CHAPTER WISE HOTS QUESTIONS.

REAL NUMBERS

1. What is the least number that is divisible by all the numbers from 1 to 10.
2. Find the HCF of 52 and 117 and express it in form $52x + 117y$.
3. If HCF of 144 and 180 is expressed in the form of $13m - 30$, find the value of m.
4. If m and n are odd positive integers, then $m^2 + n^2$ is even, but not divisible by 4. Justify.
5. If $\text{HCF}(6, a) = 2$ and $\text{LCM}(6, a) = 60$, then find $a^2 + 3a$.
6. Find the greatest number of 5 digits exactly divisible by 12,15 and 36.
7. Find the smallest number which leaves remainder 8 and 12 when divided by 28 and 32 respectively.
8. Floor of a room is to be fitted with square marble tiles of the largest possible size. The size of the floor is $10 \text{ m} \times 7 \text{ m}$. What should be the size of tiles required that has to be cut and how many such tiles are required?
9. If the HCF of 408 and 1032 is expressible in the form $\text{p} = 1032\text{x} + 408\text{ y}$, find p, x & y
10. Find HCF of 378, 180 and 420 by prime factorization method. Is HCF X LCM of three numbers is equal to the product of three numbers? Verify.

POLYNOMIALS

1. If one zero of the polynomials $5z^2 + 13z - p$ is reciprocal of the other, then find p.
2. If $f(x)$ is a polynomial such that $f(a)f(b) < 0$, then what is the least number of zeroes lying between a and b ?
3. Find all zeroes of the polynomial $2x^4 - 9x^3 + 5x^2 + 3x - 1$ if two of its zeroes are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$
4. If α and β are the zeroes of the polynomial $3x^2 - 5x - 2$, then evaluate i) $\alpha^2 + \beta^2$ ii) $\alpha^3 + \beta^3$
5. If α and β are the zeroes of the polynomial $3x^2 - 5x - 2$ then find the polynomial whose zeroes are $1/\alpha$ and $1/\beta$
6. If one zero of the quadratic polynomials $f(x) = 4x^2 - 8kx + 8x - 9$ is negative of the other then find the zeroes of $kx^2 + 3kx + 2$.
7. If the sum of the zeroes of the quadratic polynomial $ky^2 + 2y - 3k$ is equal to twice their product, find the value of k.
8. If one zero of the quadratic polynomials $4x^2 - 8kx + 8x - 9$ is negative of the other, then find the zeroes of $kx^2 + 3kx + 2$
9. If two zeroes of a cubic polynomial $px^3 + 3x^2 - qx - 6$ are -1 and -2, find the third zero and also the values of p and q.
10. Find all zeroes of the polynomial $2x^4 - 9x^3 + 5x^2 + 3x - 1$ if two of its zeroes are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$
11. If the product of two zeroes of polynomial $2x^3 + 3x^2 - 5x - 6$ is 3, then find its third zero.
12. Find the polynomial of least degree which should be subtracted from the polynomial $x^4 + 2x^3 - 4x^2 + 6x - 3$ so that it is exactly divisible by $x^2 - x + 1$.
13. If the zeroes of the polynomial $f(x) = x^3 - 12x^2 + 39x + a$ are in AP, find the value of a.
14. If m and n are the zeros of the polynomial $3x^2 + 11x - 4$, find the values of $\frac{m}{n} + \frac{n}{m}$

15. If 1 and -1 are zeroes of polynomial $Lx^4 + Mx^3 + Nx^2 + Rx + P$, show that $L + N + P = M + R = 0$
16. If $\mathrm{x} + \mathrm{a}$ is a factor of the polynomial $\mathrm{x}^2 + \mathrm{px} + \mathrm{q}$ and $\mathrm{x}^2 + \mathrm{mx} + \mathrm{n}$ prove that $\mathrm{a} = \frac{n-q}{m-p}$
17. If α, β and γ are the zeroes of the polynomial $\mathrm{f(x)} = \mathrm{ax}^3 + \mathrm{bx}^2 + \mathrm{cx} + \mathrm{d}$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.
18. If α, β are zeroes of the polynomial $\mathrm{f(x)} = \mathrm{x}^2 - \mathrm{p(x + 1)} - \mathrm{c}$, then find the value of $(\alpha + 1)(\beta + 1)$.
19. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then find the product of the other two zeroes.
20. If $a - b$, a and $a + b$ are zeroes of the polynomial $f(x) = 2x^3 - 6x^2 + 5x - 7$, write the value of a.
21. If $f(x) = x^3 + x^2 - ax + b$ is divisible by $x^2 - x$, write the value of a and b.
22. If the zeroes of the quadratic polynomial $ax^2 + bx + c, c \neq 0$ are equal, then c and a have opposite signs. Is it true or false? Justify your answer.

