Activity 2

Objective

To draw the graph of a quadratic polynomial and observe:

(i) The shape of the curve when the coefficient of x^2 is positive.

(ii) The shape of the curve when the coefficient of x^2 is negative.

(iii) Its number of zeroes.

Material Required

Cardboard, graph paper, ruler, pencil, eraser, pen, adhesive.

Method of Construction

1. Take cardboard of a convenient size and paste a graph paper on it.
2. Consider a quadratic polynomial $f(x) = ax^2 + bx + c$
3. Two cases arise:

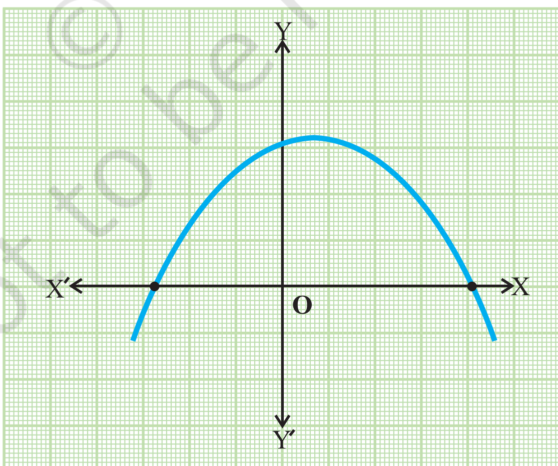

Fig. 1

(i) $a > 0$ (ii) $a < 0$

4. Find the ordered pairs $(x, f(x))$ for different values of x.

5. Plot these ordered pairs in the cartesian plane.

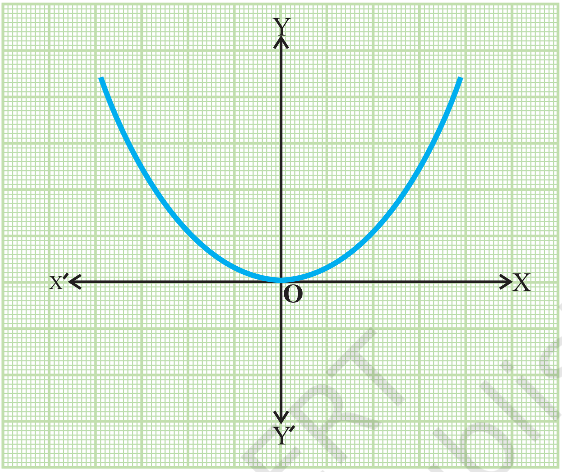

Fig. 2

6. Join the plotted points by a free hand curve [Fig. 1, Fig. 2 and Fig. 3].

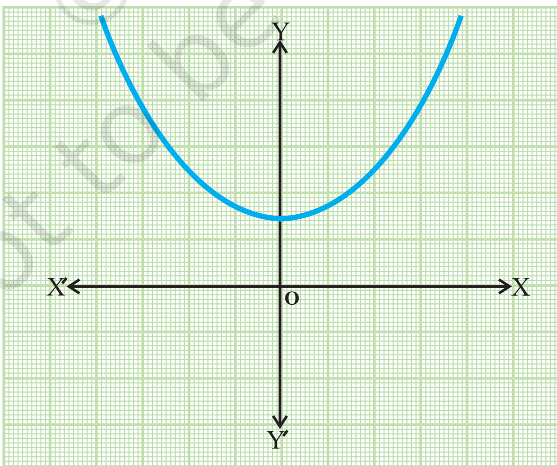

Fig. 3

DEMONSTRATION

1. The shape of the curve obtained in each case is a parabola.
2. Parabola opens upward when coefficient of x^2 is positive [see Fig. 2 and Fig. 3].
3. It opens downward when coefficient of x^2 is negative [see Fig. 1].
4. Maximum number of zeroes which a quadratic polynomial can have is 2.

OBSERVATION

1. Parabola in Fig. 1 opens ______
2. Parabola in Fig. 2 opens _______
3. In Fig. 1, parabola intersects x-axis at ______ point(s).
4. Number of zeroes of the given polynomial is ________.
5. Parabola in Fig. 2 intersects x-axis at ______ point(s).
6. Number of zeroes of the given polynomial is ______.
7. Parabola in Fig.3 intersects x-axis at ______ point(s).
8. Number of zeroes of the given polynomial is _______.
9. Maximum number of zeroes which a quadratic polynomial can have is ___________.

APPLICATION

This activity helps in

1. understanding the geometrical representation of a quadratic polynomial
2. finding the number of zeroes of a quadratic polynomial.

NOTE

Points on the graph paper should be joined by a free hand curve only.

Activity 3

OBJECTIVE

To verify the conditions of consistency/ inconsistency for a pair of linear equations in two variables by graphical method.

MATERIAL REQUIRED

Graph papers, pencil, eraser, cardboard, glue.

METHOD OF CONSTRUCTION

1. Take a pair of linear equations in two variables of the form

   $a_1x + b_1y + c_1 = 0$ (1)

   $a_2x + b_2y + c_2 = 0,$ (2)

   where a_1, b_1, a_2, b_2, c_1 and c_2 are all real numbers; a_1, b_1, a_2 and b_2 are not simultaneously zero.

There may be three cases :

Case I : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Case II: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Case III: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

2. Obtain the ordered pairs satisfying the pair of linear equations (1) and (2) for each of the above cases.

3. Take a cardboard of a convenient size and paste a graph paper on it. Draw two perpendicular lines X′OX and YOY′ on the graph paper (see Fig. 1). Plot the points obtained in Step 2 on different cartesian planes to obtain different graphs [see Fig. 1, Fig. 2 and Fig.3].

