SETH M.R. JAIPURIA SCHOOLS BANARAS PARAO CAMPUS

SUBJECT – MATHEMATICS

CLASS – XII (WORKSHEET)

Chapter 1: Relations and Functions

1. Let A = {1, 2, 3}. The number of equivalence relations containing (1, 2) is:

(a) 1 (b) 2 (c) 3 (d) 4

2. Let f: R → R be defined by $f(x) = x^3 + 5$. Then f is:

(a) one-one but not onto (b) onto but not one-one

(c) both one-one and onto (d) neither one-one nor onto

3. If R is a relation on the set N of natural numbers defined by n R m if n divides m, then R is:

(a) Reflexive and symmetric (b) Transitive and symmetric

(c) Equivalence (d) Reflexive and transitive

4. The function f: [0, ∞) → R given by $f(x) = x^2 / (x^2 + 1)$ is:

(a) one-one and onto (b) one-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

5. Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. The relation R is:

(a) reflexive and symmetric (b) reflexive and transitive

(c) symmetric and transitive (d) an equivalence relation

Subjective Questions

6. Show that the relation R on the set Z of integers given by R = {(a, b): 5 divides (a - b)} is an equivalence relation. Find the equivalence class [2].

7. Let A = R - {2} and B = R - {1}. If f: A → B is a function defined by f(x) = (x - 1) / (x - 2), show that f is one-one and onto.

8. Show that the function f: N → N defined by f(x) = x + 1 if x is odd, and f(x) = x - 1 if x is even, is a bijective function.

9. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v) if and only if xv = yu. Show that R is an equivalence relation.

10. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b): |a - b| is a multiple of 2} is an equivalence relation.

11. Check whether the relation R on R defined as R = {(a, b): $a \leq b^3$} is reflexive, symmetric, or transitive.

12. Show that the function f: R → R defined by f(x) = x / (1 + |x|) is one-one and onto.

13. Let A = {-1, 0, 1, 2} and B = {-4, -2, 0, 2}. If f(x) = 2x, show that f is a bijection from A to B.

14. Show that the relation R in the set of all polygons, given by R = {(P_1, P_2): P_1 and P_2 have the same number of sides} is an equivalence relation.

15. Determine whether the function f: R → R given by $f(x) = x^2 + 2$ is one-one and onto. Justify your answer.

16. Let A = $\{x \in Z: 0 \le x \le 12\}$. Show that R = $\{(a, b): |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

17. Show that the signum function f: R → R, given by f(x) = 1 if $x > 0$, 0 if $x = 0$, and -1 if $x < 0$, is neither one-one nor onto.

18. Prove that the greatest integer function f: R → R, given by f(x) = [x], is neither one-one nor onto.

19. Let N be the set of natural numbers and R be the relation on N × N defined by (a, b) R (c, d) ⇔ ad(b + c) = bc(a + d). Check whether R is an equivalence relation.

20. Let f: W → W be defined as f(n) = n - 1, if n is odd and f(n) = n + 1, if n is even. Show that f is invertible and find its inverse. *(Note: Adapted for bijection proof only: Show f is a bijection).*

21. Let R be the relation on the set R of real numbers defined by a R b if 1 + ab > 0. Show that R is reflexive and symmetric but not transitive.

22. Show that the relation R in the set A = $\{x \in W: 0 \le x \le 15\}$ given by R = $\{(a, b): (a - b) \text{ is a multiple of } 5\}$ is an equivalence relation.

23. Check the injectivity and surjectivity of the function f: Z → Z given by $f(x) = x^3$.

24. Prove that the relation R on set of real numbers defined as R = $\{(a, b): a = b\}$ is an equivalence relation.

25. Given a non-empty set X, let P(X) be its power set. Let R be a relation on P(X) defined by A R B if A ⊂ B. Is R an equivalence relation? Justify.