PAIR OF LINEAR EQUATION IN TWO VARIABLES

1. Sima can row down-stream 20 km in 2 hrs and upstream 4 km in in 2 hrs. find her speed of rowing in still water and the speed of the current.
2. Solve for x and y

$$(a - b)x + (a + b)y = a^2 - 2ab - b^2$$
$$(a + b)(x + y) = a^2 + b^2$$

3. 6 men and 10 women can finish making pots in 8 days, while the 4 men and 6 women can finish it in 12 days. Find the time taken by the one man alone from that of one woman alone to finish the work.
4. A boat covers 14 kms in upstream and 20 kms downstream in 7 hours. Also, it covers 22 kms upstream and 34 kms downstream in 10 hours. Find the speed of the boat in still water and that of the stream.
5. Draw the graph of $2x + y = 6$ and $2x - y + 2 = 0$. Shade the region bounded by these lines and x axis. Find the area of the shaded region
6. Find the value of k for which the system of linear equations $\mathrm{kx} + \mathrm{ky} = 12$, (k -3) $x + 3y = k$ will have infinite number of solutions.
7. In a $\triangle$ ABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find these angles.
8. In a cyclic quadrilateral ABCD, $\angle A = (2x + 4)^0, \angle B = (y + 3)^0, \angle C = (2y + 10)^0$ and $\angle D = (4x - 5)^0$. Find the four angles.
9. A number say z is exactly the four times the sum of its digits and twice the product of the digits. Find the numbers.
10. There are two points on a highway A and B. They are 70 km apart. An auto starts from A and another auto starts from B simultaneously. If they
travel in the same direction, they meet in 7 hours, but if they travel towards each other they meet in 1 hour. Find how fast the two autos are.
11. The larger of two supplementary angles exceeds thrice the smaller by 20 degrees. Find them.
12. $71x + 37y = 253$,
$37x + 71y = 287$. [CBSE 2007C]
13. $23x - 29y = 98$,
$29x - 23y = 110$.
14. $217x + 131y = 913$,
$131x + 217y = 827$.

QUADRATIC EQUATION

1. If $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$, Find x.
2. Find the value of k for which both the quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have equal roots.
3. Find the value of p for which the quadratic equation $(2p+1)x^2 - (7p+2)x + (7p-3) = 0$ has equal roots. Also find these roots.
4. $\sqrt{2x-3} + 1 = x$, solve x.
5. If $\frac{x}{x+1} + \frac{x+1}{x} = 2\frac{1}{72}$, then solve for x.
6. If the quadratic equation $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$, Then prove that $c^2 = a^2(1+m^2)$.
7. In a quadratic equation $ax^2 + \mathrm{b}x + \mathrm{c} = 0, a \neq 0$, one of its roots is four times the other root then prove that $4b^2 = 25ac$.
8. If the price of a book is reduced by Rs. 5, a person can buy 4 more books for Rs. 600. Find the original price of the book.
9. Solve for x, if $25x^{-2} - 10x^{-1} + 1 = 0, x \neq 0$.
10. What is the value of $\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 +}}}} \ldots \ldots$
11. If α *and* β are roots of $x^2 - 7x - 8 = 0$, then find i. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\alpha\beta}$ ii. $\alpha\beta^3 + \alpha^3\beta$
12. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/hr more. Find the original speed of the train.
13. If the roots of the equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal, then prove that 2 b = a + c.
14. If the roots of the equations $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ac}x + b = 0$ are simultaneously real then prove that $b^2 = ac$.
15. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are equal, then prove that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$
16. Solve for x: $2^{2x} + 32 - 3 \times 2^{x+2} = 0$
17. A polygon of n sides has $\frac{n(n-3)}{2}$ diagonals. How many sides does a polygon have with 54 diagonals?
18. One - fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.
19. Solve: $5^{(x+1)} + 5^{(2-x)} = 5^3 + 1$.
20. $3^{(x+2)} + 3^{-x} = 10$
21. $4^{(x+1)} + 4^{(1-x)} = 10$
22. $2^{2x} - 3 \cdot 2^{(x+2)} + 32 = 0$

ARITHMETIC PROGRESSION

1. How many terms of the series 54,51,48,.....be taken so that, their sum is 513? Explain the double answer.

2. If the ratio of the sum of the first n terms of the two AP is $7n+1:4n+27$, find the ratio of their m^{th} term.

3. Find the sum of the following series-

$$5+(-41)+9+(-39)+13+(-37)+17+\cdots\ldots\ldots\ldots..+(-5)+81+(-3)$$

4. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the arithmetic mean between ' a ' and ' b ', then, find the value of ' n '.