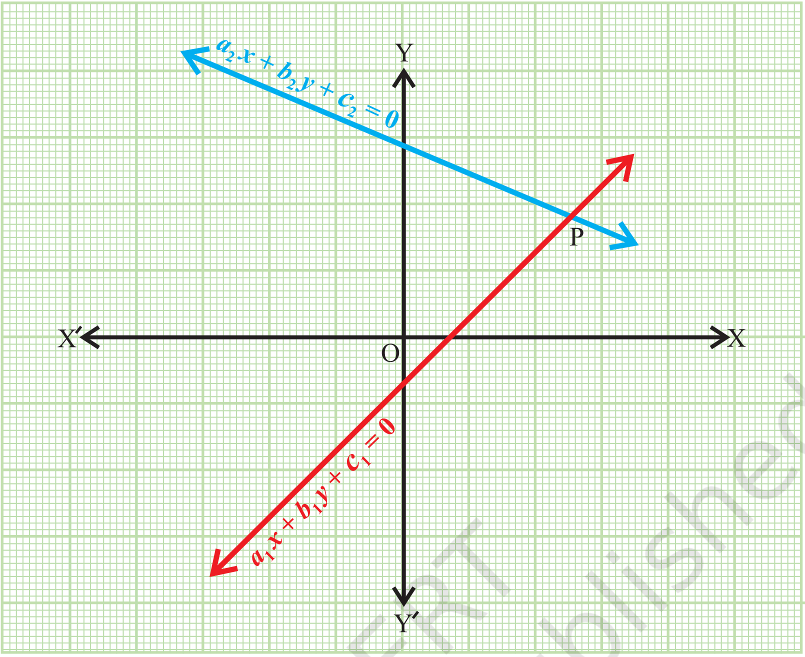

Fig. 1

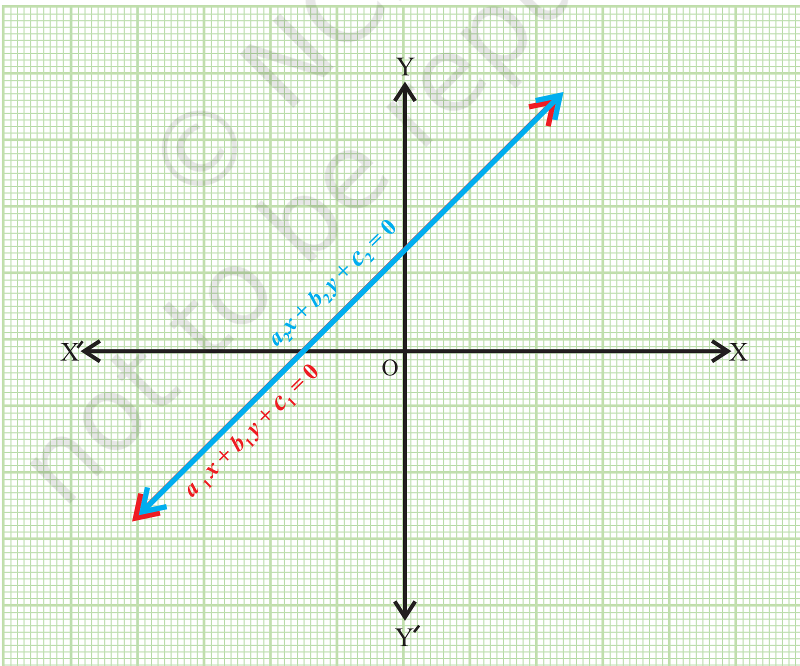

Fig. 2

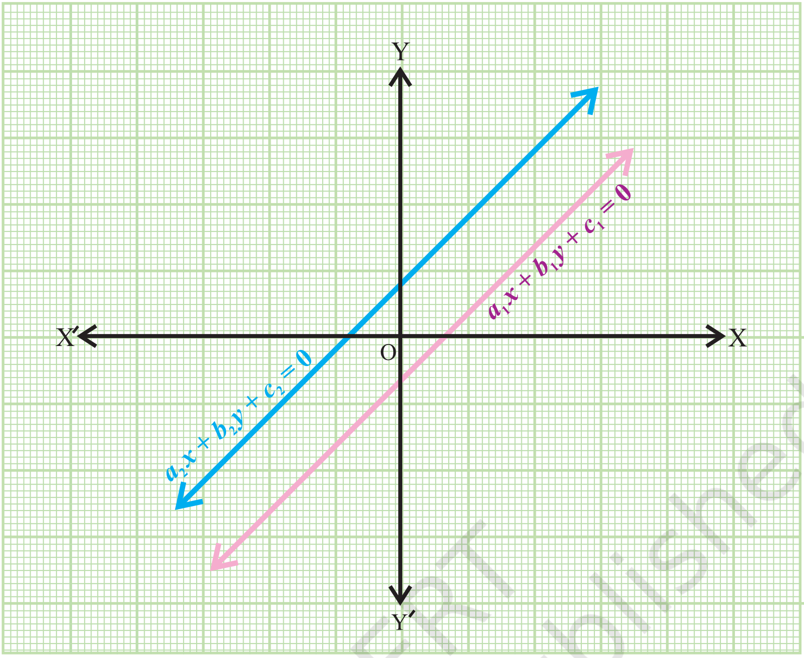

Fig. 3

DEMONSTRATION

Case I: We obtain the graph as shown in Fig. 1. The two lines are intersecting at one point P. Co-ordinates of the point P (x,y) give the unique solution for the pair of linear equations (1) and (2).

Therefore, the pair of linear equations with $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ is consistent and has the unique solution.

Case II: We obtain the graph as shown in Fig. 2. The two lines are coincident. Thus, the pair of linear equations has infinitely many solutions.

