

Finding the mean for grouped data when class intervals are not given

If x_1, x_2,. . ., x_n are the given observations and their respective frequencies are f_1, f_2, . . ., f_n respectively, then the mean of the grouped data (without class interval) is given as:

Mean= $\bar{x} = \sum x_i f_i / \sum fi$

where f_i is the frequency of i_{th} observation x_i.

For example, the marks scored by the 30 students of class 10 are given below:

Marks obtained (x_i)	10	20	36	40	50	56	60	70	72	80	88	92	95
No. of students	1	1	3	4	3	2	4	4	1	1	2	3	1

We know the formula to find the mean of the grouped data is

$\bar{x} = \bar{=} \sum x_i f_i / \sum fi$

Marks scored (x_i)	**No. of students (f_i)**	**$x_i f_i$**
10	1	10
20	1	20
36	3	108
40	4	160
50	3	150

56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	95
Sum	**$\Sigma f_i = 30$**	**$\Sigma x_i f_i = 1779$**

Now, substitute the obtained values in the formula. We get

Mean = 1779/30

Mean = 59.3

Therefore, the mean of the marks scored by the students is 59.3

Finding the mean for grouped data when class intervals are given

If $x_1, x_2, \ldots, x_n$ is the set of observations and their frequencies are $f_1, f_2, \ldots, f_n$ respectively, then the mean of the grouped data (with class interval) is given as follows:

$\text{Mean} = \bar{x} = \sum x_i f_i / \sum f_i$

Where f_i is the frequency of i_{th} class whose class mark is x_i

And, "i" varies from 1 to n.

Note: Class mark =(Upper Class Limit+ Lower Class Limit)/2

Example:

Find the mean of the following grouped data:

Class interval	10 – 25	25 – 40	40 – 55	55 – 70	70 – 85	85 – 100
No. of students	2	3	7	6	6	6

To find the mean for the grouped data, first we have to find the class mark:

The formula to find the class mark is:

Class mark = (Upper class limit + Lower Class limit)/2

Class Interval	**No.of Students (f_i)**	**Class Mark (x_i)**	**$f_i x_i$**
10 – 25	2	17.5	35.0
25 – 40	3	32.5	97.5
40 – 55	7	47.5	332.5
55 – 70	6	62.5	375.0
70 – 85	6	77.5	465.0
85 – 100	6	92.5	555.0
Sum	**$\Sigma f_i = 30$**		**$\Sigma f_i x_i = 1860.0$**

Now, substitute the obtained values in the mean formula, we get

$\bar{x} = \sum x_i f_i / \sum fi$

Mean = 1860 / 30

Mean = 62

Therefore, the mean of the given data is 62.

Direct Method to Find Mean

Step 1: Classify the **data into intervals** and find the corresponding **frequency of each class**.

Step 2: Find the **class mark** by taking the **midpoint of the upper and lower class limits.**

Step 3: Tabulate the product of the class mark and its corresponding frequency for each class. Calculate their sum ($\sum x_i f_i$).

Step 4: Divide the above sum by the sum of frequencies ($\sum f_i$) to get the mean.

The formula to find the mean using the direct method is:

$\bar{x} = \sum x_i f_i / \sum fi$

[Note: Find the class mark (x_i) from the class interval and multiply x_i by the f_i (frequency)]

Assumed Mean Method to Find Mean

Step 1: Classify the data into intervals and find the corresponding frequency of each class.

Step 2: Find the class mark by taking the midpoint of the upper and lower class limits.

Step 3: Take one of the x_i’s (usually one in the middle) as the assumed mean and denote it by ′a′.

Step 4: Find the deviation of ′a′ from each of the x′is
$d_i = x_i - a$

Step 5: Find the mean of the deviations

$\bar{d} = \sum f_i d_i / \sum f_i$

Step 6: Calculate the mean as

$\bar{x} = a + \sum f_i d_i / \sum f_i$

For example, let us consider the same example as provided above.

In this method, first, we have to choose the assumed mean (a), which lies in the centre of $x_1, x_2, \ldots x_n$. Here, we choose a = 47.5.

Secondly, we have to find the difference(di), which is obtained using the formula,

$d_i = x_i - a$

Now, let us see how to find the mean for the given data using the assumed mean method.

Class Interval	**No.of Students (f_i)**	**Class Mark (x_i)**	**$d_i = x_i - a$**	**$f_i d_i$**
10 – 25	2	17.5	-30	-60
25 – 40	3	32.5	-15	-45
40 – 55	7	47.5	0	0
55 – 70	6	62.5	15	90
70 – 85	6	77.5	30	180
85 – 100	6	92.5	45	270
Sum	**$\Sigma f_i = 30$**			**$\Sigma f_i d_i = 435$**

Now, using the assumed mean method formula, we can get

Mean = 47.5 + (435/30)

Mean = 47.5 + 14.5

Mean = 62

Thus, the mean of the given data using the assumed mean method is 62.

