SETH M.R. JAIPURIA SCHOOLS BANARAS PARAO CAMPUS

SUBJECT – MATHEMATICS

CLASS –XII (CHAPTER -1 TO 4 WORKSHEET)

EQUIVALENCE RELATION: A relation R on a set A is said to be an equivalence relation on A iff

It is ………………… i.e. $(a,a) \in R$ for all $a \in A$.

It is symmetric i.e. $(a,b) \in R \Rightarrow$ … … … . … … … … . $\in R$ for all $a,b \in A$.

It is ………………… i.e. $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$ for all $a,b,c \in A$.

Match Column I with Column II:

Column I (Relations)	Column II (Descriptions)
1. R on $\mathbb{R}$ defined by $aRb \Leftrightarrow a-b \in \mathbb{Z}$	A. Equivalence relation (defined on a finite set)
2. R on $\mathbb{R}$ defined by $aRb \Leftrightarrow a^2 = b^2$	B. Reflexive, Symmetric but not Transitive
3. R on $\mathbb{R}$ defined by $aRb \Leftrightarrow \lvert a-b \rvert < 1$	C. Equivalence relation (difference in integers)
4. R on $\mathbb{R}$ defined by $aRb \Leftrightarrow a = b$	D. Identity relation (Equivalence)
5. R on $\mathbb{Z}$ defined by $aRb \Leftrightarrow a \equiv b \bmod 3$	E. Not an equivalence relation
6. R on $A = \{1,2,3\}$ defined by $aRb \Leftrightarrow a+b$ is even	F. Equivalence relation (congruence modulo)

Match each function in Column I with the correct property in Column II:

Column- I	Column- II
1. $f(x) = x + 5$	A. One-one and Onto on $(0, \infty)$
2. $f(x) = x^2, x \in R$	B. One-one but Not Onto
3. $f(x) = \sin x$	C. One-one and Onto on $(-\infty, \infty)$
4. $f(x) = \ln(x), x \in (0, \infty)$	D. Not One-one, Not Onto
5. $f(x) = x^3$	E. One-one and Onto on $R - \{0\}$
6. $f(x) = \lvert x \rvert$	F. One-one but Not Onto on R
7. $f(x) = \tan x, x \in (-\pi/2, \pi/2)$	G. Neither One-one nor Onto
8. $f(x) = e^x$	H. One-one and Onto on R
9. f(x) = constant function	I. Many-one and Onto on [-1, 1]
10. $f(x) = 1/x, x \neq 0$	J. Many-one and Not Onto

GIVE AN EXAMPLE OF A RELATION WHICH IS -

11. Symmetric but neither reflexive nor transitive.
12. Transitive but neither reflexive nor symmetric.
13. Reflexive and symmetric but not transitive.
14. Reflexive and transitive but not symmetric.
15. Symmetric and transitive but not reflexive.

INVERSE TRIGONOMETRIC FUNCTIONS, THEIR DOMAIN AND RANGES –

Function	Domain	Principal value branch (Range)
$y = \sin^{-1} x$	$[-1,1]$	………………………………
$y = \cos^{-1} x$	…………………	$[0, \pi]$
$y = \tan^{-1} x$	R	…………………………
$y = \sec^{-1} x$	$R - (-1,1)$	… … … … … … … …
$y = \text{cosec}^{-1} x$	… … … … … …	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \cot^{-1} x$	R	… … … … … … … ….