Chapter 2: Inverse Trigonometric Functions

Multiple Choice Questions (MCQs)

26. The principal value of $\sin^{-1}(\sin(2\pi/3))$ is:

(a) $2\pi/3$ (b) $\pi/3$ (c) $4\pi/3$ (d) $5\pi/3$

27. The domain of the function $y = \sec^{-1}(2x)$ is:

(a) $[-1/2, 1/2]$ (b) $(-\infty, -1/2] \cup [1/2, \infty)$ (c) $R - \{-1/2, 1/2\}$ (d) $(-\infty, 1/2]$

28. The value of $\tan(\cos^{-1}(3/5) + \tan^{-1}(1/4))$ is:

(a) 19/8 (b) 8/19 (c) 19/12 (d) 3/4

29. The principal value of $\cot^{-1}(-\sqrt{3})$ is:

(a) $-\pi/6$ (b) $\pi/6$ (c) $5\pi/6$ (d) $2\pi/3$

30. If $\sin^{-1}x + \sin^{-1}y = \pi/2$, then $\cos^{-1}x + \cos^{-1}y$ is equal to:

(a) $\pi/2$ (b) π (c) 0 (d) 2π

Subjective Questions

31. Find the principal value of $\tan^{-1}(-1) + \cos^{-1}(-1/2) + \sin^{-1}(-1/2)$.

32. Find the domain of the function $f(x) = \sin^{-1}(\sqrt{(x - 1)})$.

33. Write the domain and range (principal value branch) of the function $f(x) = \text{cosec}^{-1}(x)$.

34. Find the value of $\cos^{-1} [\cos (13\pi / 6)] + \tan^{-1} [\tan (7\pi / 6)]$.

35. Find the value of $\sin [\pi/3 - \sin^{-1} (-1/2)]$.

36. Evaluate: $\tan^{-1} [2 \cos (2 \sin^{-1} (1/2))]$.

37. Find the value of $\tan(2 \tan^{-1}(1/5))$.

38. Write the value of $\cot (\tan^{-1} \alpha + \cot^{-1} \alpha)$.

39. Find the principal value of $\sec^{-1}(2/\sqrt{3}) + \tan^{-1}(\sqrt{3})$.

40. Evaluate: $\cos (\sec^{-1} x + \text{cosec}^{-1} x)$, $|x| \ge 1$.

41. Find the principal value of $\cos^{-1}(\cos(7\pi/6))$.

42. If $3 \sin^{-1}x = \pi$, find the value of x.

43. Find the domain of $f(x) = \cos^{-1}(x^2 - 4)$.

44. Find the value of $\sin^{-1}(\sin(3\pi/5))$.

45. Write the set of values of x for which $2 \cos^{-1}x = \cos^{-1}(2x^2 - 1)$ holds true.

46. Find the principal value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$.

47. Evaluate $\sin (1/2 \cos^{-1} (4/5))$.

48. What is the domain of $f(x) = \sin^{-1}x + \cos x$?

49. Find the exact value of $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$.

50. Find the maximum and minimum values of $f(x) = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$.

Chapter 3: Matrices

Multiple Choice Questions (MCQs)

51. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to:

(a) A (b) I - A (c) I (d) 3A

52. If A and B are symmetric matrices of the same order, then AB - BA is a:

(a) Skew-symmetric matrix (b) Symmetric matrix

(c) Zero matrix (d) Identity matrix

53. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:

(a) 27 (b) 18 (c) 81 (d) 512

54. If matrix A = [1, 2; 3, -1] and B = [1, 3; -1, 1], then AB is:

(a) [-1, 5; 4, 8] (b) [1, 5; 4, 8] (c) [-1, 5; 4, -8] (d) [1, -5; 4, 8]

55. If A is a symmetric matrix, then A^3 is:

(a) Symmetric (b) Skew-symmetric (c) Diagonal (d) Identity

Subjective Questions

56. Express the matrix A = [3, -2, -4; 3, -2, -5; -1, 1, 2] as the sum of a symmetric and a skew-symmetric matrix.

57. Find the matrix X so that X [1, 2, 3; 4, 5, 6] = [-7, -8, -9; 2, 4, 6].

58. Let A = [2, 3; -1, 2] and $f(x) = x^2 - 4x + 7$. Show that f(A) = O. Use this result to find A^3.

59. If A = [cos θ, sin θ; -sin θ, cos θ], prove that A^n = [cos nθ, sin nθ; -sin nθ, cos nθ] for all positive integers n.

