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CLASS 10 MATHEMATICS: MASTER FORMULA & CONCEPT HANDBOOK

Complete Chapter-Wise Definitions, Theorems, Formulas & Board Exam Alerts Official Resource by **mathgiri.com**

Unit	Unit Name	Included NCERT Chapters	Weightage
Unit I	Number Systems	Ch 1: Real Numbers	**06 Marks**
Unit II	Algebra	Ch 2: Polynomials, Ch 3: Linear Eq., Ch 4: Quadratic Eq., Ch 5: AP	**20 Marks**
Unit III	Coordinate Geometry	Ch 7: Coordinate Geometry	**06 Marks**
Unit IV	Geometry	Ch 6: Triangles, Ch 10: Circles	**15 Marks**
Unit V	Trigonometry	Ch 8: Intro to Trigonometry, Ch 9: Applications of Trigonometry	**12 Marks**
Unit VI	Mensuration	Ch 11: Areas Related to Circles, Ch 12: Surface Areas & Volumes	**10 Marks**
Unit VII	Statistics & Probability	Ch 13: Statistics, Ch 14: Probability	**11 Marks**

Chapter 1: Real Numbers

Unit I • 6 Marks

Theorem 1.1 (Fundamental Theorem of Arithmetic) & Theorem 1.2 (Irrationality Root Theorem)

1.1 Fundamental Theorem of Arithmetic: Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur. Example: $140 = 2^2 \times 5 \times 7$.

1.2 Prime Divisibility Theorem: Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer. *(Core foundation for proving $\sqrt{2}, \sqrt{3}, \sqrt{5}$ are irrational by contradiction).*

Key Definitions & Prime Factorisation Rules:

- **HCF (Highest Common Factor):** Product of the *smallest power* of each *common* prime factor in the numbers.
- **LCM (Lowest Common Multiple):** Product of the *greatest power* of each prime factor involved in the numbers.
- **Fundamental Relation (For Two Positive Integers a and b):** $HCF(a, b) \times LCM(a, b) = a \times b$.
- **Composite Number Proof Rule:** To prove an expression like $7 \times 11 \times 13 + 13$ is composite, factor out the common term $13 \times (7 \times 11 + 1) = 13 \times 78$ and show it has more than two factors (1, itself, and other prime factors).
- **Rational & Irrational Operations:** The sum, difference, product, and quotient of a non-zero rational number and an irrational number is *always irrational* (e.g., $3 + 2\sqrt{5}$).

15-YEAR EXPERT BOARD EXAM ALERT (CHAPTER 1)

Common Trap: Never apply $HCF \times LCM = a \times b \times c$ for **three** integers! That formula strictly holds only for **two** positive integers. Furthermore, in word problems: if the question asks for "minimum time to meet again" or "smallest common measure", compute **LCM**; if it asks for "maximum capacity of container" or "equal-sized largest rows", compute **HCF**.

Chapter 2: Polynomials

Unit II • Algebra

Standard Forms & Degree: The highest power of x in $p(x)$ is called its **degree**. Linear: $ax + b$ (deg 1, at most 1 zero); Quadratic: $ax^2 + bx + c$, $a \neq 0$ (deg 2, at most 2 zeroes); Cubic: $ax^3 + bx^2 + cx + d$ (deg 3, at most 3 zeroes). Geometrically, the zeroes of $p(x)$ are the **x-coordinates** of the points where the graph $y = p(x)$ intersects or touches the x-axis. For quadratic $y = ax^2 + bx + c$, the curve is a **parabola** opening upwards if $a > 0$ and downwards if $a < 0$.

- **Relationship Between Zeroes (α, β) & Coefficients of $ax^2 + bx + c$:** Sum of zeroes $\alpha + \beta = -b / a = -(\text{Coeff. of } x) / (\text{Coeff. of } x^2)$; Product of zeroes $\alpha \cdot \beta = c / a = \text{Constant term} / (\text{Coeff. of } x^2)$.
- **Formation of a Quadratic Polynomial:** Given zeroes α and β, $p(x) = k\,[x^2 - (\alpha + \beta)x + \alpha\beta]$, where k is any non-zero real constant.
- **Essential Symmetric Identities (High-Frequency Board Questions):** $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ | $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ | $1/\alpha + 1/\beta = (\alpha + \beta) / \alpha\beta$ | $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$.