Therefore, the pair of linear equations with $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ is also consistent as well as dependent.

Case III: We obtain the graph as shown in Fig. 3. The two lines are parallel to each other.

This pair of equations has no solution, i.e., the pair of equations with $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ is inconsistent.

OBSERVATION

1. a_1 = ___________, a_2 = ___________,

b_1 = ___________, b_2 = ___________,

c_1 = ___________, c_2 = ___________,

So, $\frac{a_1}{a_2}$ = ..., $\frac{b_1}{b_2}$ = ..., $\frac{c_1}{c_2}$ = ...

$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Case I, II or III	Type of lines	Number of solution	Conclusion Consistent/ inconsistent/ dependent

APPLICATION

Conditions of consistency help to check whether a pair of linear equations have solution (s) or not.

In case, solutions/solution exist/exists, to find whether the solution is unique or the solutions are infinitely many.

Activity 5

Objective

To identify Arithmetic Progressions in some given lists of numbers (patterns).

Material Required

Cardboard, white paper, pen/pencil, scissors, squared paper, glue.

Method of Construction

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Take two squared papers (graph paper) of suitable size and paste them on the cardboard.

Fig. 1

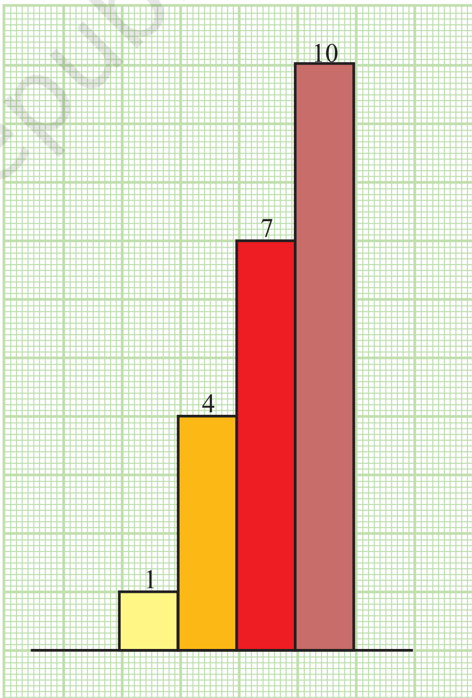

Fig. 2

3. Let the lists of numbers be

   (i) 1, 2, 5, 9, ...   (ii) 1, 4, 7, 10, ...

4. Make strips of lengths 1, 2, 5, 9 units and strips of lengths 1, 4, 7, 10 units and breadth of each strip one unit.

5. Paste the strips of lengths 1, 2, 5, 9 units as shown in Fig. 1 and paste the strips of lengths 1, 4, 7, 10 units as shown in Fig. 2.

DEMONSTRATION

1. In Fig. 1, the difference of heights (lengths) of two consecutive strips is not same (uniform). So, it is not an AP.

2. In Fig. 2, the difference of heights of two consecutive strips is the same (uniform) throughout. So, it is an AP.

OBSERVATION

In Fig. 1, the difference of heights of first two strips = _____________

the difference of heights of second and third strips = _____________

the difference of heights of third and fourth strips = _____________

Difference is _____________ (uniform/not uniform)

So, the list of numbers 1, 2, 5, 9 ____________ form an AP. (does/does not)

Write the similar observations for strips of Fig.2.

Difference is _____________ (uniform/not uniform)

So, the list of the numbers 1, 4, 7, 10 _________ form an AP. (does/does not)

APPLICATION

This activity helps in understanding the concept of arithmetic progression.

NOTE

Observe that if the left top corners of the strips are joined, they will be in a straight line in case of an AP.

Activity 6

Objective

To find the sum of first n natural numbers.

Material Required

Cardboard, coloured papers, white paper, cutter, adhesive.

Method of Construction

1. Take a rectangular cardboard of a convenient size and paste a coloured paper on it. Draw a rectangle ABCD of length 11 units and breadth 10 units.
2. Divide this rectangle into unit squares as shown in Fig. 1.
3. Starting from upper left-most corner, colour one square, 2 squares and so on as shown in the figure.

Demonstration

1. The pink colour region looks like a stair case.
2. Length of 1st stair is 1 unit, length of 2nd stair is 2 units, length of 3rd stair 3 units, and so on, length of 10th stair is 10 units.

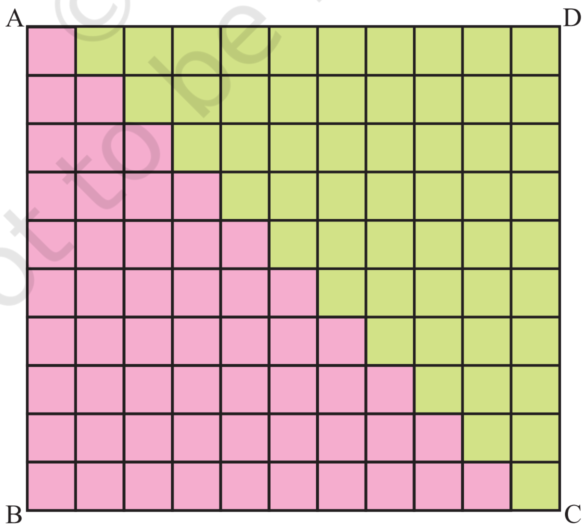

Fig. 1

3. These lengths give a pattern

   1, 2, 3, 4, ..., 10,

   which is an AP with first term 1 and common difference 1.

4. Sum of first ten terms

   $= 1 + 2 + 3 + ... + 10 = 55$ (1)

   Area of the shaded region $= \frac{1}{2}$ (area of rectangle ABCD)

   $= \frac{1}{2} \times 10 \times 11$, which is same as obtained in (1) above. This shows that the sum of the first 10 natural numbers is $\frac{1}{2} \times 10 \times 11 = \frac{1}{2} \times 10(10+1)$.