60. Find the values of x, y, and z if the matrix A = [0, 2y, z; x, y, -z; x, -y, z] satisfies the equation A'A = I.

61. If A = [2, -1; 3, 2] and B = [0, 4; -1, 7], find $3A^2 - 2B + I$.

62. Show that the elements on the main diagonal of a skew-symmetric matrix are all zero.

63. If A and B are symmetric matrices, prove that AB is symmetric if and only if AB = BA.

64. Find the matrix A such that [2, -1; 1, 0; -3, 4] A = [-1, -8, -10; 1, -2, -5; 9, 22, 15].

65. If A = [1, -1; -1, 1], find A^2 and hence deduce A^n.

66. Construct a 3 × 4 matrix whose elements are given by $a_{ij} = (1/2) \mid -3i + j \mid$.

67. Let A = [1, 2; 3, 4]. Find a matrix B such that AB = BA.

68. If A = [α, β; γ, -α] is such that $A^2 = I$, then prove that $1 - \alpha^2 - \beta\gamma = 0$.

69. Prove that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix.

70. Find x, y if 2 [1, 3; 0, x] + [y, 0; 1, 2] = [5, 6; 1, 8].

71. If A is a skew-symmetric matrix of odd order, prove that its determinant is zero. *(Applicable using matrix transpose properties)*.

72. Let A = [0, -tan(α/2); tan(α/2), 0] and I be the identity matrix of order 2. Show that I + A = (I - A) [cos α, -sin α; sin α, cos α].

73. If A, B are square matrices of the same order and A is symmetric, show that B'AB is also symmetric.

74. Find x if [x, -5, -1] [1, 0, 2; 0, 2, 1; 2, 0, 3] [x; 4; 1] = O.

75. Determine the matrix X such that 2A + B + X = O, where A = [-1, 2; 3, 4] and B = [3, -2; 1, 5].

Chapter 4: Determinants

Multiple Choice Questions (MCQs)

76. If A is a square matrix of order 3 and |A| = 4, then |adj A| is:

(a) 4 (b) 16 (c) 64 (d) 12

77. Area of a triangle with vertices (k, 0), (4, 0), (0, 2) is 4 square units. The value of k is:

(a) 0 or 8 (b) 0 or -8 (c) 8 or -8 (d) 4 or -4

78. If A is an invertible matrix of order 2, then det(A^{-1}) is equal to:

(a) det(A) (b) 1 / det(A) (c) 1 (d) 0

79. Let A be a non-singular matrix of order 3 × 3. Then |3A| is equal to:

(a) 3|A| (b) 9|A| (c) 27|A| (d) |A|

80. For any 2 × 2 matrix A, if A(adj A) = [10, 0; 0, 10], then |A| is:

(a) 100 (b) 10 (c) 1 (d) 0

Subjective Questions

81. Find the inverse of the matrix A = [1, -1, 2; 0, 2, -3; 3, -2, 4] and hence solve the system: x - y + 2z = 1; 2y - 3z = 1; 3x - 2y + 4z = 2.

82. If A = [1, 2, -3; 2, 3, 2; 3, -3, -4], find A^{-1}. Using A^{-1}, solve the system of equations: x + 2y - 3z = -4; 2x + 3y + 2z = 2; 3x - 3y - 4z = 11.

83. Use the product of matrices [1, -1, 2; 0, 2, -3; 3, -2, 4] and [-2, 0, 1; 9, 2, -3; 6, 1, -2] to solve the system of equations: x - y + 2z = 1; 2y - 3z = 1; 3x - 2y + 4z = 2.

84. Given A = [2, 2, -4; -4, 2, -4; 2, -1, 5] and B = [1, -1, 0; 2, 3, 4; 0, 1, 2], find BA and use this to solve the system of equations: y + 2z = 7; x - y = 3; 2x + 3y + 4z = 17.

