CHAPTER - 3

LINEAR EQUATIONS IN TWO VARIABLES

Basic Concepts:

- **Equation:** An algebraic expression with 'equal to'(=) sign is called the equation. It is an algebraic expression of equality.

  Example: 4x=5, 3y-4=5, 3x+4y=5 etc.

- **Linear Equation:** If the greatest exponent of the variables in an equation is one, the equation is said to be a linear equation.

  Example: 3x-4=2, 4x+6y=7 etc.

- **Linear Equation in One Variable:** An equation of the form ax+b=0, where a, b are real numbers and $a \neq 0$ is called linear equation in one variable.

  Example: 2x+7=0.

- **Linear Equation in Two Variables':** An equation of the form ax+by+c=0, where a, b and c are real numbers where a, $b \neq 0$ is called linear equation in two variables.

  Example: 9x-2y+3=0

- **Solution of an Equation:** That value/values of variable/variables used in equation, which make(s) two sides of equation equal *or* satisfy the equation is called solution of the equation.

  Example: For equation 3x-4=2 , x=2 is a solution.

  For equation 5x-2y=4, (x,y)=(2,3) is one of solutions.

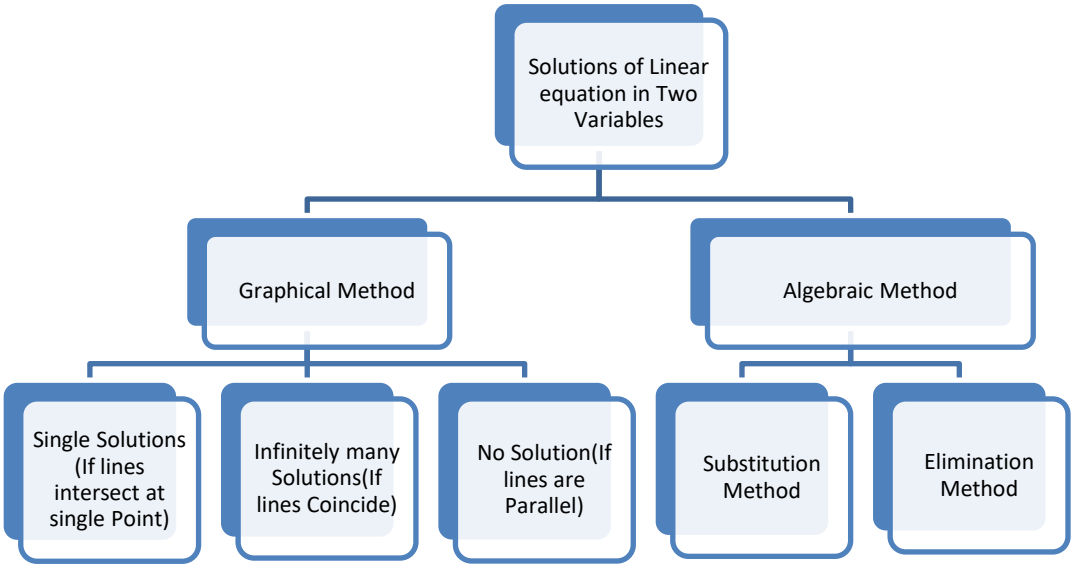
- **Pair of Linear Equations in Two Variables:** Two linear equations of the form ax+by+c=0 taken together form a pair of linear equations in two variables.

  Example: 2x-3y+4=0 and 3x+5y-7=0.

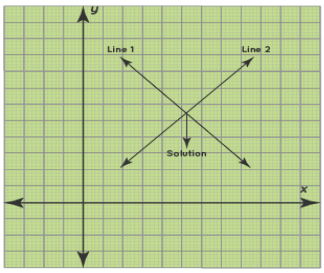
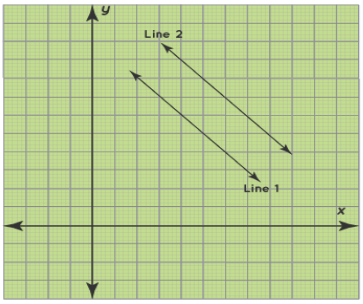
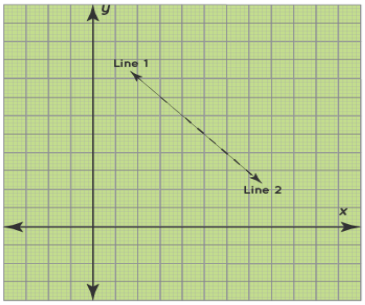
- **Solutions of Linear equation in Two Variables:** The values of x and y satisfying each one of the given pair of linear equation is called their solutions.