   This can be generalised to find the sum of first n natural numbers as

   $$S_n = \frac{1}{2} n(n+1) \qquad (2)$$

Observation

For $n = 4$, S_n = ...

For $n = 12$, S_n = ...

For $n = 50$, S_n = ...

For $n = 100$, S_n = ...

Application

Result (2) may be used to find the sum of first n terms of the list of numbers:

1. $1^2, 2^2, 3^2, ...$
2. $1^3, 2^3, 3^3, ...$

   to be studied in Class XI.

Activity 10

Objective

To verify the distance formula by graphical method.

Material Required

Cardboard, chart paper, graph paper, glue, pen/pencil and ruler.

Method of Construction

1. Paste a chart paper on a cardboard of a convenient size.
2. Paste the graph paper on the chart paper.
3. Draw the axes X′OX and Y′OY on the graph paper [see Fig. 1].
4. Take two points A(a, b) and B(c,d) on the graph paper and join them to get a line segment AB [see Fig. 2].

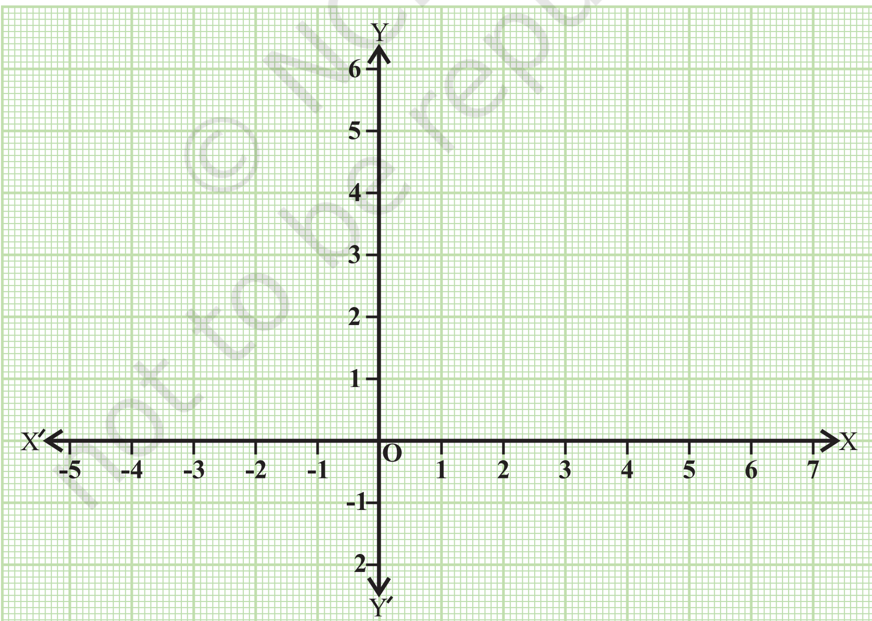

Fig. 1

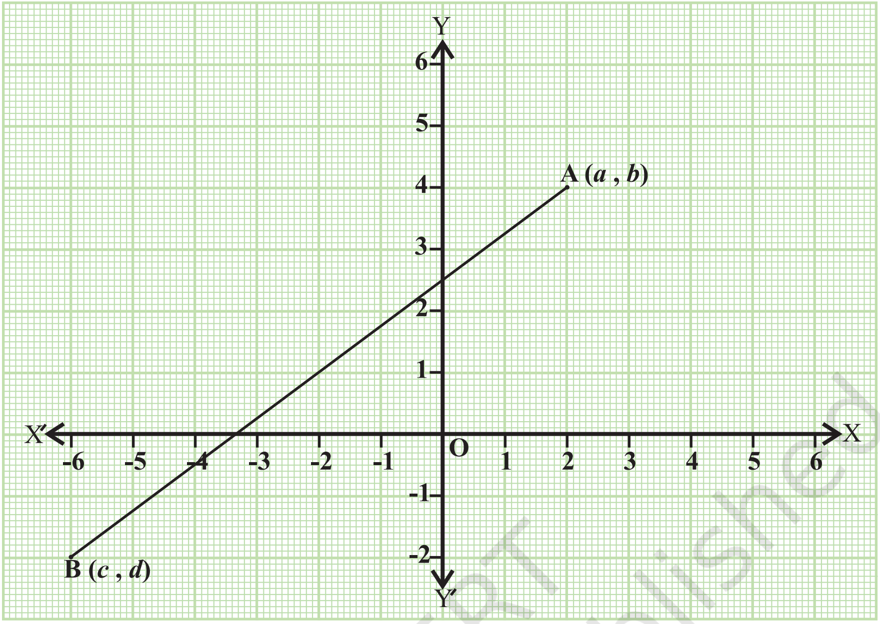

Fig. 2

DEMONSTRATION

1. Calculate the distance AB using distance formula.
2. Measure the distance between the two points A and B using a ruler.
3. The distance calculated by distance formula and distance measured by the ruler are the same.

OBSERVATION

1. Coordinates of the point A are ________________.

   Coordinates of the point B are ________________.
2. Distance AB, using distance formula is _______________.
3. Actual distance AB measured by ruler is _______________.
4. The distance calculated in (2) and actual distance measured in (3) are _____.

APPLICATION

The distance formula is used in proving a number of results in geometry.

Activity 11

OBJECTIVE

To verify section formula by graphical method.

MATERIAL REQUIRED

Cardboard, chart paper, graph paper, glue, geometry box and pen/pencil.