Chapter 3: Pair of Linear Equations in Two Variables

Unit II • Algebra

Standard Form: $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. Comparing coefficient ratios determines the nature of lines:

Ratio Condition	Graphical Representation	Algebraic Interpretation	System Consistency
$a_1/a_2 \neq b_1/b_2$	Intersecting Lines (at 1 point)	Unique Solution (Exactly 1)	**Consistent** (Independent)
$a_1/a_2 = b_1/b_2 = c_1/c_2$	Coincident Lines (Overlapping)	Infinitely Many Solutions	**Consistent** (Dependent)
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	Parallel Lines (Never meet)	No Solution	**Inconsistent**

Algebraic Methods: (1) **Substitution Method:** Express one variable in terms of the other from one equation and substitute into the second. (2) **Elimination Method:** Multiply equations by suitable constants to make coefficients of one variable numerically equal, then add/subtract. **Expert Tip:** When coefficients of x and y are interchanged (e.g., $47x + 31y = 63$ and $31x + 47y = 15$), first *add* both equations to get $x + y$, then *subtract* them to get $x - y$.

Chapter 4: Quadratic Equations

Unit II • Algebra

Quadratic Formula (Sridharacharya's Formula) & Discriminant (D)

For standard quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$), the Discriminant is $D = b^2 - 4ac$, and the roots are given by:

$$x = [\, -b \pm \sqrt{(b^2 - 4ac)}\,] / (2a) = (\, -b \pm \sqrt{D}\,) / (2a)$$

- **Case 1 ($D > 0$):** Two **distinct real roots** exist ($\alpha \neq \beta$).
- **Case 2 ($D = 0$):** Two **equal real roots** (coincident roots) exist, each equal to $x = -b / (2a)$.
- **Case 3 ($D < 0$): No real roots** exist (roots are imaginary/complex).

Speed & Stream Word Problem Rule: If speed of boat in still water is u km/h and speed of stream is v km/h ($u > v$), then **Upstream Speed** = $(u - v)$ km/h and **Downstream Speed** = $(u + v)$ km/h. Time = $\text{Distance} / \text{Speed}$.

Chapter 5: Arithmetic Progressions (AP)

Unit II • Algebra

1. General Term (n^{th} Term) of an AP

Sequence with first term a and common difference d = $a_{k+1} - a_k$:

$a_n = a + (n-1)d$

n^{th} term from the end (where l is last term): $l - (n-1)d$.

Arithmetic Mean: If a, b, c are in AP, then $2b = a + c$.

2. Sum of First n Terms (S_n)

$S_n = (n/2)[2a + (n-1)d]$ or $S_n = (n/2)[a + l]$

Finding a_n from S_n: $a_n = S_n - S_{n-1}$ (for $n \geq 2$), and $a_1 = S_1$.

Term Selection: 3 terms in AP: $a - d, a, a + d$; 4 terms: $a - 3d, a - d, a + d, a + 3d$.

Chapter 7: Coordinate Geometry

Unit III • 6 Marks

1. Distance Formula & Collinearity

Between $P(x_1, y_1)$ and $Q(x_2, y_2)$:

$PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

From Origin O(0,0): $OP = \sqrt{(x^2 + y^2)}$.

Equidistant on Axes: Point on x-axis is $(x, 0)$; on y-axis is $(0, y)$.

2. Section, Mid-Point & Centroid Formulae

Internal division of AB in ratio $m_1 : m_2$:

$P(x, y) = ((m_1x_2 + m_2x_1)/(m_1 + m_2), (m_1y_2 + m_2y_1)/(m_1 + m_2))$

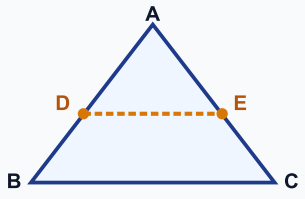
Mid-Point: $((x_1 + x_2)/2, (y_1 + y_2)/2)$.