  Example: The solution of pair of linear equations

  2x+3y-8=0 and 5x-2y-9=0 is x=1,y=2



Types of Solutions and their Graphs

Type of solution	Conditions	Graphical Representation
Unique Solution (Consistent and Independent)	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	 Unique Solution (Consistent and independent)
No Solution (Inconsistent and Independent)	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	 No Solution (Inconsistent and independent)
Infinite Number of Solutions (Consistent and Dependent)	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	 Infinite number of solutions (consistent and dependent)

➢ **Interpretation of the pair of linear Equations:**

The general form of a pair of linear equations is

$$a_1x+b_1y+c_1=0$$
$$a_2x+b_2y+c_2=0$$

where $a_1 \neq 0$, $b_1 \neq 0$, $a_2 \neq 0$, $b_2 \neq 0$

Solved Questions:

1. **Solve : the following pair of linear equations:**

3x – 5y = 4,
2y + 7 = 9x.

Sol- Given, 3x – 5y = 4.....(1),
9x - 2y = 7 .....(2)

Multipy eq (1) by 3 and eq (2) by 1 and subtracting eq (2) from (1) we get

(3x – 5y)3 – (9x - 2y)1= 4 X 3 – 7 X 1

i.e. -13y = 5 $\therefore y = \frac{-5}{13}$

Then put $y = \frac{-5}{13}$ in eq. (1) then We get $x = \frac{9}{13}$

Ans : $x = \frac{9}{13}$, $y = \frac{-5}{13}$

2. **Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?**

Sol- Let the present age of Jacob and his son be x and y respectively.
According to the question,
(x + 5) = 3(y + 5)
Or, x – 3y = 10 …….. (1)
Or, (x – 5) = 7(y – 5)
Or,x – 7y = -30 ……. (2)
From (1), we get x = 3y + 10 ……. (3)
Substituting the value of x in (2), we get
3y + 10 – 7y = -30
Or,-4y = -40
∴ y = 10 …… (4)
Substituting the value of y in (3), we get
x = 3 x 10 + 10 = 40
Hence, the present age of Jacob's and his son is 40 years and 10 years respectively.

3. **Raman's age is three times he sum of the ages of his two sons. After 5 years his age will be twice the sum of the ages of his two sons. Find the age of Raman.**

Sol- Let x be the present age of Raman's and y be the present ages of sum of his two sons.
ATQ,

$x = 3y$ -----(1)

After 5 years :

Raman's age $= x + 5$

The sum of the ages of his two sons $= y + 5 + 5$

$= y + 10$

ATQ,

$x + 5 = 2(y + 10)$

Or ,$x + 5 = 2y + 20$

Or ,$x - 2y = 20 - 5$

Or, $x - 2y = 15$ --------(2)

By applying the value of x in (2), we get

$3y - 2y = 15$ {from eqn 1}

$\therefore \quad y = 15$

Put the value of y in eqn (1) $x = 3(15)$

$\therefore \quad x = 45$

Answer : So, Raman present age is 45 years old.

4. The middle digit of a number between 100 and 1000 is zero and the sum of the other digit is 13. If the digits are reversed, the number so formed exceeds the original number by 495. Find the number.

Sol-The required number will be in the form X 0 Y
Middle digit = 0

$$x + y = 13 \text{ -------(1)}$$
$$Y\,0\,X = X0Y + 495$$
$$100y + x = 100x + 1y + 495$$
$$x - 100x + 100y - y = 495$$
$$-99x + 99y = 495$$
$$-x + y = 5 \text{ -------(2)}$$
$$x = y - 5$$

By applying the value of x in (1), we get

$$y - 5 + y = 13$$
$$2y = 13+5$$
$$2y = 18$$
$$y = 9$$

When $y = 9$,

$$x = 9 - 5$$
$$x = 4$$

So, the required number is 409.