5. If p^{th} term of an A.P. is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$ prove that the sum of the first ' pq ' terms is $\frac{1}{2}[pq+1]$.

6. If S_n denotes the sum of the first n terms of an AP Prove that $S_{30}=3(S_{20}-S_{10})$

7. Find the sum of the integers between 100 and 200 that are-

   i)divisible by 9

   ii)not divisible by 9

8. Two APs have the same common difference. The first terms -1 and -8 respectively. Find the difference between the 4^{th} terms.

9. In a polygon the smallest interior angle is $120°$. Angles are increased by 5^0. Find the number of sides of the polygon.

   10. Show that the sum of an AP whose 1^{st} term is a, second term is b and the last term is c, is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$

10. If $\frac{1}{b+c},\frac{1}{c+a},\frac{1}{a+b}$ are in A.P., prove that $a^2,\ b^2,c^2$ are also in A.P.

11. The 24^{th} term of an AP is twice its 10^{th} term. Show that its 72^{th} term is 4 times its 15^{th} term.

12. 150 workers were engaged to complete a work in certain days. After first day 4 workers left the jobs, after 2^{nd} day 4 more workers left the job and so on. The assigned work took 8 more days to be finished. In how many days the work was completed.

13. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. Calculate the total volume of concrete required to build the terrace.

14. The houses of a row are numbered consecutively from 1 to 49. Show that there is the value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find the value of x.

15. The sum of $n, 2n, 3n$ terms of an AP is S_1, S_2 and S_3 respectively. Prove that $S_3=3(S_2-S_1)$

16. A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

17. Solve the equation: $1+4+7+10+\cdots+x=287$

18. Find three numbers in A.P. whose sum is 21 and their product is 231.

19. The sum of the first p, q and r terms of an AP is a, b, and c respectively.

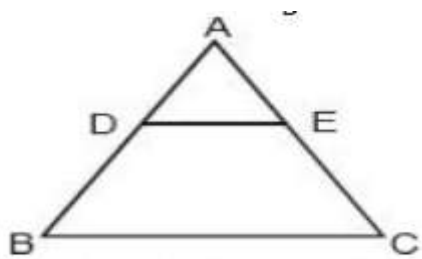
20. Prove that $\frac{a}{p}(q - r) + \frac{b}{q}(r - p) + \frac{c}{r}(p - q) = 0$

10. TRIANGLES

1. Two isosceles triangles have equal angles and their areas are in the ratio 81: 25. Find the ratio of their corresponding heights.
2. ABC is a triangle right-angled at C and ' p ' is the length of the perpendicular from C to AB. By expressing the area of the triangle in the two ways, show that

$$\text{i) } pc = ab \quad \text{ii) } \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

3. The perimeter of two similar triangles ABC and LMN are 60 cm and 48 cm respectively. If LM = 8 cm, then what is the length of AB?
4. If one diagonal of a trapezium divides the other diagonal in the ratio 1: 3. Prove that one of the parallel sides is three times the other.
5. In the given figure, ABC is a triangle in which AB = AC, D and E are points on the sides AB and AC respectively, such that AD = AE. Show that the points B, C, E and D are concyclic.



6. ABCD is a trapezium with AB||DC in which diagonals AC and BD intersect at E and △ AED ~△ BEC. Prove that AD = BC.
7. ABC is a triangle. PQ is a line segment intersecting AB in P and AC in Q such that PQ||BC and divides △ ABC into two parts equal in area. Find BP/AB,
8. ABC is a triangle in which AB = AC and D is any point in BC. Prove that: $(AB)^2 - (AD)^2 = BD.CD$.
9. AD is the median of △ ABC, O is any point on AD. BO and CO produced meet AC and AB in E and F respectively. AD is produced to X such that OD = DX. Prove that AO: AX = AF: AB.
10. In a triangle ABC, P divides the sides AB such that AP: PB = 1: 2, Q is a point on AC such that PQ||BC. Find the ratio of the areas of △ APQ and trapezium BPQC.
11. If A be the area of a right triangle and b one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^2+4A^2}}$.
12. A point D is on the side BC of an equilateral △ ABC such that $DC = \frac{1}{4}BC$. Prove that $AD^2 = 13CD^2$

n △ ABC, A line XY parallel to BC cuts AB at X and AC at Y, if BY bisects < XYZ, show that BC = CY.