85. If A = [3, 1; -1, 2], show that A^2 - 5A + 7I = O. Hence find A^{-1}.

86. Find the equation of the line joining A(1, 3) and B(0, 0) using determinants and find k if D(k, 0) is a point such that the area of triangle ABD is 3 sq units.

87. Find the matrix P such that P = A(adj A) where A = [1, 2; 3, 4]. Find |P|.

88. The sum of three numbers is 6. If we multiply the third number by 3 and add the second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

89. Let A = [1, sin α, 1; -sin α, 1, sin α; -1, -sin α, 1], where 0 ≤ α ≤ 2π. Evaluate det(A) and find the interval in which its value lies.

90. If the points (2, -3), (λ, -1) and (0, 4) are collinear, find the value of λ using determinants.

91. Solve the following system of equations by matrix method: $2/x + 3/y + 10/z = 4$; $4/x - 6/y + 5/z = 1$; $6/x + 9/y - 20/z = 2$.

92. If A = [2, 3; 1, -4] and B = [1, -2; -1, 3], verify that $(AB)^{-1} = B^{-1}A^{-1}$.

93. Find the adjoint of the matrix A = [1, 2, 3; 0, 5, 0; 2, 4, 3] and verify A(adj A) = |A|I.

94. If A = [cos θ, sin θ; -sin θ, cos θ], show that A is orthogonal (i.e., $A^{-1} = A'$).

95. Find the value of x if det([x, 2; 18, x]) = det([6, 2; 18, 6]).

96. Find the inverse of A = [2, 1, 3; 4, -1, 0; -7, 2, 1].

97. Using determinants, find the area of the triangle whose vertices are (-2, -3), (3, 2), and (-1, -8).

98. Determine whether the system of equations is consistent: $3x - y - 2z = 2$, $2y - z = -1$, $3x - 5y = 3$.

99. If A = [3, 2; 7, 5] and B = [6, 7; 8, 9], verify that adj(AB) = (adj B)(adj A).

100. Write the cofactor of a_{12} in the matrix [2, -3, 5; 6, 0, 4; 1, 5, -7].

Chapter 5: Continuity and Differentiability

Multiple Choice Questions (MCQs)

101. The function f(x) = |x - 1| is:

(a) Continuous and differentiable at x=1

(b) Continuous but not differentiable at x=1

(c) Differentiable but not continuous at x=1

(d) Neither continuous nor differentiable at x=1

102. If y = 2^(x), then dy/dx is:

(a) x 2^(x-1)

(b) 2^(x) / log 2

(c) 2^(x) log 2

(d) 2^(x)

103. If $x = t^2$ and $y = t^3$, then d^2y/dx^2 is:

(a) 3/2

(b) 3/(4t)

(c) 3/(2t)

(d) 3t/2

104. The value of c in Rolle's Theorem for $f(x) = x^2$ in [-1, 1] is: *(Note: Excluded in rationalised syllabus, but conceptually limits to:)* Derivative of $f(x) = x^2$ at x=0 is:

(a) 1

(b) -1

(c) 0

(d) 2

105. Derivative of $\sin^{-1}(2x / (1 + x^2))$ w.r.t x is:

(a) $2 / (1 + x^2)$

(b) $-2 / (1 + x^2)$

(c) $1 / (1 + x^2)$

(d) $-1 / (1 + x^2)$

Subjective Questions

106. Find the values of a and b such that the function f(x) defined by: f(x) = 5 for $x \le 2$; f(x) = ax + b for $2 < x < 10$; and f(x) = 21 for $x \ge 10$, is a continuous function.

107. Find the relationship between a and b so that f(x) = ax + 1 if $x \le 3$, and f(x) = bx + 3 if $x > 3$, is continuous at x = 3.