METHOD OF CONSTRUCTION

1. Paste a chart paper on a cardboard of a convenient size.
2. Paste a graph paper on the chart paper.
3. Draw the axes X′OX and Y′OY on the graph paper [see Fig. 1].

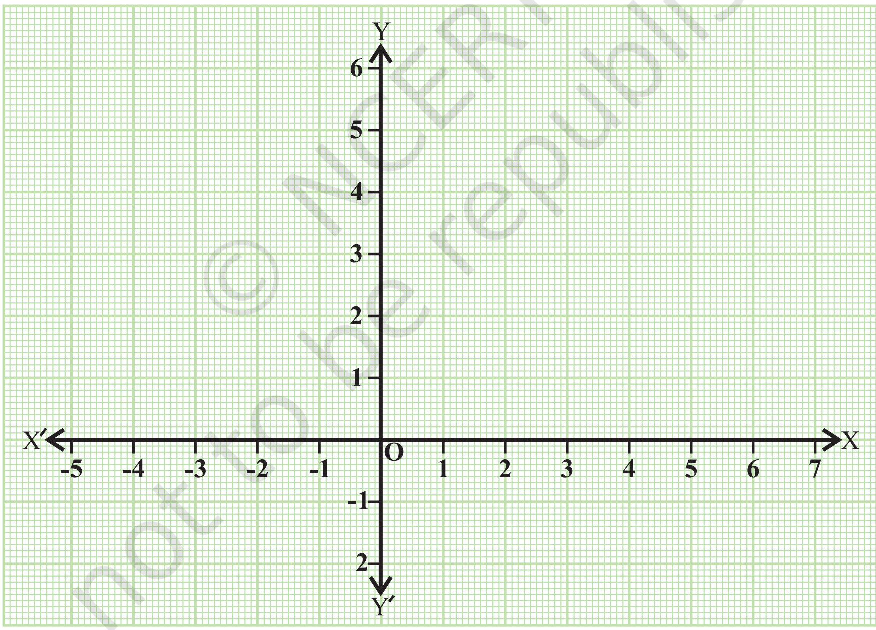

Fig. 1

4. Take two points A (x_1, y_1) and B (x_2, y_2) on the graph paper [see Fig. 2].
5. Join A to B to get the line segment AB.

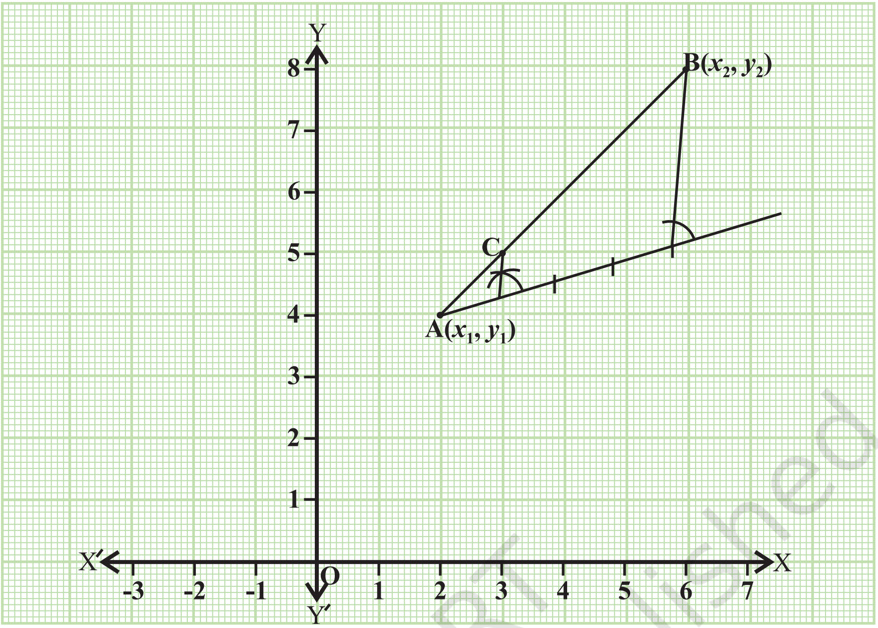

Fig. 2

DEMONSTRATION

1. Divide the line segment AB internally (in the ratio $m:n$) at the point C [see Fig. 2].
2. Read the coordinates of the point C from the graph paper.
3. Using section formula, find the coordinates of C.
4. Coordinates of C obtained from Step 2 and Step 3 are the same.

OBSERVATION

1. Coordinates of A are __________.
2. Coordinates of B are __________.
3. Point C divides AB in the ratio __________.
4. Coordinates of C from the graph paper are __________.
5. Coordinates of C by using section formula are __________.
6. Coordinates of C from the graph paper and from section formula are ______.

APPLICATION

This formula is used to find the centroid of a triangle in geometry, vector algebra and 3–dimensional geometry.

Activity 13

Objective

To establish the criteria for similarity of two triangles.

Material Required

Coloured papers, glue, sketch pen, cutter, geometry box .

Method of Construction

I

1. Take a coloured paper/chart paper. Cut out two triangles ABC and PQR with their corresponding angles equal.

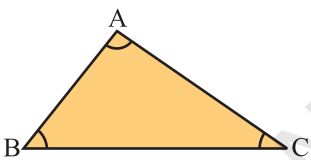

Fig. 1

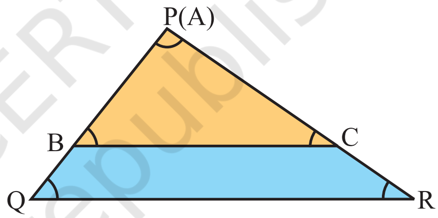

Fig. 2

2. In the triangles ABC and PQR, ∠A = ∠P; ∠B = ∠Q and ∠C = ∠R.
3. Place the ΔABC on ΔPQR such that vertex A falls on vertex P and side AB falls along side PQ (side AC falls along side PR) as shown in Fig. 2.