13. Through the midpoint M of the side CD of a parallelogram $ABCD$, the line BM is drawn, intersecting AC in L and AD produced in E. Prove that $EL = 2BL$. [CBSE 2006C, '08, '09]

11. COORDINATE GEOMETRY

1. The coordinates of A, B, C and D are (6,3), (-3,5), (4, -2) and (k,3k) respectively. If area $(\triangle DBC)$:area $(\triangle ABC) = 1:2$, find k.
2. Find the vertices of the triangle, the midpoints of whose sides are $(3,1), (5,6)$ and $(-3,2)$.
3. If the points $(x, 0), (0, y)$ and $(1,1)$ are collinear, show that $x^{-1} + y^{-1} = 1$.
4. If the midpoint of the line joining $(3,4)$ and $(k, 7)$ is (x, y) and $2x + 2y + 1 = 0$, find the value of k.
5. If the point $\mathrm{C(a, b)}$ is equidistant from the points $\mathrm{A}(x + y, y - x)$ and $\mathrm{B}\ (x - y, x + y)$ prove that $ay = by$.
6. Prove that area of a triangle with the vertices $(\ p, p - 2\ ), (\ p + 2, p + 2\ )$ and $(p + 3, p)$ is independent of p.
7. Points $\mathrm{A}(-1, y)$ and $\mathrm{B}(5,7)$ lie on a circle with centre $\mathrm{O}(2, -3y)$. Find the values of y. Hence find the radius of the circle.
8. $A(4, -6), B(3, -2)$ and $C(5,2)$ are the vertices of triangle ABC and AD is its median. Prove that the median AD divides triangle ABC into two triangles of equal areas.
9. The mid-point P of the line segment joining the points $A(-10,4)$ and $B(-2,0)$ lies on the line segment joining the points $C(-9, -4)$ and $D(-4, y)$. Find the ratio in which P divides CD. Also find the value of y.
10. Determine the ratio in which a line $3x + y - 9 = 0$, divides the segment joining points $(1,3)$ and $(2,7)$.

12. INTRODUCTION TO TRIGONOMETRY

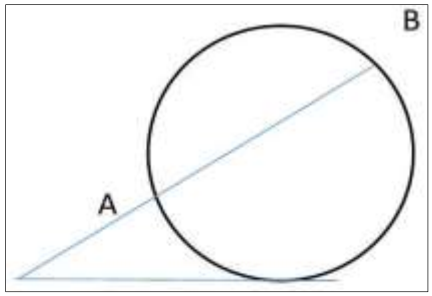
1. If $\tan\,(\theta + \emptyset) = \frac{\tan\,\theta + \tan\,\emptyset}{1 - \tan\,\theta \cdot \tan\,\phi}$ such that $\tan\,\theta = 1/2$ and $\tan\,\emptyset = 1/3$ find the value of $\tan\,(\theta + \emptyset)$.
2. If $\tan\,\mathrm{A} = \mathrm{ntanB}$ and $\sin\,\mathrm{A} = \mathrm{msin}\,\,\mathrm{B}$, prove that $\cos^2\,\,\mathrm{A} = \frac{m^2 - 1}{n^2 - 1}$.
3. If $2\cos\,\theta - \sin\,\theta = x$ and $\cos\,\theta - 3\sin\,\theta = y$, prove that $2\mathrm{x}^2 + \mathrm{y}^2 - 2\mathrm{xy} = 5$
4. If $\frac{x}{a}\cos\,\theta + \frac{y}{b}\sin\,\theta = 1$ and $\frac{x}{a}\sin\,\theta - \frac{y}{b}\cos\,\theta = 1$, prove that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.
5. If $\sqrt{3}\cot^2\,\theta - 4\cot\,\theta + \sqrt{3} = 0$, find the value of $\cot^2\,\theta + \tan^2\,\theta$.
6. Prove that $(\sin\,\theta + \mathrm{cosec}\theta)^2 + (\cos\,\theta + \sec\,\theta)^2 - \tan^2\,\theta - \cot^2\,\theta = 7$.
7. If $\mathrm{a}\mathrm{cosec}\theta + \mathrm{b}\sec\theta + c = 0$ and $p\mathrm{cosec}\theta + q\sec\,\theta + r = 0$ then prove that $\frac{1}{(br - cq)^2} + \frac{1}{(cp - ar)^2} = \frac{1}{(aq - bp)^2}$.
8. If $7\mathrm{cosec}\theta - 3\cot\,\theta = 7$, prove that $7\cot\,\theta - 3\mathrm{cosec}\theta = \pm 3$.
9. If $\cot\,\theta + \tan\,\theta = \mathrm{x}$ and $\sec\,\theta + \cos\,\theta = \mathrm{y}$, prove that $(x^2 y)^{2/3} - (xy^2)^{2/3} = 1$.
10. Prove that : $\frac{\sec\,\theta + \tan\,\theta - 1}{\tan\,\theta - \sec\,\theta + 1} = \frac{\cos\,\theta}{1 - \sin\,\theta}$.
11. $\cos\,\theta + \sin\,\theta = \sqrt{2}\cos\,\theta$, show that $\cos\,\theta - \sin\,\theta = \sqrt{2}\sin\,\theta$.
12. If $\mathrm{a}\cos\theta - b\sin\,\theta = c$, prove that $\mathrm{a}\sin\theta + b\cos\,\theta = \sqrt{(a^2 + b^2 - c^2)}$.
13. If $\frac{1 - \tan\,\theta}{1 + \tan\,\theta} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$, show that $\frac{\sin\,\theta}{\cos\,2\theta} = 1$