Centroid of ΔABC: $G = ((x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3)$.

Quadrilateral Proofs: To prove vertices A, B, C, D form a **Parallelogram**, show diagonals share the same mid-point (**Mid-point of AC = Mid-point of BD**). For a **Rectangle**, show parallelogram + equal diagonals ($AC = BD$). For a **Rhombus**, show all four sides equal ($AB = BC = CD = DA$).

Chapter 6: Triangles

Unit IV • Geometry

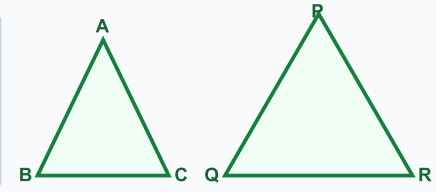


Thales Theorem (BPT):

If DE ∥ BC in ΔABC, then:

AD / DB = AE / EC

Corollaries: AD/AB = AE/AC = DE/BC



ΔABC ~ ΔPQR

∠A=∠P, ∠B=∠Q

AB/PQ = BC/QR = AC/PR

Theorem 6.1 (Basic Proportionality Theorem / Thales Theorem) & Similarity Criteria

Theorem 6.1 (BPT): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio ($AD/DB = AE/EC$). **Theorem 6.2 (Converse of BPT):** If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Criteria for Similarity of Triangles: (1) **AAA / AA Similarity:** If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. (2) **SSS Similarity:** Three sides proportional ($AB/PQ = BC/QR = CA/RP$). (3) **SAS Similarity:** One angle equal and the two sides including these angles are proportional ($\angle A = \angle P$ and $AB/PQ = AC/PR$). Note: Ratio of perimeters of similar triangles equals the ratio of their corresponding sides.

Chapter 10: Circles

Unit IV • Geometry

Mandatory Circle Theorems (Guaranteed Board Proof Questions)

Theorem 10.1 (Radius-Tangent Perpendicularity): The tangent at any point of a circle is perpendicular to the radius through the point of contact ($\mathbf{OP \perp XY}$).

Theorem 10.2 (Equal Tangents from External Point): The lengths of tangents drawn from an external point to a circle are equal ($\mathbf{PA = PB}$). Furthermore, for centre $\mathbf{O}$: (i) $\mathbf{PO}$ bisects $\mathbf{\angle APB}$ and $\mathbf{\angle AOB}$, (ii) $\mathbf{\angle APB + \angle AOB = 180^\circ}$ (supplementary).

Important Standard Properties: (1) A parallelogram circumscribing a circle is always a **rhombus**. (2) Opposite sides of a quadrilateral circumscribing a circle subtend **supplementary angles** ($\mathbf{180^\circ}$) at the centre of the circle.

Chapter 8: Introduction to Trigonometry

Unit V • 12 Marks

1. Trigonometric Ratios in Right ΔABC (Right-angled at B, for acute angle θ):
$\mathbf{\sin\theta = P/H}$ | $\mathbf{\cos\theta = B/H}$ | $\mathbf{\tan\theta = P/B = \sin\theta/\cos\theta}$ | $\mathbf{\operatorname{cosec}\theta = H/P = 1/\sin\theta}$ | $\mathbf{\sec\theta = H/B = 1/\cos\theta}$ | $\mathbf{\cot\theta = B/P = \cos\theta/\sin\theta}$.

2. Three Pythagorean Trigonometric Identities & Conjugate Forms:

- **Identity I:** $\mathbf{\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta}$ and $\mathbf{\cos^2\theta = 1 - \sin^2\theta}$.
- **Identity II:** $\mathbf{1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1 \Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1}$ (for $\mathbf{0^\circ \le \theta < 90^\circ}$).
- **Identity III:** $\mathbf{1 + \cot^2\theta = \operatorname{cosec}^2\theta \Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow (\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1}$ (for $\mathbf{0^\circ < \theta \le 90^\circ}$).