5. **Solve $2x + 3y = 11$ and $2x – 4y = – 24$ and hence find the value of 'm' for which $y = mx + 3$.**

Sol- $2x + 3y = 11$……..(I)
$2x – 4y = -24$…… (II)
From equation (II), we get

x = (11-3y)/2 …….(III)

Substituting the value of x in equation (II), we get

2(11-3y)/2 – 4y = 24

11 – 3y – 4y = -24

-7y = -35

y = 5……..(IV)

Putting the value of y in equation (III), we get

x = (11-3×5)/2 = -4/2 = -2

Hence, x = -2, y = 5

Also,

y = mx + 3

5 = -2m +3

-2m = 2

m = -1

Therefore the value of m is -1.

6. **The sum of numerator and denominator of a fraction is 3 less than twice the denominator. If each of the numerator and denominator is decreased by 1, the fraction becomes ½. Find the fraction**.

Solution: Let the numerator be x and denominator be y.

Let the fraction be x/y

ATQ,

x+y+3=2y

⇒x−y=−3

x=y−3 ……(i)

Also,

(x-1)/(y-1) = ½

⇒ (x-1) = (y-1)/2

⇒2x=y−1+2=y+1

2x−y=1 ……..(ii)

Substituting (i) in equation (ii), we get;

2(y−3)−y=1

⇒2y−6−y=1

⇒y=7

Now, put y=7 in (i) we get;

x=7−3=4

Thus, x=4,y=7

Therefore, the required fraction is 4/7.

7. **If the system of equations has a unique solution, find the value of k.**

6x + 2y = 3 and kx + y = 2

Solution: Given,

6x + 2y = 3 and kx + y = 2 have unique solutions

Thus, a1/a2 ≠ b1/b2

6/k ≠ 2/1 k ≠ 3

Therefore, k will have any real value apart from 3.

Q.8 Read the following text and answer the following questions on the basis of the same: Places A and B are 100 km apart on a highway . One car starts from A and another from B at the same time .If the cars travel in the same direction at different speeds , they meet in 5 hours . If they travel towards each other ,they meet in 1 hour.

a) What is the actual speed of the other car ?

i) 40 km/hr ii) 60km/hr iii) 20km/hr iv))100km/hr

b) What is the relative speed of both cars while they are travelling towards each other?

i) u+v km/hr ii)u-v km/hr iii)u/v km/hr iv)uv km/hr

c) The given problem is based on which mathematical concept

i)polynomial ii)pair of linear equations iii)quadratic iv) none of these

Answer		
a	b	c
ans (i)	ans (i)	ans (ii)

SECTION-A (MCQ - 1 Marks)

1*. Graphically, the pair of equation

6x-3y+10=0

2x-y+9=0

Represents two lines which are

(A) Intersecting at exactly one point (B) Intersecting at exactly two point

(C) Coincident (D) Parallel

2*. The pair of equation x+2y+5=0 and -3x-6y+1=0 have :

(A) A unique solution (B) Exactly two solutions

(C) Infinitely many solutions (D) No Solution

3**. If a pair of linear equations is consistent, then the lines will be:

(A) Parallel (B) Always coincident

(C) Intersecting or coincident (D) Always interesting

4**. The pair of equation x=a and y=b graphically represents lines which are:

(A) Parallel (B) Intersecting at (b, a)

(C) Coincident (D) Intersecting at (a, b)

5**. The pair of equation y = 0 and y = -7 has:

(A) One solution (B)Two solutions

(C) Infinitely many solutions (D) No solution

6*. One equation of a pair of dependent linear equations is -5x+7y=2. The second equation can be:

(A) 10x+14y+4=0 (B) -10x-14y+4=0

(C) -10x+14y+4=0 (D) 10x-14y = -4

7***. For what value of k, do the equations 3x-y+8=0 and 6x-ky=-16 represents coincident lines?