108. Differentiate $x^{(x \cos x)} + (x^2 + 1)/(x^2 - 1)$ with respect to x.

109. If $y = (\sin x)^x + \sin^{-1}(\sqrt{x})$, find dy/dx.

110. If x = a(θ - sin θ) and y = a(1 + cos θ), find d^2y/dx^2 at θ = π/2.

111. If $x\sqrt{(1 + y)} + y\sqrt{(1 + x)} = 0$, for $-1 < x < 1$ and $x \neq y$, prove that $dy/dx = -1 / (1 + x)^2$.

112. If cos y = x cos(a + y), with cos a ≠ ±1, prove that $dy/dx = [\cos^2(a + y)] / \sin a$.

113. If y = 3 cos(log x) + 4 sin(log x), show that $x^2y'' + xy' + y = 0$.

114. If $e^y(x + 1) = 1$, show that $d^2y/dx^2 = (dy/dx)^2$.

115. Differentiate $\tan^{-1} [(3x - x^3) / (1 - 3x^2)]$ with respect to x.

116. Discuss the continuity of f(x) = |x| / x if $x \neq 0$, and f(x) = 0 if x = 0.

117. Find dy/dx if $y^x + x^y + x^x = a^b$.

118. Differentiate $\sin^2 x$ with respect to $e^{(\cos x)}$.

119. If $y = \log [x + \sqrt{(x^2 + a^2)}]$, show that $(x^2 + a^2) y'' + x y' = 0$.

120. Determine all points of discontinuity for $f(x) = [x]$ (greatest integer function) in the interval (-3, 3).

121. Find dy/dx when $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$.

122. If $y = [\log(x + \sqrt{(x^2 + 1)})]^2$, show that $(x^2 + 1)(d^2y / dx^2) + x(dy / dx) - 2 = 0$.

123. Show that the function $f(x) = |x - 3|$ is continuous but not differentiable at $x = 3$.

124. Differentiate $f(x) = (x + 1/x)^x + x^{(1 + 1/x)}$ with respect to x.

125. If $x^y = e^{(x - y)}$, prove that $dy/dx = \log x / (1 + \log x)^2$.

Chapter 6: Applications of Derivatives

Multiple Choice Questions (MCQs)

126. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is:

(a) 10π (b) 12π (c) 8π (d) 11π

127. The function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing in:

(a) (2, 3) (b) $(-\infty, 2) \cup (3, \infty)$ (c) (1, 5) (d) [-2, 3]

128. Maximum value of $f(x) = \sin x + \cos x$ is:

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) $1/\sqrt{2}$

129. The interval in which $y = x^2 e^{-x}$ is increasing is:

(a) $(-\infty, \infty)$ (b) (-2, 0) (c) $(2, \infty)$ (d) (0, 2)

130. Point on the curve $x^2 = 2y$ which is nearest to the point (0, 5) is:

(a) $(2\sqrt{2}, 4)$ (b) $(2\sqrt{2}, 0)$ (c) (0, 0) (d) (2, 2)

Subjective Questions

131. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 m^3/hr. Find the rate at which the level of the water is rising when the depth of water is 4 m.

132. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

133. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}(\sqrt{2})$.

134. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval [1, 5].

135. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.

136. Find the intervals in which the function $f(x) = (3/10)x^4 - (4/5)x^3 - 3x^2 + (36/5)x + 11$ is strictly increasing or strictly decreasing.

137. An open topped box is to be constructed by removing equal squares from each corner of a 3 meter by 8 meter rectangular sheet of aluminum and folding up the sides. Find the volume of the largest such box.

138. Show that the right circular cylinder of given surface area and maximum volume is such that its height is equal to the diameter of the base.

139. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

140. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2R/\sqrt{3}$.

141. Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is strictly increasing or decreasing.

142. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

143. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

144. Find the local maxima and local minima, if any, of the function $f(x) = \sin x + \cos x$, $0 < x < \pi/2$.

145. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

146. Find the points of local maxima and local minima of the function $f(x) = x^3 - 6x^2 + 9x + 15$.

147. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

148. The total cost C(x) in Rupees associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$. Find the marginal cost when 17 units are produced.

149. Find the intervals in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing and strictly decreasing.

150. An open box with a square base is to be made out of a given quantity of cardboard of area c^2. Show that the maximum volume of the box is $c^3 / (6\sqrt{3})$.