Ratio \ Angle (θ)	0°	30°	45°	60°	90°
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not Defined (∞)
$\operatorname{cosec}\theta$	Not Defined (∞)	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Not Defined (∞)
$\cot\theta$	Not Defined (∞)	$\sqrt{3}$	1	$1/\sqrt{3}$	0

Chapter 9: Some Applications of Trigonometry (Heights & Distances)

Unit V • Trigonometry

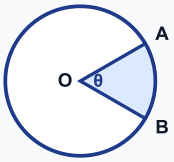
KEY DEFINITIONS & 15-YEAR EXPERT STRATEGY FOR HEIGHTS & DISTANCES

Line of Sight: Line drawn from the eye of an observer to the point in the object viewed. **Angle of Elevation:** Angle between horizontal level and line of sight when object is *above* horizontal. **Angle of Depression:** Angle between horizontal level and line of sight when object is *below* horizontal (always draw a horizontal dotted line at the top observer and mark alternate interior angles equal!).

Golden Ratio Shortcuts for Verification: In a $\mathbf{30^\circ\text{–}60^\circ\text{–}90^\circ}$ triangle, sides opposite to $\mathbf{30^\circ, 60^\circ, 90^\circ}$ are in the ratio $\mathbf{1 : \sqrt{3} : 2}$. In a $\mathbf{45^\circ\text{–}45^\circ\text{–}90^\circ}$ triangle, the perpendicular equals the base ($\mathbf{P = B}$) because $\mathbf{\tan 45^\circ = 1}$. Use $\mathbf{\sqrt{3} = 1.732}$ only when specified.

Chapter 11: Areas Related to Circles

Unit VI • Mensuration

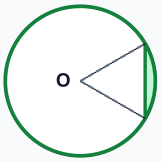


1. Area of Sector OAPB:

A = (θ / 360°) × πr²

Arc Length (l) = (θ / 360°) × 2πr

Relation: Area = (1/2) × l × r



2. Area of Minor Segment:

= Area(Sector OAPB) − Area(ΔOAB)

Where Area(ΔOAB) = (1/2) r² sin θ

(Or √3/4 r² when θ = 60°; 1/2 r² when θ = 90°)

- **Major Sector & Major Segment:** **Area of Major Sector = πr^2 − Area of Minor Sector = $[(360^\circ - \theta)/360^\circ] \times \pi r^2$**. Similarly, **Area of Major Segment = πr^2 − Area of Minor Segment**.
- **Perimeter of a Sector:** **Perimeter = $l + 2r = (\theta / 360^\circ) \times 2\pi r + 2r$**. *(Do not forget to add the two radii $2r$!)*
- **Clock Hand Angles:** Angle swept by the **Minute Hand** in 60 mins = **$360^\circ \Rightarrow 6^\circ$ per minute**. Angle swept by the **Hour Hand** in 12 hours = **$360^\circ \Rightarrow 30^\circ$ per hour** or **0.5° per minute**.
- **Wheel Revolution Formula:** **Distance Covered in 1 Revolution = Circumference = $2\pi r$**; **Number of Revolutions = Total Distance / $(2\pi r)$**.

Chapter 12: Surface Areas and Volumes

Unit VI • Mensuration

3D Solid Shape	Curved / Lateral Surface Area (CSA/LSA)	Total Surface Area (TSA)	Volume (V) & Diagonal / Slant
Cuboid (l, b, h)	$2h(l + b)$ *(Area of 4 walls)*	$2(lb + bh + hl)$	$V = l \times b \times h$; Diag = $\sqrt{(l^2 + b^2 + h^2)}$
Cube (edge a)	$4a^2$	$6a^2$	$V = a^3$; Diagonal = $\sqrt{3}\,a$
Right Circular Cylinder	$2\pi rh$	$2\pi r(h + r)$	$V = \pi r^2 h$
Right Circular Cone	πrl	$\pi r(l + r)$	$V = (1/3)\pi r^2 h$; $l = \sqrt{(r^2 + h^2)}$
Sphere (radius r)	$4\pi r^2$	$4\pi r^2$	$V = (4/3)\pi r^3$
Solid Hemisphere	$2\pi r^2$	$3\pi r^2$ *(CSA + base πr^2)*	$V = (2/3)\pi r^3$

15-YEAR EXPERT RULE FOR COMBINATION OF SOLIDS (CHAPTER 12)

Crucial Distinction: When two solids are joined (or a cavity is scooped out), the **Total Volume** is simply the sum (or difference) of individual volumes. However, the **Total Surface Area (TSA) of the resulting solid is NEVER the sum of individual TSAs!** Always visualize what you can touch/paint from the outside: e.g., for a wooden block with a hemispherical depression scooped out from one face, **TSA = $6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$** (you *add* the inner curved surface area $2\pi r^2$ because it is newly exposed!). Also, take π and r common first before multiplying decimals.