(A) 12 (B) −12 (C) 2 (D) -2

8***. If the lines given by 3x+2ky=2 and 2x+5y+1=0 are parallel, then the value of k is

(A) −5/4 (B) 2/5 (C) 15/4 (D) 3/2

9**. A pair of linear equation which has a unique solution x=2, y=-3 is

(A) x+y=-1 and 2x-3y=-5 (B) 2x+5y=-11 and 4x+10y=-22

(C) 2x-y=1 and 3x+2y=0 (D) x-4y-14=0 and x-y-13=0

10***. Shweta has only ₹1 and ₹2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹75, then the number of ₹1 and ₹2 coins are, respectively

(A) 35 and 15 (B) 35 and 20 (C) 15 and 35 (D) 25 and 25

11**. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father are, respectively:

(A) 4 and 24 (B) 5 and 30 (C) 6 and 36 (D) 3 and 24

12**. If x=a, y=b is the solution of the equations x-y=2 and x+y=4, then the values of a and b are, respectively

(A) 3 and 5 (B) 5 and 3 (C) 3 and 1 (D) -1 and 3

13*. The larger of the two supplementary angles exceed the smaller by 18°, then the angles are:

(A) 99°, 81° (B) 98°, 82° (C) 97°, 83° (D) None of these

.

14**. x and y are 2 different digits. If the sum of the two digit numbers formed by using both the digits is a perfect square, then value of x +y is

(a) 10 (b) 11 (c) 12 (d) 13

15***. In a number of two digits, unit's digit is twice the tens digit. If 36 be added to the number, the digits are reversed. The number is

(a) 36 (b) 63 (c) 48 (d) 84

16**.The value of k for which the system of equation x+y-4=0 and 2x + ky =3 has no solution is

(a) -2 (b) ≠2 (c) 3 (d) 2

SECTION-B (2 Marks)

1*. Given the linear equation 3x+4y = 9. Write another linear equation in these two variables such that the geometrical representation of the pair so formed is:

(1) intersecting lines (2) coincident lines.

2***. For what value of 'p' does the pair of linear equations given below has unique solution?

4x+ py +8 = 0 and 2x+2y + 2 =0 .

3**. Is the system of linear equations 2 x+3 y − 9 =0 and 4 x+6 y − 18 =0 consistent? Justify your answer.

4*. Two lines are given to be parallel. The equation of one of the lines is 4x + 3y = 14, then find the equation of the second line.

5***. Find the value(s) of k for which the pair of linear equations kx + y = k2 and x + ky = 1 have infinitely many solutions.

6***. Find the value of 'c' for which the pair of equations cx-y =2 and 6x-2y =3 will have infinitely many solutions.

7**. If the lines given by 4x+5ky=10 and 3x+y+1=0 are parallel, then find value of 'k'.

8*. Do the equations 4x+3y-1=5 and 12x + 9y =15 represent a pair of coincident lines?

9**. For what value of 'k', does the systems of linear equations 2x + 3y = 7 ,

(k-1)x + (k + 2)y = 3k have an infinite number of solutions?

10*. Solve the pair of linear equations.

3x + 4y = 10 and 2x – 2y = 2.

SECTION-C (3 marks)

1**. Solve graphically: 2x − 3y + 13 = 0; 3x − 2y + 12 = 0

2**. Find the value of k for which the following pair of equations has no solution :

x + 2y = 3, (k-1)x+(k+1)y=(k+2)

3*. Solve x + y = 5 and 2x − 3y = 4 by elimination method and the substitution method.

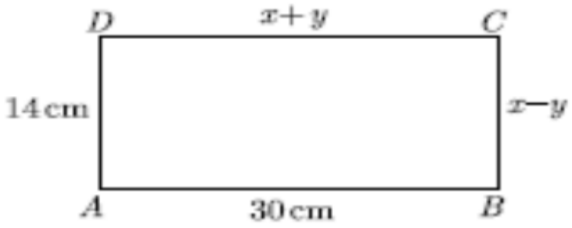
4***. Draw the graph of the following equations:

2x - y = 1, x + 2y = 13 ,

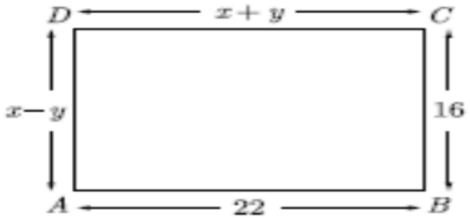
Find the solution of the equations from the graph and shade the triangular region formed by the lines and the y -axis.