1. PROPERTY I

(i) $\sin^{-1} (\dots\dots\dots..) = \theta; \theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(ii) $\cos^{-1} (\cos\ \theta) = \theta; \theta \in [\dots\dots\dots\dots\dots]$

(iii) $\tan^{-1} (\dots\dots\dots\dots) = \theta; \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iv) $\text{cosec}^{-1}(\text{cosec}\theta) = \theta; \theta \in [\dots\dots\dots\dots], \theta \neq 0$

(v) $\sec^{-1} (\sec\ \theta) = \theta; \theta \in [0, \pi], \theta \neq \frac{\pi}{2}$

(vi) $\cot^{-1} (\cot\ \theta) = \theta; \theta \in (0, \pi)$

2. PROPERTY II

(i) $\sin (\sin^{-1} x) = x; x \in [\dots\dots\dots\dots]$

(ii) $\cos (\dots\dots\dots\dots..) = x; x \in [-1,1]$

(iii) $\tan(\tan^{-1} x) = x; x \in\ \dots\dots\dots\dots$

(iv) $\text{cosec}(\text{cosec}^{-1}x) = x; x \in (\dots\dots\dots\dots\dots\dots\dots)$

(v) $\sec (\dots\dots\dots\dots.) = x; x \in (-\infty, -1| \cup |1, \infty)$

(vi) $\cot(\cot^{-1} x) = x; x \in\ \dots\dots\dots\dots..$

3. PROPERTY III

(i) $\sin^{-1}(-x) =\ \dots\dots\dots\dots\dots; x \in [-1,1]$

(ii) $\cos^{-1}(-x) =\ \dots\dots\dots\dots\dots\dots\dots..; x \in [-1,1]$

(iii) $\tan^{-1}(-x)\ \dots\dots\dots\dots\dots\dots.; x \in R$

(iv) $\text{cosec}^{-1}(-x) =\ \dots\dots\dots\dots..; x \in (-\infty, -1] \cup [1, \infty)$

(v) $\dots\dots\dots\dots\dots.. = \pi - \sec^{-1}\ x; x \in (-\infty, -1] \cup [1, \infty)$

(vi) $\cot^{-1}(-x) =\ \dots\dots\dots\dots\dots\dots.; x \in R$

Draw y = sin⁻¹x

Draw y = tan⁻¹x

Draw y = sec⁻¹x

If A = $\begin{bmatrix} 2 & 5 \\ 3 & -7 \end{bmatrix}$, express A as sum of symmetric and skew symmetric matrices -

4. PROPERTY V

(i) $\sin^{-1} x + \cos^{-1} x = \ \dots\dots\dots\dots \ ; x \in [-1,1]$
(ii) $\tan^{-1} x + \ \dots\dots\dots\dots\dots\dots\dots = \frac{\pi}{2}; x \in R$
(iii) $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}; x \in \ \dots\dots\dots\dots\dots\dots\dots\dots.$

Match Column I with Column II:

Column I	Column II
1. Order of a 3x2 matrix -	A. Matrix with only one row
2. Determinant of a 2×2 matrix -	B. Scalar quantity
3. Row matrix -	C. 3 rows, 2 columns
4. Column matrix -	D. Only square matrices
5. Identity matrix of order 2 -	E. Always has equal number of rows and columns
6. Singular matrix -	F. Determinant = 0
7. |A| = 1 -	G. Invertible matrix
8. Matrix multiplication possible when -	H. No. of columns in first = no. of rows in second
9. Adjoint of a matrix is defined for -	I. I = [1 0; 0 1]
10. Square matrix -	J. Matrix with only one column

Match Column I with Column II:

Column I	Column II
1. $(A^T)^T$ = -	A. $A^T = A$
2. A matrix is symmetric if -	B. $A^T = -A$
3. A matrix is skew-symmetric if -	C. Are all zero
4. Transpose of a column matrix -	D. $B^T A^T$
5. Transpose of a row matrix -	E. Columns of A = Rows of B
6. A symmetric matrix has -	F. A
7. Diagonal elements of skew-symmetric matrix -	G. A column matrix
8. If AB is defined, then multiplication possible when -	H. A row matrix
9. $(AB)^T$ = -	I. $A^T + B^T$
10. $(A + B)^T$ = -	J. Mirror image across the diagonal

Write a symmetric and skew symmetric matrix-

Symmetric:

Skew-symmetric:

5. PROPERTIES OF ADJOINT OF A MATRIX

If A and B are two non-singular matrices of same order n, then

(i) $A(\text{adj}A) = (\text{adj}A)A =$ … … … … … … …

(ii) $\text{adj}(A') =$ … … … … … … … … … ….