SOME APPLICATION OF TRIGONOMETRY

1. Find the height of a chimney, when it is found that on walking towards it 50 m in a horizontal line through its base, the angular elevation of its top changes from 30° to 45°.
2. Two poles of equal heights are standing opposite to each other on either side of a road, which is 100 m wide. From a point between them on the road, the angles of elevation of the tops are 30° and 60°. Find the height of each pole.
3. At a point on a level ground, the angle of elevation of a vertical tower is found to be such that it's tangent is $5/12$. On walking 192 m towards the tower, the tangent of the angle of elevation is $3/4$. Find the height of the tower.
4. An aeroplane is flying 300 m high passes vertically above another aeroplane at a distance when the angles of elevation of two aeroplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two aeroplanes.
5. A bird is sitting on the top of a tree which is 80 m high. The angle of elevation of the bird, from a point on the ground is 45^0. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird becomes 30°. Find the speed of the flying bird.
6. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of
depression of the base of the hill as 30°.calculate the distance of the hill from the ship and height of the hill.
7. The angle of elevation of the top of a tower from a point on the same level as the foot of the tower is α.On advancing ' p ' meters towards the foot of the tower, the angle of elevation becomes β. Show that the height ' h ' of the tower given by
$$\mathrm{h}=\frac{p\tan \alpha \cdot \tan \beta}{\tan \beta-\tan \alpha}.$$
8. If the angle of elevation of a cloud from a point h metre above a lake is α and the angle of depression of its reflection in the lake is β. Prove that the height of the cloud is $\frac{h(\tan \alpha+\tan \beta)}{\tan \beta-\tan \alpha}$.
9. The angle of elevation of the top of a tower from a point A due south of the tower is α and from B due east of the tower is β. If $AB=d$, show that the height of the tower is $\frac{d}{\sqrt{\cot^2 \alpha+\cot^2 \beta}}$.
10. The elevation of a tower at a station A due north of it is α and at a station B due west of A is β. Prove that the height of the tower is $\frac{AB\sin \alpha\sin \beta}{\sqrt{\sin^2 \alpha-\sin^2 \beta}}$.
11. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height h. At a point on the plane, the angles of elevation of the bottom and the top of the flag-staff are α and β respectively. Prove that the height of the tower is $\frac{h\tan \alpha}{\tan \beta-\tan \alpha}$.

CIRCLES

1. QR is a tangent at Q. PR||AQ, where AQ is a chord through A and P is a centre, the end point of the diameter AB. Prove that BR is a tangent at B.
2. A circle is touching the side BC of triangle ABC at P and touching AB and AC produced at Q and R respectively. Prove that: $\mathrm{AQ}=\frac{1}{2}$(Perimeter of $\triangle$ ABC)
3. In the given figure PT is a tangent to the circle at T. If $\mathrm{PA}=4$ cm and $\mathrm{AB}=5$ cm, find PT.