5**. Solve the following pair of linear equations graphically: x - y = 1, 2x + y = 8. Also find the co-ordinates of the points where the lines represented by the above equation intersect y - axis.

6*. In Figure, ABCD is a rectangle. Find the values of x and y



7*. In Figure, ABCD is a rectangle. Find the values of x and y



8**. Half the perimeter of a rectangular garden, whose length is 4 m more then its width, is 36 m. Find the dimensions of garden.

9***. Determine graphically whether the following pair of linear equations :

3x - y = 7 ; 2x + 5y + 1 = 0 has : unique solution infinitely many solutions or no solution.

10**. Solve : 99x + 101y = 499, 101x + 99y = 501

SECTION-D (5 Marks)

1*. The ratio of incomes of two persons is 11:7 and the ratio of their expenditures is 9:5. If each of them manages to save Rs 400 per month, find their monthly incomes.

2***. A fraction becomes 9/11 if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator it becomes 5/6. Find the fraction.

3***. 2 man and 7 boys can do a piece of work in 4 days. It is done by 4 men and 4 boys in 3 days. How long would it take for one man or one boy to do it?

4**. Solve the following pair of equations graphically: 2x + 3y = 12, x − y − 1 = 0. Shade the region between the two lines represented by the above equations and the X -axis.

5**. A chemist has one solution which is 50% acid and a second which is 25% acid. How much of each should be mixed to make 10 litre of 40% acid solution.

6**. If 2x+y =23 and 4x-y=19, find the value of (5y-2x) and ($\frac{y}{x}$ -2).

7***. A train covered a certain distance at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time and if the train were slower by 6km/h, it would have taken 6 hours more than the scheduled time. Find the length of the journey.

8***. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

9*. Students of a class x are made to stand in rows. If one student is extra in each row, there would be 2 rows less. If one student is less in each row, there would be 3 rows more. Find the number of students in the class x.

10*. Draw the graph of pair of linear equations :

$x + y = 5$, $2x + 2y = 10$

SECTION-E (Case Study Questions – 4 marks)

1. *** It is common that governments revise travel fare from time to time based on various factors such as inflation (a general increase in prices and fall in purchasing value of money) on different types of vehicles like auto, rickshaws, taxis, radio cab etc. The auto charges in a city comprise of a fixed charge together with the charge for the distance covered. Study the following situations:**

Name of the city	Distance travelled (km)	Amount Paid (Rs)
City A	10	75
	15	110
City B	8	91
	14	145

Situation1: In city A, for a journey of 10 km, the charge paid is Rs 75 and for a journey of 15 km, the charge paid is Rs 110.

Situation2: In a city B, for a journey of 8 km, the charge paid is Rs 91 and for a journey of 14 km, the charge paid is Rs 145.

i) If the fixed charges of auto rickshaw be Rs *x* and the running charges be Rs *y* km/h , the pair of linear equations representing the situation is

a) $x + 10y = 110$, $x + 15y = 75$
b) $x + 10y = 75$, $x + 15y = 110$
c) $10x + y = 110$, $15x + y = 75$
d) $10x + y = 75$, $15x + y = 110$