(iii) … … … … … . . $= (\text{adj}B)(\text{adj}A)$

(iv) $\text{adj}(kA) =$ … … … … … . $(\text{adj}A), k \in R$

(v) $\text{adj}(A^m) =$ … … … … … … …

(vi) adj (adj A) $=$ … … … … … … …., where A is a non-singular matri

(vii) … … … … … $= |A|^{n-1}$, where A is a non-singular matrix.

(viii) $|\text{adj}(\ldots\ldots)| = |A|^{(n-1)^2}$, where A is a non-singular matrix.

(ix) adj(… … … .) $= I_n$, adj(… … … … …) $= O$

Draw y = cot⁻¹x

Draw y = cos⁻¹x

Note
(i) Adjoint of a diagonal matrix is a ……………………… matrix.
(ii) Adjoint of a triangular matrix is a ……………………… matrix.
(iii) Adjoint of a symmetric matrix is a ………………………… matrix.

6. PROPERTIES OF INVERSE OF A MATRIX

Let A and B be two square matrices of same order n. Then,

(i) $(A^{-1})^{-1} =$ … … … … … … … … ….

(ii) $(AB)^{-1} =$ … … … … … … … … … …. In general, $(A_1A_2A_3 \ldots A_n)^{-1} = A_n^{-1}A_{n-1}^{-1} \ldots A_3^{-1}A_2^{-1}A_1^{-1}$

(iii) $(A')^{-1} =$ … … … … … … … … … … … ..

(iv) … … … … … … … . . $= |A|^{-1}$

(v) $AA^{-1} = A^{-1}A =$ … … … … … … … ….

(vi) … … … … … … … … … . $= (A^{-1})^k, k \in N$

(vii) If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ and $abc \neq 0$, then $A^{-1} = \begin{bmatrix} \ldots\ldots\ldots\ldots.. & 0 & 0 \\ 0 & \ldots\ldots\ldots\ldots\ldots & 0 \\ 0 & 0 & \ldots\ldots\ldots\ldots\ldots. \end{bmatrix}$.

(viii) If A, B and C are square matrices of the same order and A is a non-singular matrix, then

(a) $AB = AC \Rightarrow$ … … … . . $= C$

[left cancellation law]

(b) $BA = CA \Rightarrow B =$ … … … … … … ..

[right cancellation law]

Draw y = coses⁻¹x

8. PROPERTIES OF TRANSPOSE

For any two matrices A and B of suitable orders,

(i) $(A')' =$ ………………..

(ii) $(A \pm B)' =$ ……………………………….

(iii) $(\ldots\ldots\ldots\ldots.)' = kA'$

(iv) $(AB)' =$ ………………………..

(v) $(A^n)' =$ ……………………..

(vi) $(ABC)' =$ ……………………………

Show a diagrammatic form of one- one but not onto function -

9. SYMMETRIC AND SKEW-SYMMETRIC MATRICES

(i) A square matrix $A = [a_{ij}]_{n\times n}$ is said to be symmetric, if $A' = A$. i.e. ………………..$= a_{ji}$, $\forall i$ and j.

(ii) A square matrix A is said to be skew-symmetric, if $A' = -A$, i.e. $a_{ij} =$ ……………..,$\forall i$ and j.