Demonstration I

1. In Fig. 2, ∠B = ∠Q. Since corresponding angles are equal, BC||QR
2. By BPT, $\frac{PB}{BQ} = \frac{PC}{CR}$ or $\frac{AB}{BQ} = \frac{AC}{CR}$

or $\frac{BQ}{AB} = \frac{CR}{AC}$

or $$\frac{BQ + AB}{AB} = \frac{CR + AC}{AC}$$ [Adding 1 to both sides]

or $$\frac{AQ}{AB} = \frac{AR}{AC} \text{ or } \frac{PQ}{AB} = \frac{PR}{AC} \text{ or } \frac{AB}{PQ} = \frac{AC}{PR} \qquad (1)$$

II

1. Place the ΔABC on ΔPQR such that vertex B falls on vertex Q, and side BA falls along side QP (side BC falls along side QR) as shown in Fig. 3.

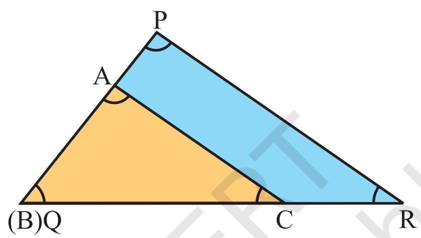

Fig. 3

DEMONSTRATION II

1. In Fig. 3, $\angle C = \angle R$. Since corresponding angles are equal, $AC \parallel PR$

2. By BPT, $\frac{AP}{AB} = \frac{CR}{BC}$; or $\frac{BP}{AB} = \frac{BR}{BC}$ [Adding 1 on both sides]

or $$\frac{PQ}{AB} = \frac{QR}{BC} \text{ or } \frac{AB}{PQ} = \frac{BC}{QR} \qquad (2)$$

From (1) and (2), $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

Thus, from Demonstrations I and II, we find that when the corresponding angles of two triangles are equal, then their corresponding sides are proportional. Hence, the two triangles are similar. This is AAA criterion for similarity of triangles.

Alternatively, you could have measured the sides of the triangles ABC and PQR and obtained

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}.$$

From this result, ΔABC and ΔPQR are similar, i.e., if three corresponding angles are equal, the corresponding sides are proportional and hence the triangles are similar. This gives AAA criterion for similarity of two triangles.

III

1. Take a coloured paper/chart paper, cut out two triangles ABC and PQR with their corresponding sides proportional.

   i.e., $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

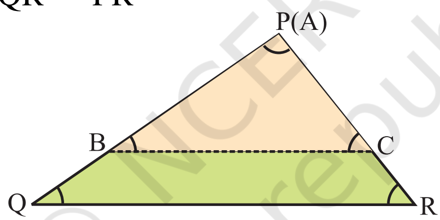

Fig. 4

2. Place the ΔABC on ΔPQR such that vertex A falls on vertex P and side AB falls along side PQ. Observe that side AC falls along side PR [see Fig. 4].

Demonstration III

1. In Fig. 4, $\frac{AB}{PQ} = \frac{AC}{PR}$. This gives $\frac{AB}{BQ} = \frac{AC}{CR}$. So, BC||QR (by converse of BPT)

   i.e., ∠B = ∠Q and ∠C = ∠R. Also ∠A = ∠P. That is, the corresponding angles of the two triangles are equal.

   Thus, when the corresponding sides of two triangles are proportional, their corresponding angles are equal. Hence, the two triangles are similar. This is the SSS criterion for similarity of two triangles.

Alternatively, you could have measured the angles of ΔABC and ΔPQR and obtained ∠A = ∠P, ∠B = ∠Q and ∠C = ∠R.

From this result, ΔABC and ΔPQR are similar, i.e., if three corresponding sides of two triangles are proportional, the corresponding angles are equal, and hence the triangles are similar. This gives SSS criterion for similarity of two triangles.

IV

1. Take a coloured paper/chart paper, cut out two triangles ABC and PQR such that their one pair of sides is proportional and the angles included between the pair of sides are equal.

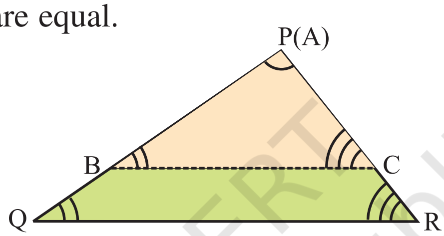

Fig. 5

i.e., In ΔABC and ΔPQR, $\frac{AB}{PQ} = \frac{AC}{PR}$ and ∠A = ∠P.

2. Place triangle ABC on triangle PQR such that vertex A falls on vertex P and side AB falls along side PQ as shown in Fig. 5.

Demonstration IV

1. In Fig. 5, $\frac{AB}{PQ} = \frac{AC}{PR}$. This gives $\frac{AB}{BQ} = \frac{AC}{CR}$. So, BC||QR (by converse of BPT)

   Therefore, ∠B = ∠Q and ∠C = ∠R.

   From this demonstration, we find that when two sides of one triangle are proportional to two sides of another triangle and the angles included between the two pairs of sides are equal, then corresponding angles of two triangles are equal.

Hence, the two triangles are similar. This is the SAS criterion for similarity of two triangles.

Alternatively, you could have measured the remaining sides and angles of ΔABC and ΔPQR and obtained ∠B = ∠Q, ∠C = ∠R and

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}.$$

From this, ΔABC and ΔPQR are similar and hence we obtain SAS criterion for similarity of two triangles.