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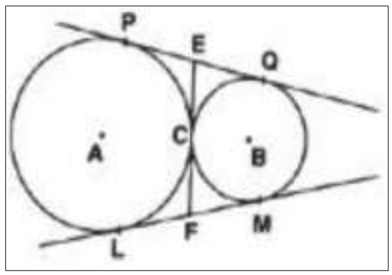
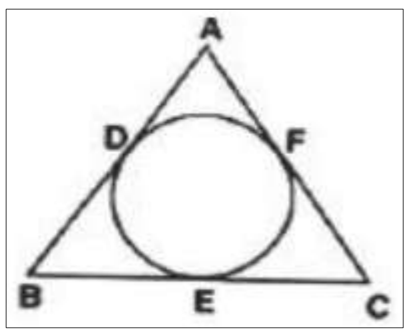
4. In the following figure, two circle touch each other externally at C. Prove that the common tangent at C bisects the other two common tangents.

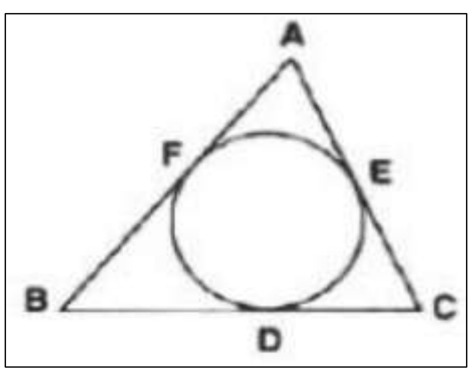


5. In the figure, if $AB = AC$, prove that $BE = CE$.

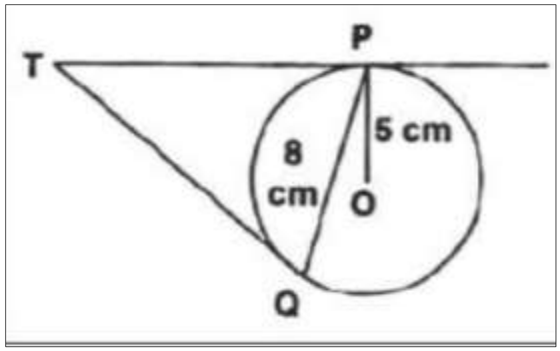


6. The incircle of triangle ABC touches the sides BC, CA and AB at D, E and F respectively.

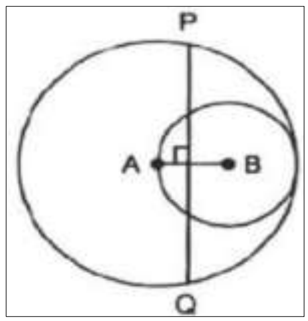
Show that $\mathrm{AF} + \mathrm{BD} + \mathrm{CD} = \mathrm{AE} + \mathrm{BF} + \mathrm{CE} = \frac{1}{2}$ (Perimeter of $\triangle$ ABC)



8. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of TP.

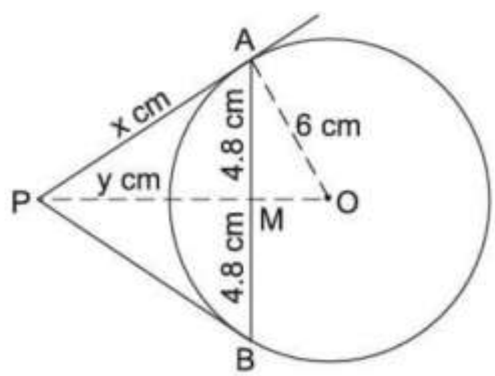


9. In the figure, two circles with centres A and B and radii 5 cm and 3 cm touching each other internally. If the perpendicular bisector of segment AB, meets the bigger circle at P and Q, find the length of PQ.

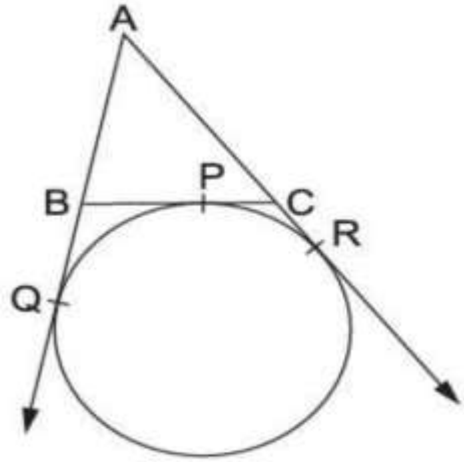


10. Two tangents making an angle of $120°$ with each other, are drawn to a circle of radius 6 cm. Show that the length of each tangent is $2\sqrt{3}$ cm.