ii) A person travels a distance of 50km. The amount he has to pay is

a) Rs 155 b) Rs 255 c) Rs 355 d) Rs 455

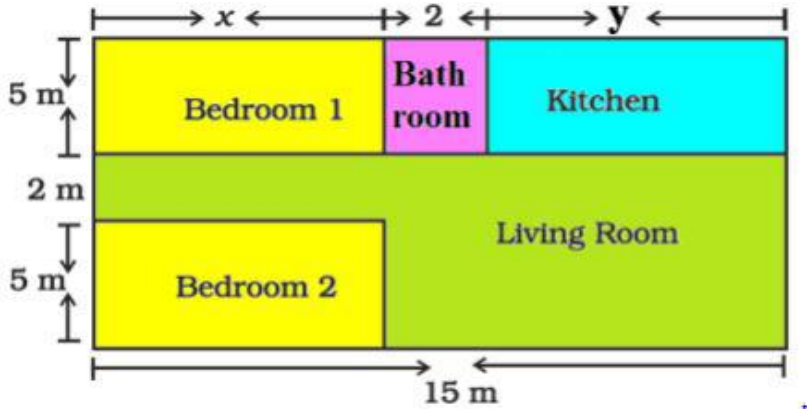
iii)What will a person have to pay for travelling a distance of 30 km?

a) Rs 185 b) Rs 289 c) Rs 275 d) Rs 305

iv)Out of both the city, which one has cheaper fare?

a) City A b) City B c) Both are same d) cannot be decided

2. * Amit is planning to buy a house and the layout is given below. The design and the measurement has been made such that areas of two bedrooms and kitchen together is 95m^2.**



i). Form the pair of linear equations in two variables from this situation.

ii). Find the length of the outer boundary of the layout.

iii). Find the area of each bedroom and kitchen in the layout.

iv). Find the area of living room in the layout.

v). Find the cost of laying tiles in kitchen at the rate of Rs. 50 per sq.m.

3. ** A test consists of 'True' or 'False' questions. One mark is awarded for every correct answer while ¼ mark is deducted for every wrong answer. A student knew answers to some of the questions. Rest of the questions he attempted by guessing. He answered 120 questions and got 90 marks.

Type of Question	Marks given for correct Answers	Marks deducted for wrong Answers
True/ False	1	0.25

i) If answer to all questions he attempted by guessing were wrong, then how many questions did he answer correctly?

ii) How many questions did he guess?

iii) If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got?

iv) If answer to all questions he attempted by guessing were wrong, then how many questions answered correctly to score 95 marks?

ANSWER KEY

CHAPTER-3 PAIR OF LINEAR EQUATION IN TWO VARIABLES

Answers Section A

1. d	2. d	3. c	4.d	5. d	6. d	7.c	8.c	9.b,d	10.d	11.c	12.c	13.a	14.b	15.c	16.d

Answers (Section B)

1. One of the possible equation 3x-5y=10
One of the possible equation 6x+8y=18
2. $p \neq 4$
3. Consistent
4. One of the possible solution 12x+9y=5
5. K=1 or k= -1
6. There is no value of 'c' for which lines have infinitely many solutions.
7. k = $\frac{4}{15}$
8. Given pair of lines do not represent a pair of coincident lines.
9. k = 7
10. x=2, y=1.

Answers (Section C)

1. x= -2 and y=3 2.k=3 3.x=19/5 and y=6/5 4.Draw the graph 5.Draw the graph
6.x=22 and y=8 7.x=19 and y=3 8.Length =20m and width =16m 9.Unique solution 10.x=2 , y=3

Answer (Section D)

1. 2200 and1400 2. 7/9 3. 15 days 4. Draw the Graph 5. x=6 ,y=4 6. x=7, y=9 , 31, $\frac{-5}{7}$ 7. 720 km 8. Speed of the stream is 3 km/h and speed of boat is 8km/h
9. 60 Students

Answer (Section E)

1. Answer (Case Study QN-1)

i) b ii) c iii) b iv) a

2. Answer (Case Study QN-2)

i) 2x + y =19, x + y = 13
ii) Length of outer boundary = 12 + 15 + 12 + 15 = 54m
iii) area of bedroom = 5 x 6 = $30m^2$, area of kitchen = 5 x 7 = $35m^2$
iv) Area of living room = (15 x 7) – 30 = 105 – 30 = 75 sq.m
v) Total cost of laying tiles in the kitchen = Rs50 x 35 = Rs1750

3. Answer (Case Study QN-3)

i) 96 ii) 24 iii) 70 iv) 100

******