Observation

By actual measurement:

I. In ΔABC and ΔPQR,

∠A = ______, ∠P = ______, ∠B = ______, ∠Q = ______, ∠C = ______, ∠R = ______,

$\frac{AB}{PQ}$ = _______; $\frac{BC}{QR}$ = _________; $\frac{AC}{PR}$ = _________

If corresponding angles of two triangles are _________, the sides are ________. Hence the triangles are _________.

II. In ΔABC and ΔPQR

$\frac{AB}{PQ}$ = _______; $\frac{BC}{QR}$ = _________; $\frac{AC}{PR}$ = _________

∠A = _______, ∠B = _______, ∠C = _______, ∠P = _______,

∠Q = _______, ∠R = _______.

If the corresponding sides of two triangles are ______, then their corresponding angles are ________. Hence, the triangles are ________.

III. In ΔABC and ΔPQR ,

$\frac{AB}{PQ}$ = ________; $\frac{AC}{PR}$ = __________

$\angle A$ = ________, $\angle P$ = ________, $\angle B$ = ________, $\angle Q$ = ________,

$\angle C$ = ________, $\angle R$ = ________.

If two sides of one triangle are _______ to the two sides of other triangle and angles included between them are ________, then the triangles are ________.

Application

The concept of similarity is useful in reducing or enlarging images or pictures of objects.

Activity 16

OBJECTIVE

To verify Basic Proportionality Theorem (Thales theorem).

MATERIAL REQUIRED

Two wooden strips (each of size 1 cm wide and 30 cm long), cutter, adhesive, hammer, nails, bard board, white paper, pulleys, thread, scale and screw etc.

METHOD OF CONSTRUCTION

1. Cut a piece of hardboard of a convenient size and paste a white paper on it.

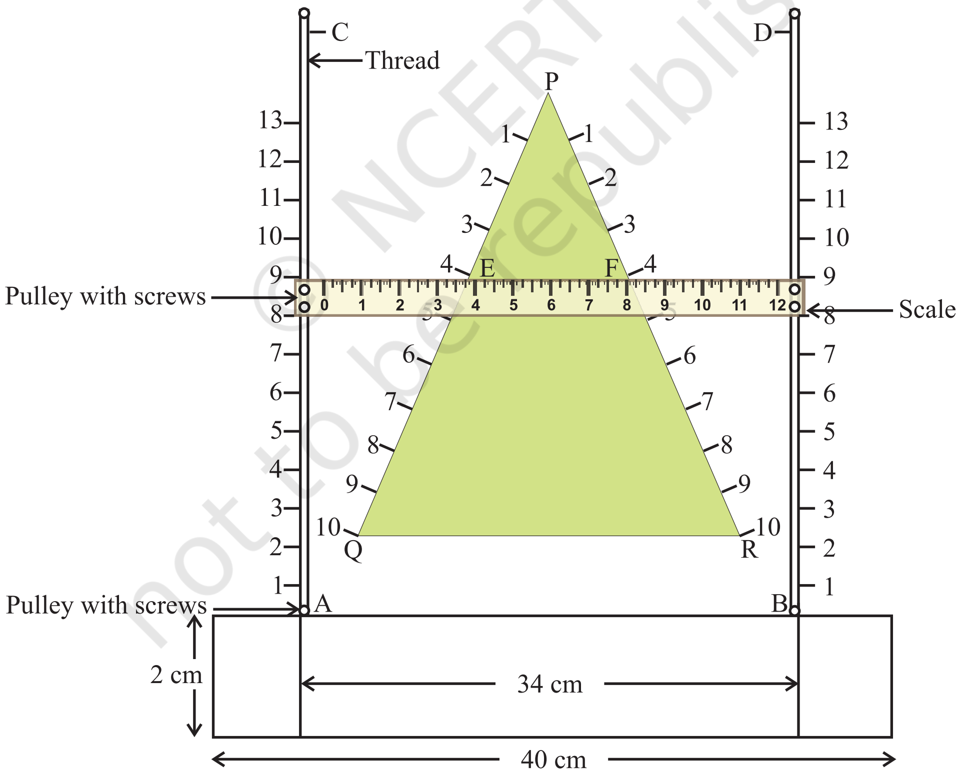

Fig. 1

2. Take two thin wooden strips with markings 1, 2, 3, ... at equal distances and fix them vertically on the two ends of the horizontal strip as shown in Fig. 1 and call them AC and BD.
3. Cut a triangular piece PQR from hardboard (thickness should be negligible) and paste coloured glazed paper on it and place it between the parallel strips AC and BD such that its base QR is parallel to the horizontal strip AB as drawn in Fig. 1.
4. Graduate the other two sides of the triangular piece as shown in the figure.
5. Put the screws along the horizontal strip and two more screws on the top of the board at the points C and D such that A, B, D and C become four vertices of a rectangle.
6. Take a ruler (scale) and make four holes on it as shown in the figure and fix four pulleys at these holes with the help of screws.
7. Fix the scale on the board using the thread tied to nails fixed at points A, B, C and D passing through the pulleys as shown in the figure, so that the scale slides parallel to the horizontal strip AB and can be moved up and down over the triangular piece freely.

Demonstration

1. Set the scale on vertical strips parallel to the base QR of D PQR, say at the points E and F. Measure the distances PE and EQ and also measure the distance PF and FR. It can be easily verified that

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

   This verifies Basic Proportionality Theorem (Thales theorem).

2. Repeat the activity as stated above, sliding the scale up and down parallel to the base of the triangle PQR and verify the Thales theorem for different positions of the scales.

OBSERVATION

By actual measurement:

PE = ___________, PF = ___________, EQ = ___________,

FR = ___________

$\frac{PE}{EQ}$ = _______, $\frac{PF}{FR}$ = ___________

Thus, $\frac{PE}{EQ} = \frac{PF}{FR}$. It verifies the Theorem.