11. In the given figure, AB is a chord of length 9.6 cm of a circle with centre O and radius 6 cm. The tangents at A and B intersect at P. Find the length of PA. [CBSE 2009C, '13C, '15]

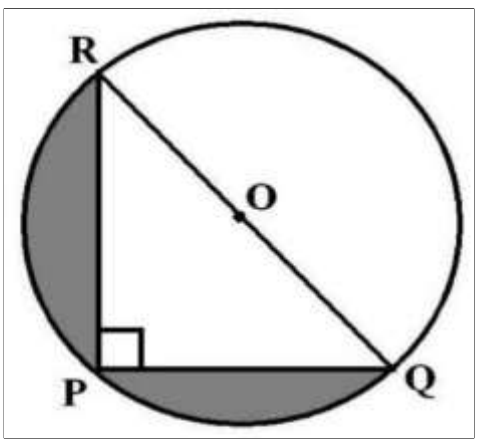


12. A circle is touching the side BC of $\triangle ABC$ at P and touching AB and AC produced at Q and R respectively. Prove that $AQ = \frac{1}{2}$(perimeter of $\triangle ABC$). [CBSE 2001, '02, '06, '14, '17]

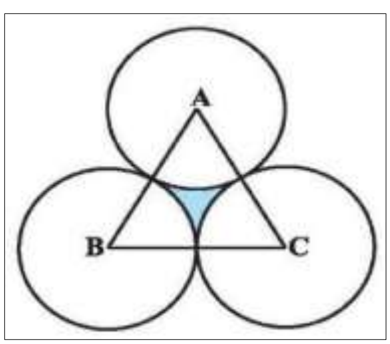


AREAS RELATED TO CIRCLES

1. If the perimeter and the area of a circle are numerically equal, then find the radius of the circle.
2. Find the area of the shaded region in figure, if $\mathrm{RP} = 24$ cm and $\mathrm{PQ} = 10$ cm and O is the centre of circle. (take $\pi = 3.14$)

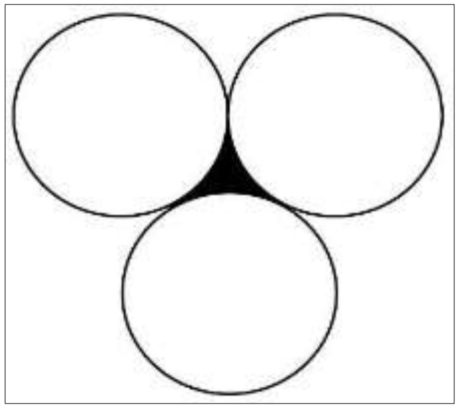


3. The area of an equilateral triangle is $49\sqrt{3}$ cm^2. Taking each angular points centre, circles are drawn with radius equal to half the length of the side of the triangle. Find the area of the triangle

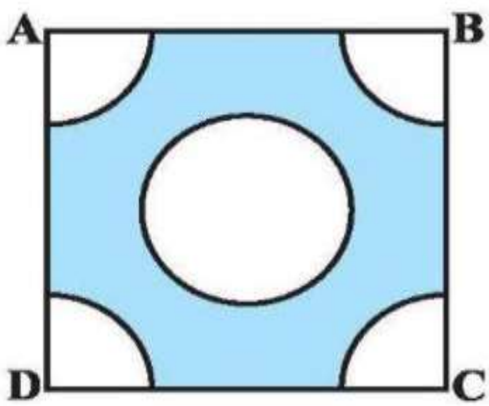


not included in the circles. (Take $\sqrt{3} = 1.73$)

4. In the adjoining figure, three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these three circles.



5. From each corner of a square of side 4 cma quadrant of a circle of radius 1 cm is cutand also a circle of diameter 2 cm is cut as shown in above sided Fig. Find the area of the remaining portion of the square.