Step-Deviation Method to Find Mean

Step 1: Classify the data into intervals and find the corresponding frequency of each class.

Step 2: Find the class mark by taking the midpoint of the upper and lower class limits.

Step 3: Take one of the x′is (usually one in the middle) as the assumed mean and denote it by ′a′.

Step 4: Find the deviation of a from each of the x′is
$d_i = x_i - a$

Step 5: Divide all deviations $-d_i$ by the class width (h) to get u′is.

ui=xi−ah

Step 6: Find the mean of u′is

u¯=∑fi ui /∑fi

Step 7: Calculate the mean as

x¯=a+ h×(∑fi ui/∑fi)

Considering the same grouped data with class interval, now let us discuss how to find the mean using the step deviation method.

Using the formula $u_i = (x_i – a)/h$

Here, a = 47.5, which is the assumed mean

Class size, h = 15.

Class Interval	**No.of Students (f_i)**	**Class Mark (x_i)**	**$d_i = x_i – a$**	**$u_i = (x_i – a)/h$**	**f_iu_i**
10 – 25	2	17.5	-30	-2	-4
25 – 40	3	32.5	-15	-1	-3
40 – 55	7	47.5	0	0	0
55 – 70	6	62.5	15	1	6
70 – 85	6	77.5	30	2	12
85 – 100	6	92.5	45	3	18
Sum	$\Sigma f_i = 30$				$\Sigma f_iu_i = 29$

Now, substituting the obtained values in the formula, we get

Mean = 47.5 + (15)(29/30)

Mean = 47.5 + (15)(0.967)

Mean = 47.5 + 14.5

Mean = 62

Important Relations Between Methods of Finding Mean

- All three methods of finding mean yield the same result.
- Step deviation method is easier to apply if all the deviations have a common factor.
- Assumed mean method and step deviation method are simplified versions of the direct method.

Median

Finding the median of grouped data when class intervals are not given

Step 1: Tabulate the observations and the corresponding frequency in ascending or descending order.

Step 2: Add the cumulative frequency column to the table by finding the cumulative frequency up to each observation.

Step 3: If the number of observations is odd, the median is the observation whose cumulative frequency is just greater than or equal to (n+1)/2

If the number of observations is even, the median is the average of observations whose cumulative frequency is just greater than or equal to n/2 and (n/2)+1.

The marks scored by 100 students out of 50 marks are given below. Find the median of the given data:

Marks scored	20	29	28	33	42	38	43	25
No.of students	6	28	24	15	2	4	1	20

Now, arrange the marks scored by students in ascending order.

Marks Scored	**No. of students**
20	6
25	20
28	24
29	28

33	15
38	4
42	2
43	1
Sum	**100**

Since the number of observations is 100, the average of the 50th and 51st observations is the median of the given data.

To find the value of the 50^{th} and 51^{st} observations, we have to construct the frequency table with a cumulative frequency column.

Marks Scored	**No. of students**	**Cumulative frequency**
20	6	6
25	20	26
28	24	50
29	28	78
33	15	93
38	4	97
42	2	99
43	1	100

From the above-given table, we can observe that,

50^{th} observation = 28 and hence 51^{st} observation = 29

Therefore, Median = (28 + 29)/2

Median = 57/2

Median = 28.5

Therefore, the median of the given data is 28.5.

Cumulative Frequency

Cumulative frequency is obtained by adding all the frequencies up to a certain point.

Finding the median for grouped data when class intervals are given

Step 1: find the cumulative frequency for all class intervals.
Step 2: the median class is the class whose cumulative frequency is greater than or nearest to n2, where n is the number of observations.
Step 3: Median = l + [(N/2 – cf)/f] × h

Where,
l = lower limit of median class,
n = number of observations,
cf = cumulative frequency of class preceding the median class,
f = frequency of median class,
h = class size (assuming class size to be equal).

CH.13 STATISTICS
NCERT C.W – EXE 13.1: Q.1,3,5,7,9 EXE 13.2 –Q.1,3,5 EXE 13.3 –Q.1,2,4,7
NCERT H.W- EXE 13.1: Q.2,4,6,8 EXE 13.2 – Q.2,4,6 EXE 13.3 –Q.2,3,5
NCERT EXEMPLAR C.W- EXE-13.3 Q.12,15,17
NCERT EXEMPLAR H.W- EXE-13.3 Q.6,10,15
WORKBOOK C.W - CASE STUDY(Q.38)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)

CH.14 PROBABILITY

What Is Probability?