BEST QUESTIONS FROM CHAPTER 1 TO 4

1. Let A = {1,2,3}. Then, the number of relations containing (1,2) and (1,3) which are reflexive and symmetric but not transitive is
   a) 1 b) 2 c) 3 d) 4

2. Let A = {1,2,3}. Then the number of equivalence relations containing (1,2) is
   a) 1 b) 2 c) 3 d) 4

3. The maximum number of equivalence relations on the set A = {1,2,3} are
   a) 1 b) 2 c) 3 d) 5

4. If A and B are square matrices of the same order, then $(A+B)(A-B)$ is equal to
   (a) $A^2 - B^2$ (b) $A^2 - BA - AB - B^2$
   (c) $A^2 - B^2 + BA - AB$ (d) $A^2 - BA + B^2 + AB$

5. If A and B are matrices of same order, then $(AB' - BA')$ is a
   (a) skew-symmetric matrix (b) null matrix
   (c) symmetric matrix (d) unit matrix

6. Given that $\boldsymbol{A}$ is a square matrix of order 3 and $|\boldsymbol{A}| = -\mathbf{2}$, then $|\boldsymbol{adj}(\mathbf{2}\boldsymbol{A})|$ is equal to
   a) -2^6 b) $+4$ c) -2^8 d) 2^8

7. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) or 'x' is/are
   a) 3 b) $\sqrt{3}$ c) $-\sqrt{3}$ d) $\sqrt{3}, -\sqrt{3}$

8. Let A be a 3×3 square matrix such that $A(\text{adj } A) = 2I$, where I is the identity matrix. The value of $|\text{adj } A|$ is
   (a) 4 (b) -4 (c) 0 (d) None of these

9. If A is square matrix of order 3×3 such that $|A| = 2$, then write the value of $|\text{adj}\ (\text{adj}\ A)|$.
(a) -16 (b) 16 (c) 0 (d) 2

10. If A is a square matrix of order 3, such that $A(\text{adj}\ A) = 10I$, then $|\text{adj}\ A|$ is equal to
(a) 1 (b) 10 c) 100 (d) 101 [CBSE 2020 (65/5/1)]

11. Domain of $f(x) = \cos^{-1} x + \sin x$ is: 2025 (65/7/3)
(A) R (B) $(-1,1)$ (C) $[-1,1]$ (D) ϕ

12. Domain of $\sin^{-1}(2x^2 - 3)$ is: 2025 (65/4/3)
(A) $(-1,0) \cup (1,\sqrt{2})$ (B) $(-\sqrt{2},-1) \cup (0,1)$
(C) $[-\sqrt{2},-1] \cup [1,\sqrt{2}]$ (D) $(-\sqrt{2},-1) \cup (1,\sqrt{2})$

13. Which of the following relations is symmetric but neither reflexive nor transitive for a set A = {1, 2, 3}.
a) R = {(1, 2), (1, 3), (1, 4)} b) R = {(1, 2), (2, 1)}
c) R = {(1, 1), (2, 2), (3, 3)} d) R = {(1, 1), (1, 2), (2, 3)}

14. Let R be a relation in the set N given by R= {(a, b): a + b= 5, b > 1}. Which of the following will satisfy the given relation?
a) (2, 3) ∈ R b) (4, 2) ∈ R c) (2, 1) ∈ R d) (5, 0) ∈ R

15. If S = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, adjA is
a) $\begin{bmatrix} -d & -b \\ -c & a \end{bmatrix}$ b) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ c) $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$ d) $\begin{bmatrix} d & c \\ b & a \end{bmatrix}$

16. If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|adj\ A|$ is
a) a^{27} b) a^9 c) a^6 d) a^2

17. If A = $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det(adj(adjA)) is
a) 14^4 b) 14^3 c) 14^2 d) 14

18. If A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then A^5 =
a) 5A b) 10A c) 16A d) 32A

19. If A = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and c_{ij} is cofactor of a_{ij} in A, then the value of $|A|$ is given by
a) $a_{11}C_{31}+a_{12}C_{32}+a_{13}C_{33}$ b) $a_{11}C_{11}+a_{12}C_{21}+a_{13}C_{31}$
b) $a_{21}C_{11}+a_{22}C_{12}+a_{23}C_{13}$ c) $a_{11}C_{11}+a_{21}C_{21}+a_{31}C_{31}$