Chapter 13: Statistics

Unit VII • 11 Marks

Preliminary Definitions for Grouped Data: Class Mark (Mid-point) x_i = (Upper Class Limit + Lower Class Limit) / 2; Class Size h = Upper Limit − Lower Limit (ensure classes are continuous before computing Median or Mode!).

Measure & Method	Master Formula	Symbol Meaning & Selection Rule
1. Mean ($\bar{x}$) *Direct / Assumed / Step-Deviation*	Direct: $\bar{x} = (\sum f_i x_i) / (\sum f_i)$ Assumed: $\bar{x} = a + (\sum f_i d_i) / (\sum f_i)$ Step-Dev: $\bar{x} = a + h \times [(\sum f_i u_i) / (\sum f_i)]$	a = Assumed mean (middle x_i); $d_i = x_i - a$; $u_i = (x_i - a) / h$. Use Step-Deviation when x_i values are large with equal class width h.
2. Mode *(Grouped Data)*	$\text{Mode} = l + [(f_1 - f_0) / (2f_1 - f_0 - f_2)] \times h$	**Modal Class:** Class with highest frequency. l = lower limit of modal class; f_1 = freq. of modal class; f_0 = preceding freq.; f_2 = succeeding freq.
3. Median *(Grouped Data)*	$\text{Median} = l + [(n/2 - cf) / f] \times h$	**Median Class:** Class whose cumulative freq. ≥ $n/2$. l = lower limit of median class; f = freq. of median class; cf = cumulative freq. of **preceding** class.

Empirical Relationship Between Mean, Median, and Mode (1-Mark & 2-Mark Favourite)

$$3 \text{ Median} = \text{Mode} + 2 \text{ Mean} \quad \Leftrightarrow \quad \text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

Chapter 14: Probability

Unit VII • Statistics & Probability

Theoretical Probability of an Event E: P(E) = (Number of outcomes favourable to E) / (Total number of all possible outcomes) = n(E) / n(S). Range: $0 \le P(E) \le 1$. **Sure Event: P(E) = 1**; **Impossible Event: P(E) = 0**. **Complementary Event (not E or $\bar{E}$): $P(E) + P(\bar{E}) = 1 \Rightarrow P(\bar{E}) = 1 - P(E)$.**

Standard Deck of 52 Playing Cards

- **Total Cards: 52 — 26 Red** (13 Hearts ♥ + 13 Diamonds ♦) and **26 Black** (13 Spades ♠ + 13 Clubs ♣).
- **4 Suits (13 each):** Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack (J), Queen (Q), King (K).
- **Face Cards (12 Total):** 4 Kings + 4 Queens + 4 Jacks (6 Red Face cards, 6 Black Face cards). *Note: Ace is NOT a face card!*
- **Honour Cards (16 Total):** 12 Face Cards + 4 Aces.

Coins, Dice & Leap Year Calendar

- **Coins:** 1 Coin (2: **H, T**); 2 Coins (4: **HH, HT, TH, TT**); 3 Coins (8 outcomes: $2^3 = 8$).
- **Dice:** 1 Die (6 outcomes: 1 to 6); **2 Dice Thrown Simultaneously (36 outcomes: $6^2 = 36$)**; Doublets = 6 **(1,1)** to **(6,6)**.
- **Non-Leap Year (365 days):** 52 full weeks + **1 odd day → P(53 Sundays) = 1/7**.
- **Leap Year (366 days):** 52 full weeks + **2 odd days → P(53 Sundays) = 2/7**.

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