The branch of mathematics that measures the uncertainty of the occurrence of an event using numbers is called probability. The chance that an event will or will not occur is expressed on a scale ranging from 0-1.
It can also be represented as a percentage, where 0% denotes an impossible event and 100 % implies a certain event.
Probability of an Event E is represented by P(E).
For example, the probability of getting a head when a coin is tossed is equal to 1/2. Similarly, the probability of getting a tail when a coin is tossed is also equal to 1/2.
Hence, the total probability will be:
P(E) = 1/2 + 1/2 = 1

Event and outcome

An **Outcome** is a result of a random experiment. For example, when we roll a dice getting six is an outcome.
An **Event** is a set of outcomes. For example, when we roll dice, the probability of getting a number less than five is an event.
Note: An event can have a single outcome.

Experimental Probability

Experimental probability can be applied to any event associated with an experiment that is repeated a large number of times.
A trial is when the experiment is performed once. It is also known as **empirical probability**.
Experimental or empirical probability: P(E) =Number of trials where the event occurred/Total Number of Trials
Example: In a day, a shopkeeper is able to sell 15 balls, out of which 6 were red balls. Find the probability of selling red balls on the next day of his sales.
Given, the total number of balls sold = 15
Number of red balls sold = 6
Probability of red balls = 6/15 = 2/5

Theoretical Probability

Theoretical Probability, P(E) = Number of Outcomes Favourable to E / Number of all possible outcomes of the experiment

Here we assume that the outcomes of the experiment are **equally likely**.
Example: Find the probability of picking up a red ball from a basket that contains 5 red and 7 blue balls.
Solution: Number of possible outcomes = Total number of balls = 5+7 = 12
Number of favourable outcomes = Number of red balls = 5
Hence,
Probability, P(red) = 5/12

Elementary Event

An event having only **one outcome** of the experiment is called an **elementary event**.
Example: Take the experiment of tossing a coin n number of times. One trial of this experiment has two possible outcomes: Heads(H) or Tails(T). So for an individual toss, it has only one outcome, i.e. Heads or Tails.

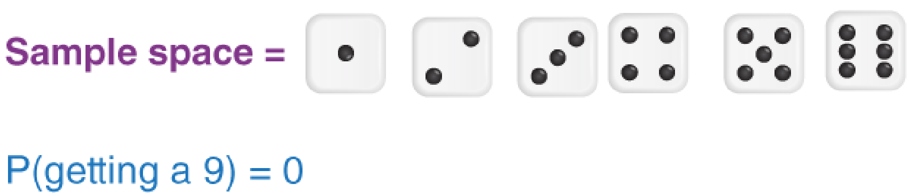
Sum of Probabilities

The **sum** of the probabilities of all the **elementary events** of an experiment is **one**.
Example: take the coin-tossing experiment. P(Heads) + P(Tails)

= (1/2)+ (1/2) =1

Impossible Event

An event that has **no chance of occurring** is called an **Impossible event**, i.e. P(E) = 0.
E.g., The probability of getting a 7 on a roll of a die is 0. As 7 can never be an outcome of this trial.



Sure event

An event that has a **100% probability** of occurrence is called a **sure event**. The probability of occurrence of a **sure event** is **one**.

E.g., What is the probability that a number obtained after throwing a die is less than 7?

So, P(E) = P(Getting a number less than 7) = 6/6= 1

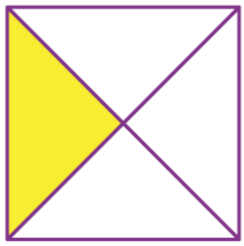
Range of Probability of an event

Probability can range between 0 and 1, where 0 probability means the event to be an impossible one and probability of 1 indicates a certain event i.e. $0 \leq P(E) \leq 1$.

Geometric Probability

Geometric probability is the calculation of the likelihood that one will hit a particular area of a figure. It is calculated by dividing the desired area by the total area. In the case of Geometrical probability, there are infinite outcomes.

$$\text{Probability} = \frac{\text{shaded region}}{\text{entire shape}}$$



Complementary Events

Complementary events are two outcomes of an event that are the only two possible outcomes. This is like flipping a coin and getting heads or tails.

$P(E)+P(\bar{E})=1$

where E and $\bar{E}$ are complementary events.

The event $\bar{E}$, representing 'not E', is called the complement of the event E.

The best example of complementary events is flipping a coin, where 'getting a head' complement the event of 'getting a tail'.

CH.14 PROBABILITY
NCERT C.W – EXE 14.1: Q.1,5,7,9,12,14,15,17,18,22,23
NCERT H.W- EXE 13.1: Q.2,3,4,6,8,10,11,13,16,19,20,21
NCERT EXEMPLAR C.W- EXE-13.3 Q.27,28,30,32,41,42
NCERT EXEMPLAR H.W- EXE-13.3 Q.19,20,24,25
WORKBOOK C.W - CASE STUDY(Q.38)
WORKBOOK H.W- MCQs(Q.1 TO Q.15)