APPLICATION

The theorem can be used to establish various criteria of similarity of triangles. It can also be used for constructing a polygon similar to a given polygon with a given scale factor.

Activity 24

OBJECTIVE

To verify that the lengths of tangents to a circle from some external point are equal.

MATERIAL REQUIRED

Glazed papers of different colours, geometry box, sketch pen, scissors, cutter and glue.

METHOD OF CONSTRUCTION

1. Draw a circle of any radius, say a units, with centre O on a coloured glazed paper of a convenient size [see Fig. 1].
2. Take any point P outside the circle.
3. Place a ruler touching the point P and the circle, lift the paper and fold it to create a crease passing through P [see Fig. 2].

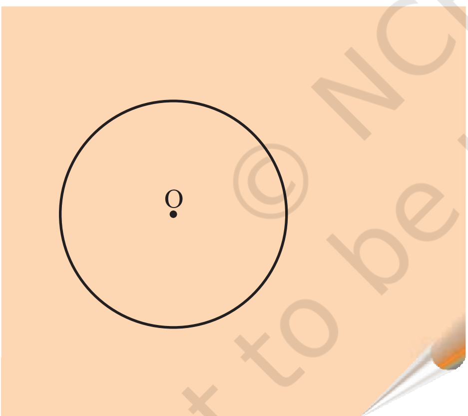

Fig. 1

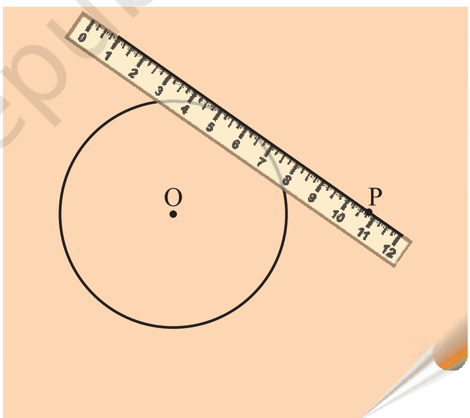

Fig. 2

4. Created crease is a tangent to the circle from the point P. Mark the point of contact of the tangent and the circle as Q. Join PQ [see Fig. 3].
5. Now place ruler touching the point P and the other side of the circle, and fold the paper to create a crease again [see Fig. 4].

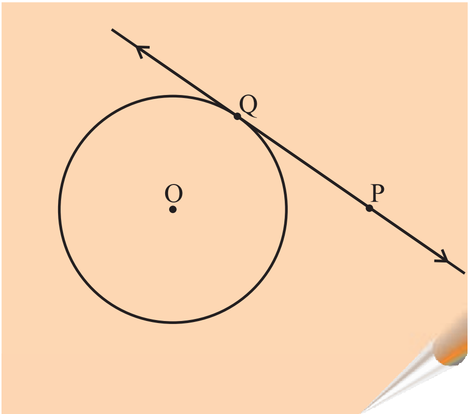

Fig. 3

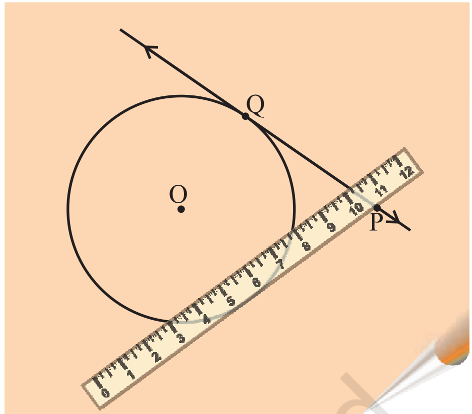

Fig. 4

6. This crease is the second tangent to the circle from the point P. Mark the point of contact of the tangent and the circle as R. Join PR [see Fig. 5].
7. Join the centre of the circle O to the point P [see Fig. 6].

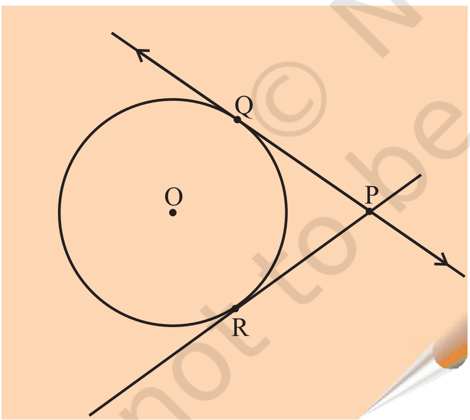

Fig. 5

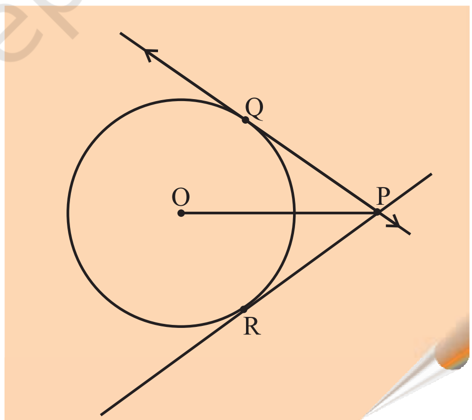

Fig. 6

DEMONSTRATION

1. Fold the circle along OP.

2. We observe that Q coincides with R. Therefore, QP = RP, i.e., length of the tangent QP = length of the tangent RP.

   This verifies the result.

OBSERVATION

On actual measurement:

1. Length of tangent QP = ...
2. Length of tangent RP = ...

   So, length of tangent QP = length of tangent ....

APPLICATION

This result is useful in solving problems in geometry and mensuration.