SURFACE AREAS AND VOLUMES

1. A cone, a hemisphere and a cylinder stand on equal bases and have the same height what is the ratio of their volumes?
2. The slant height of the frustum of a cone is 5 cm if the difference between the radii of its two circular ends is 4 cm, find the height of the frustum.
   3. The radius and height of a solid right circular cone are in the ratio of 5: 12. If its volume is 314 cm^3, find its total surface area.
3. A well of diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly all around it to a width of 5 m to form an embankment. Find the height of the embankment.
4. An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part in 36 cm high. Find the weight of the pillar if one cu. cm of iron weighs 7.8 grams.
5. An open container made up of a metal sheet is in the form of a frustum of a cone of height 8 cm with radii of its lower and upper ends as 4 cm and 10 cm respectively. Find the cost of oil which can

completely fill the container at the rate of Rs. 50 per litre. Also, find the cost of metal used, if it costs Rs. 5 per 100 cm^2.

6. A building is in the form of a cylinder surmounted by a hemispherical dome as shown in the figure. The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}$ m^3 of air.
7. The diameter of a sphere is 28 cm. Find the cost of painting it all around at Rs. 0.10 per square cm.
8. The perimeter of one face of a wooden cube is 20 cm. Find its weight if 1 cm^3 of wood weighs 8.25 g.
9. The radii of two cylinders are in the ratio of $1:\sqrt{3}$. If the volumes of two cylinders be same, find the ratio of their respective heights.
10. If the radius of the base of a cone is doubled keeping the height same. What is the ratio of the volume of the larger cone to the smaller cone?
11. If the length, breadth and height of a solid cube are in the ratio $4:3:2$ and total surface area is $832\ cm^2$. Find its volume.
12. Three cubes of a metal whose edges are in the ratio $3:4:5$ are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.
13. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and diameter of the base is 8 cm. Determine the volume of the toy. If the cube circumscribes the toy, then find the difference of the volumes of the cube and the toy. Also, find the total surface area of the toy.

4. STATISTICS

1. What measure of central tendency is used to obtain graphically as the x coordinate of meeting point of the two ogives for grouped data?

2. The average weight of students in 4 sections A, B, C and D is 60 kg. The average weights of the students of A, B, C and D individually are 45 kg, 50 kg, 72 kg and 80 kg respectively. If the average weight of the students of section A and B together is 48 kg and that of the students of B and C together is 60 kg, what is the ratio of the number of students in section A and D ?

3. The mode of a distribution is 55 & the modal class is $45 - 60$ and the frequency preceding the modal class is 5 and the frequency after the modal class is 10. Find the frequency of the modal class.

4. The Median and Mode of the following wage distribution are known to be Rs. 33.5 and Rs. 34 respectively. Three frequency values from the table, however are missing. Find the missing frequencies.

Wages	0 – 10	10 - 20	20 - 30	30 - 40	40 - 50	50 60	60 70	Total 70
No.of persons	4	16	X	Y	Z	6	4	230

5. The mode of a distribution is 55 and mode class is 45-60 and the frequency preceding the modal class is 5 and the frequency after the modal class is 10. Find the frequency of the modal class.

5. PROBABILITY

1. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.

2. Two dice are thrown simultaneously. What is the probability that
(i) 5 will not come up on either of them?
(ii) 5 will come up at least one?
(iii) 5 will come up at both dice?

3. From a well shuffled pack of playing cards, black jacks, black kings and black aces are removed. A card is then drawn from the pack. Find the probability of getting.
(i) a red card
(ii) not a diamond card.

4.A bag contains cards which are numbered from 2 to 90. A card is drawn at random from the bag. Find the probability that it bears.
(i) a two-digit number
(ii) a number which is a perfect cube.

5. Find the probability of getting 53 Mondays in
I) a leap year
ii) a non-leap year.

6.A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.

7.A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue balls in the jar.

8. An integer is chosen between 0 and 100. what is the probability that it is divisible by 2 or 3.

9. Two dice are numbered 1,2,3,4,5,6 and 1,2,2,3,3,4 respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting-
i) sum 7
ii) sum as perfect square

10.A bag contains 24 balls out of which x are white. If one ball is drawn at random the probability of drawing a white ball is y. 12 more white balls are added to the bag. Now if a ball is drawn from the bag, the probability of drawing the white ball is $\frac{5}{3}y$. Find the value of x.

11.A number is selected at random from the numbers 3,5,5,7,7,7,9,9,9,9. Find the probability that the selected number is their average.

12.A number x is chosen from the numbers 1,2,3 and a number y is selected from the numbers 1,4,9. Find the probability that $xy = 10$.

13.The probability of guessing the correct answer to a certain test is $\frac{p}{12}$. If the probability of not guessing the correct answer to this question is $\frac{1}{3}$, finf the value of p.